Department of Mathematics

Birla Institute of Technology Mesra, Ranchi

MA103 Mathematics-I, Session: (MONSOON)

Tutorial - 1 (Module I)

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1. Determine whether the sequence $\{a_n\}$ is monotonic, bounded and convergent, where

   a) $a_n = \dfrac{3n+1}{n+1}$

   b) $a_n = \dfrac{(2n+3)!}{(n+1)!}$

   c) $a_n = \dfrac{n!}{n^n}$

   d) $a_n = \dfrac{4^{n+1}+3^n}{4^n}$

2. Discuss the convergence of the sequence $\{a_n\}$ where

   a) $a_n = \dfrac{n+1}{n}$

   b) $a_n = \dfrac{n}{n^2+1}$

   c) $a_n = 1 + \dfrac{1}{3} + \dfrac{1}{3^2} + \ldots \dfrac{1}{3^n}$

3. Test the behavior of the following infinite series

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{n!}$

   b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

   c) $\displaystyle\sum_{n=1}^{\infty} \left(2^{n+(-1)^n}\right)^{-1}$

   d) $\displaystyle\sum_{n=1}^{\infty} \left((n^3+1)^{\frac{1}{3}} - n\right)$

   e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2}{n^3+1}\right) x^{n-1}$

   f) $\displaystyle\sum_{n=1}^{\infty} \frac{5n^3-3n}{n^2(n-2)(n^2+5)}$

   g) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$

   h) $\displaystyle\sum_{n=1}^{\infty} \frac{1+2+3+\ldots+n}{1^2+2^2+3^2+\ldots+n^2} x^n$

   i) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot (2n-3)}{2\cdot 4\cdot 6\cdots(2n-2)} \cdot \frac{1}{2n-1}$

j) $\displaystyle\sum_{n=1}^{\infty} \frac{\log n}{n^{\frac{3}{2}}}$

k) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+1}}{n \cdot 3^{n-1}}$

l) $\displaystyle\sum_{n=1}^{\infty} \frac{n \cdot 5^n}{(2n+3)\log(n+1)}$

4. Test the convergence of the series

a) $1 + \dfrac{x}{1!} + \dfrac{2^2x^2}{2!} + \dfrac{3^3x^3}{3!} + \cdots, \qquad x > 0$

b) $\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 + \left(\dfrac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10}\right)^2 + \cdots$

5. Examine the convergence of the series

$$1 + \frac{2^2}{3^2} + \frac{2^2 \cdot 4^2}{3^2 \cdot 5^2} + \frac{2^2 \cdot 4^2 \cdot 6^2}{3^2 \cdot 5^2 \cdot 7^2} + \cdots$$

6. Check the following positive term series for convergence and divergence

a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 \cdot 2^n}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \ldots (2n-1)}{4^n 2^n n!}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2 \cdot 4 \cdot 6 \ldots 2n(3^n+1)}$

e) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^{\frac{n}{3}}}$

7. Examine the behaviour of the following infinite series of positive terms.

a) $\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1 \cdot 3}{2 \cdot 4}\right)^2 + \left(\dfrac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 + \ldots$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \ldots (2n+1)}{n^2 2^n} x^{n+1}$

c) $\left(\dfrac{1}{3}\right)^2 + \left(\dfrac{1 \cdot 4}{3 \cdot 6}\right)^2 + \left(\dfrac{1 \cdot 4 \cdot 7}{3 \cdot 6 \cdot 9}\right)^2 + \ldots$

d) $\dfrac{x}{2} + \left(\dfrac{1 \cdot 3}{2 \cdot 4}\right)\dfrac{x^3}{6} + \left(\dfrac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\right)\dfrac{x^5}{10} + \ldots$

e) $x^2 + \dfrac{2^2}{3 \cdot 4}x^4 + \dfrac{2^2 \cdot 4^2}{3 \cdot 4 \cdot 5 \cdot 6}x^6 + \ldots$

f) $1 + \frac{x}{1!} + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \ldots, \quad x > 0$

g) $)1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \ldots$

h) $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots$

i) $1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \ldots$

8. Using integral test show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent when $p > 1$ and diverges for $p \leq 1$.

9. Check the following series for absolutely or conditionally convergence,

a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$

b) $\sum_{n=1}^{\infty} \frac{\sin n\alpha}{n^2}$

c) $\sum_{n=1}^{\infty} \frac{(-1)^n(n+2)}{2^n + 5}$

d) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(1+n)}$