Department of Mathematics

Birla Institute of Technology Mesra, Ranchi

Branch-All | MA103 Mathematics-I | Session:MO/2023

Assignment-2 (Module II)

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1. Evaluate the rank of the following matrices

   a) $\begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -1.5 \end{bmatrix}$

   b) $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$

   c) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

   d) $\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 7 & 2 \\ 8 & 1 \end{bmatrix}$

   e) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$

   f) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

2. Find the value of k such that rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2

3. Using rank method, find whether the following equations are consistent or not, $x + y + 2z = 4,\ 2x - y + 3z = 9,\ 3x - y - z = 2$. If consistent, solve them.

4. Find the values of a and b for which the system $x+2y+3z = 6,\ x+3y+5z = 9,\ 2x+5y+az = b$ has (i) no solution (ii) unique solution (iii) infinite number of solutions. Also, find the solutions in case (i) and (ii).

5. Find the value of λ, for which the system $3x - y + 4z = 3,\ x + 2y - 3z = -2,\ 6x + 5y + \lambda z = -3$ will have infinite number of solutions and solve them with that λ value.

6. Determine k such that system $2x + y + 2z = 0,\ x + y + 3z = 0,\ 4x + 3y + kz = 0$ has (i) trivial solution (ii) non-trivial solution.

7. Check whether the following equations will have a non-trivial solution or not:

   $$4x + 2y + z + 3w = 0,\ 6x + 3y + 4z + 7w = 0,\ 2x + y + w = 0$$

   If non-trivial solution exists, find the solution.

8. Using Row-Echelon form technique, solve the the following system of equations

a) $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \\ 13 \end{pmatrix}$.

b) $\begin{pmatrix} 2 & 2 & 1 \\ 4 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

c) $\begin{pmatrix} 10 & -1 & 2 \\ 1 & 10 & -1 \\ 2 & 3 & 20 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix}$

9. Examine the following vectors for linear dependence: $(1, 0, 3, 1)$, $(0, 1, -6, -1)$ and $(0, 2, 1, 0)$ in $\mathbb{R}^4$

10. Show that the given system of vectors: $(2, 2, 1)$, $(1, 3, 1)$, $(1, 2, 2)$ are linearly independent.

11. Find the value of λ for which the vectors $(-1, -2, \lambda)$, $(2, -1, 5)$ and $(3, -5, 7\lambda)$ are linearly dependent. Find the relation between the vectors.

12. Using matrix, show that the set of vectors $(1, 2, -3, 4)$, $(3, -1, 2, 1)$ and $(1, -5, 8, -7)$are linearly dependent. Find the relation between the vectors.

13. Find the eigenvalues and eigenvectors of the matrix

a) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

e) $\begin{pmatrix} k & k & k \\ k & k & k \\ k & k & k \end{pmatrix}$, for fixed real k.

f) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

g) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

h) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

14. Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Also, find the inverse using this theorem.

15. Using Cayley Hamilton theorem find A^{-2}, where $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

16. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$. Verify Cayley Hamilton theorem. Also, find A^{-1} and A^4

17. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ then, using Cayley Hamilton theorem, express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in A.

18. Write the characteristic equation of the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. Verify Cayley Hamilton theorem. Hence express $A^5 - 3A^4 + A^2 - 4I$ into a linear polynomial in A.

19. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Using Caley-Hamilton theorem evaluate A^{-1} and the matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.