MA106-Ordinary Differential Equations

SP Semester 2020

Module-I

1. Find the differential equation of the family of curves $y = e^{mx}$, where m is an arbitrary constant. Ans: $y' = y \log_e y$.

2. Obtain the differential equation satisfied by the family of circles $x^2 + y^2 = a^2$, where a is an arbitrary constant. Ans: $x + yy' = 0$.

3. Solve $\frac{dy}{dx} = e^{x+y} + x^2 e^y$. Ans: $-e^{-y} = e^x + x^3/3 + c$.

4. Solve $3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$. Ans: $-3 \log(1 - e^x) + \log(\tan y) = \log c$.

5. Solve $\frac{dy}{dx} = \sec(x + y)$ Ans: $y - \tan \frac{1}{2}(x + y) = c$.

6. Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$. Ans: $cx = \sin\left(\frac{y}{x}\right)$.

7. Solve $\frac{dy}{dx} = \frac{(x+y+4)}{(x-y-6)}$ Ans: $(x-1)^2 + (y+5)^2 = ce^{2\tan^{-1}((y+5)/(x-1))}$.

8. Solve $x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1, \quad 0 < x < \pi/2$ Ans: $\tan x + c$.

9. Solve $(1 + x^2)\frac{dy}{dx} = x(1 - y)$ Ans: $y = 1 + c(1 + x^2)^{-1/2}$.

10. Solve $\frac{dy}{dx} \cos y + 2x \sin y = x$ Ans: $y = \sin^{-1}\left(\frac{1}{2} + ce^{-x^2}\right)$.

11. Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ Ans: $ve^{x^2} = \frac{1}{2}(x^2 - 1)e^{x^2} + c$.

12. Solve $x\frac{dy}{dx} + y = y^2 \ln x$ Ans: $y^{-1} = 1 + \ln x + cx$.

13. Solve $y' + 4xy + xy^3 = 0$ Ans: $y = \left(-\frac{1}{4} + ce^{4x^2}\right)^{-1/2}$.