Department of Mathematics

Birla Institute of Technology Mesra, Ranchi

MA103 Mathematics-I, Session: (MO2023)

Tutorial - 3 (Module III)

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1. Examine wheatear the limit exist or not in each case

   a) $\lim_{(x,y)\to(0,0)} \frac{xy^3}{(x^2+y^6)}$

   b) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{(x-y)}$

   c) $\lim_{(x,y)\to(0,0)} \frac{xy}{(x^2+y^2)}$

   d) $\lim_{(x,y)\to(0,0)} \frac{2xy^2}{(x^2+y^4)}$

   e) $\lim_{(x,y)\to(1,2)} \left(x^2+2y\right)$

   f) $\lim_{(x,y)\to(0,0)} \frac{xy(x^2-y^2)}{(x^2+y^2)}$

2. Investigate the continuity at $(0,0)$ of

   a) $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

   b) $f(x,y) = \begin{cases} \frac{2x^4+3y^4}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

   c) $f(x,y) = \begin{cases} \frac{2x(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

3. Show that the function $f(x,y) = \begin{cases} \frac{x^3+y^3}{x-y}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$ is discontinuous at origin but possesses partial derivatives f_x and f_y at every point, including origin.

4. Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & x\neq y \\ 0, & x=y \end{cases}$
   (i) show that $f_x(0,0)$ and $f_y(0,0)$ exist (ii) but the function f is not continuous at $(0,0)$.

5. Let $f(x,y) = \begin{cases} \frac{x^2+y^2}{|x|+|y|}, & x\neq y \\ 0, & x=y \end{cases}$
   (i) show that $f_x(0,0)$ and $f_y(0,0)$ do not exist (ii) but the function f is continuous at $(0,0)$.

6. Let $f(x,y) = \begin{cases} \frac{xy(x^2-y^2}{x^2+y^2}, & x\neq y \\ 0, & x=y \end{cases}$, show that $f_{xy}(0,0)\neq f_{yx}(0,0)$.

7. If $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

8. If $u = \sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u$

9. If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

10. If $u = \sin^{-1}\left(\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right)$, show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$

11. Find $\frac{\partial w}{\partial t}$, when $t = 1,\ s = -1$, where $w = (x+y+z)^2$ and $x = t-s,\ y = \cos(t+s),\ z = \sin(t+s)$

12. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at the given point (u, v)

    a) $w = (xy + yz + zx),\ x = u+v,\ y = u-v,\ z = uv;\ (u,v) = \left(\frac{1}{2}, 1\right)$

    b) $w = \ln(x^2+y^2+z^2),\ x = ue^v \sin u,\ y = ue^v \cos u,\ z = ue^v;\ (u,v) = (-2, 0)$

13. If $\phi = x^2 - 2y^2, \psi = 2x^2 - y^2$ and $x = r\cos\theta,\ y = r\sin\theta$ then $\frac{\partial(\phi,\psi)}{\partial(r,\theta)} = 6r^2\sin(2\theta)$

14. If $f = x + 3y^2 - z^3,\ g = 2x^2yz$ and $h = 2z^2 - xy$. Evaluate $\frac{\partial(f,g,h)}{\partial(x,y,z)}$ at $(1,-1,0)$.

15. If $u = \frac{x^2+y^2+z^2}{x},\ v = \frac{x^2+y^2+z^2}{y}$ and $w = \frac{x^2+y^2+z^2}{z}$. Find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

16. Using Taylor's Theorem expand

    a) $f(x,y) = e^x \cos y$, in powers of x and y upto and including third degree

    b) $f(x,y) = e^x \sin y$, in powers of x and y upto and including third degree

    c) $f(x,y) = \sin(xy)$ in power of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$ upto and including third degree

    d) $f(x,y) = x^2y + e^x + \sin(y)$ in power of $(x-1)$ and $(y-\pi)$ up to the terms containing second degree.

17. Examine the following functions for extreme values

    a) $f = x^3 + y^3 - 63(x+y) + 12xy$

    b) $f = x^3 + y^3 + 3xy$

    c) $f = x^3 - y^3 - 2xy + 6$

    d) $f = \frac{1}{x} + xy + \frac{1}{y}$

    e) $f = y^2 + x^2y + x^4$ at $(0,0)$