DEPARTMENT OF MATHEMATICS

BIRLA INSTITUTE OF TECHNOLOGY MESRA, RANCHI

MA103 Mathematics-I, Session: (MO-2023)

Assignment - 5 (Module IV)

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1. If $\vec{A} = 5t^2\hat{\imath} + t^3\hat{\jmath} - t\hat{k}$ and $\vec{B} = 2(\sin t)\hat{\imath} - (\cos t)\hat{\jmath} + 5t\hat{k}$, find

   a) $\dfrac{d}{dt}\left(\vec{A}\cdot\vec{B}\right)$　　b) $\dfrac{d}{dt}\left(\vec{A}\times\vec{B}\right)$

2. Find a vector normal to the surface at the given point

   a) $f(x,y) = y\ln x + xy^2$
   b) $f(x,y) = 2z^3 - 3\left(x^2+y^2\right)x + \tan^{-1}(xz)$ at $(1,1,1)$
   c) $f(x,y,z) = e^{x+y}\cos z + (y+1)\sin^{-1}(x)$ at $\left(0,0,\dfrac{\pi}{6}\right)$

3. Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ is orthogonal to the surface $4x^2 - yz + z^3 = 4$ at the point $(1,1,-2)$.

4. Find the directional derivative of the function at the given point P_0 in the direction of the vector $\vec{A}$

   a) $f(x,y) = 2xy - 3y^2$, $P_0(5,5)$, $\vec{A} = 4\hat{\imath} + 3\hat{\jmath}$
   b) $f(x,y,z) = 3e^x\cos(yz)$, $P_0(0,0,0)$, $\vec{A} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

5. Find the direction in which the functions increase and decrease most rapidly at the given point P_0 . Find also the directional derivative of the function in that direction

   a) $f(x,y) = x^2 + xy + y^2$, $P_0(-1,1)$
   b) $f(x,y,z) = \ln(x^2+y^2+1) + y + 6z$, $P_0(1,1,0)$

6. Find the directional derivative of $f(x,y) = x^2yz^3$ along the curve $x = e^{-u}, y = 2\sin u + 1, z = u - \cos u$ at the point P where $u = 0$

7. In what direction from $(3,1,-2)$ is the directional derivative of $\phi = x^2y - 2x^4$ maximum. Find also the magnitude of this maximum.

8. Evaluate div R and curl R and div (curl R) where

   a) $\vec{R} = (x^2y^3 - z^4)\hat{\imath} + 4x^5y^2z\hat{\jmath} - y^3z^6\hat{k}$
   b) $\vec{R} = (x-y)^3\hat{\imath} + e^{yz}\hat{\jmath} + xye^{2y}\hat{k}$

9. Find the work done in moving a particle once around a circle C in the XY plane, of the circle has centre at the origin and radius 2 and if the force field is given by $\vec{F} = (2x-y+2z)\hat{\imath} + (x+y+z)\hat{\jmath} + (3x-2y-5z)\hat{k}$.

10. Find the circulation of F around the curve C where $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ and C is the circle $x^2+y^2 = 1, z = 0$.

11. Find the work done by the force If $\vec{F} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$ acting along the curve given by $\vec{R} = t^3\hat{\imath} + t^2\hat{\jmath} + t\hat{k}$ from $t = 1$ to $t = 3$.

12. Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. Find also the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$.

13. Evaluate $\int_{(0,0)}^{(2,1)} (10x^4 - 2xy^3)dx - (3x^2y^2)dy$ along the path $x^4 - 6xy^3 = 4y^2$.

14. Applying Green's theorem to evaluate $\oint_C e^{2x}\sin(2y)dx + e^{2x}\cos(2y)dy$, where C is the ellipse $9(x-1)^2 + 4(y-3)^2 = 36$.

15. Applying Green's theorem to evaluate $\oint_C (x^5 + 3y)dx + (2x - e^{y^3})dy$, where C is the circle $(x-1)^2 + (y-5)^2 = 4$.

16. Verify Green's theorem $\oint_C (xy + y^2)dx + x^2dy$, where C is bounded by the curve $y = x, y = x^2$.

17. Find the surface area of the portion of the cylinder $x^2 + z^2 = 4$ lying inside the cylinder $x^2 + y^2 = 4$.

18. Find the surface area of the portion of the sphere $x^2+y^2+z^2 = 9$ lying inside the cylinder $x^2+y^2-3z = 0$

19. Evaluate $\iint_S yz\hat{i} + zx\hat{j} + xy\hat{k}$, where S is the surface of the sphere $x^2+y^2+z^2 = a^2$ in the first octant.

20. By Gauss's Divergence theorem evaluate $\iint_S x^2dydz + y^2dzdx + 2z(xy - x - y)dxdy$ where S is the surface of the cube $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$.

21. By transforming to a triple integral evaluate, $\iint_S x^3dydz + x^2zdxdy$ where S is the closed surface bounded by the planes $x = 0, x = 6$ and cylinder $y^2 + z^2 = a^2$.

22. Apply divergence theorem to evaluate $\iint_S (x+z)dydz + (y+z)dzdx + (x+y)dxdy$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$.