Tutorial Sheets MA311 Numerical Techniques

Module-1

1. Define errors, truncation and rounding error.
2. Do five iterations of Regula-Falsi method to find the root of $f(x) = 2x - \cos x - 3 = 0$.
3. Define the rate of convergence.
4. Discuss the convergence of the Regula-Falsi method.
5. Define the rate of convergence. Discuss the convergence of the secant.
6. Discuss the convergence of the Newton-Raphson method.
7. Do five iteration of secant method to find the root of
   i. $f(x) = x^3 - 3x + 1 = 0$
   ii. $f(\mathrm{x}) = \mathrm{x}^3 - 5x + 1 = 0$
8. Do three iterations of bisection method to find a root of $f(\mathrm{x}) = \mathrm{x}^3 + x - 1 = 0$ lies in $(0,1)$.

Module-2

9. Use *Jacobi's iteration method* and *the Gauss-Seidel method* to solve the system of equations:
   i. $\begin{pmatrix} 4 & 1 & 1 \\ 1 & 5 & 2 \\ 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \\ -4 \end{pmatrix}$, with $x^{(0)} = (0.5, -0.5, -0.5)$
   ii. $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ 1 \end{pmatrix}$ with $x^{(0)} = (0,0,0)$ perform three iterations.
10. Use the Gauss elimination method to solve the following system of equations:
    i. $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \\ 13 \end{pmatrix}$,
    ii.

$$\begin{pmatrix} 2 & 2 & 1 \\ 4 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

11. Define the LU- Decomposition method for solving n equations and n unknowns.
12. Define a power method for numerical Eigenvalues.
13. Determine the Euclidean and the maximum absolute row sum norms of

$$A = \begin{pmatrix} 1 & 7 & -4 \\ 4 & -4 & 9 \\ 12 & -1 & 3 \end{pmatrix}$$

14. If A is a strictly diagonally dominant matrix, then the Jacobi iteration scheme converges for any initial guess.
15. Find the largest eigenvalue of**,** using power method up-to five steps

$$A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

Module-3

16. Find the cubic spline approximation for the function defined by the data $f(0) = 1, f(1) = 2, f(2) = 33, f(3) = 244$, with $M(0) = 0, M(3) = 0$. And estimate the value of $f(1.5)\ and\ \ f(2.5)$.
17. The function $\boldsymbol{f(x) = sin(x)}$ is defined on $\boldsymbol{[1, 3]}$**.** Find the Lagrange linear interpolating polynomial in$\boldsymbol{[1, 3]}$**.** Also find the bound on the truncation error $\boldsymbol{E_1(f, x) = f(x) - P_1(x)}$**.**
18. Find the unique polynomial of degree 2 or less, such that $\boldsymbol{f(0) = 1, f(1) = 3, f(3) = 55}$, using the Newton divided difference interpolation.
19. Calculate the nth divided difference of $\boldsymbol{1/x}$ based on the points $\boldsymbol{x_0, x_1, x_2, \ldots, x_n}$.
20. Define finite differences.
21. Define forward differences.
22. Evaluate $\Delta^4 f(x)\ where\ f(x) = 5x^4 + 2x^3 - 8x^2 + 3x - 13, taking\ h = 2$
23. Define backward and shift differences operators.
24. Show that $E = 1 + \Delta$ and $E^{-1} = 1 - \nabla$.
25. Construct a divided difference table for given data.

x	2	4	5	7	8

$F(x)$	3	43	138	778	1515

Module-4

26. Find $\frac{dy}{dx}; \frac{d^2y}{dx^2}$ $at\ x = 51$from the following data.

x	*50*	*60*	*70*	*80*	*90*
y	19.96	*36.65*	*58.81*	77.21	*94.61*

27. Find $y'(x)$ for given data:

x	0	1	2	3	4
Y(x)	*1*	*1*	*15*	*40*	*85*

28. Define numerical integration.
29. Define trapezoidal rule for numerical integration.
30. Discuss the truncation error for trapezoidal rule.
31. Define Simpson 1/3rd and 3/8 rule for numerical integration.
32. Evaluate $\int_{1.0}^{1.3} \sqrt{x}dx$ by taking $h = 0.05$ by trapezoidal rule.
33. Find the numerical value of $I = \int_{-1}^{1} x^2 e^{-x}\, dx$. Using trapezoidal, 1/3rd Simpson and 3/8th Simpson rules.
34. Find the numerical value of $I = \int_{0}^{2} \frac{e^{2x}}{1+x^2}\, dx$. Using trapezoidal, 1/3rd Simpson and 3/8th Simpson rules.
35. The following table gives the values of $f(x)$ at equal intervals of x

x	0	0.5	1.0	1.5	2.0
$f(x)$	0.399	0.352	.242	0.129	0.054

$Evaluate \int_0^2 f(x)dx$ using Simpson's rule.

36. Evaluate $\int_0^4 5/(4x+5)dx$ by trapezoidal rule using 11 coordinates.

Module-5

37. Define Euler's method.
38. Solve $y' = 1 - y, y(0) = 0$, using Euler's method. Find $y\ at\ x = 0.1\ and\ x = 0.2$. Compare the results with results of the exact solution.
39. Solve $y' = xy, y(0) = 1$. Find $y(0.4)$ using Euler's method with $h = 0.1$.
40. Define second order R-K Method.
41. Define fourth order R-K Method.
42. Using fourth order R-K Method, evaluate y(1.1) given that $y' + y/x = 1/x^2, y(1) = 1$
43. Solve $y' = y^2 + x^2,\ y(2) = 3$. Find $y(2.1)$ and $y(2.2)$ by fourth order Runge-Kutta method.
44. Define predictor corrector methods.
45. Given $dy/dx = 1/(x + y);\ y(0) = 2$. If $y(0.2) = 2.09, y(0.4) = 2.17, y(0.6) = 2.24$. Find $y(0.8)$ using Milne's method
46. Define finite difference method for first and second order derivatives.
47. Using the finite difference method, solve the boundary value problems: $y'' + y = 0, y(0) = 0, y(1) = 1, taking\ h = 0.25$.
48. Using the finite difference method, solve the boundary value problems: $y'' - y = 0, y(0) = 0, y(2) = 3.63, taking\ h = 0.5$.
49. Define shooting methods.
50. Solve BVPs $y'' + 4y = x\ , y(0) = 1/2\ and\ y(\pi/4) = 1$by shooting method.