

DEPARTMENT OF MATHEMATICS
BIRLA INSTITUTE OF TECHNOLOGY MESRA, RANCHI
MA107 Mathematics-II, Session: (SP-2020)
Tutorial - 5 (Module V)

1. Let X be a random variable with probability density function

$$f(x) = k(2x - 1) \text{ for } x = 1, 2, 3, \dots, 12$$

for some constant k . What is the value of k . Also, find the cumulative distribution of X .

2. Find the probability density function of the random variable X whose cumulative distribution function is

$$F(x) = \begin{cases} 0.00 & \text{if } x < -1 \\ 0.25 & \text{if } -1 \leq x < 1 \\ 0.50 & \text{if } 1 \leq x < 3 \\ 0.75 & \text{if } 3 \leq x < 5 \\ 1.0 & \text{if } x \geq 5. \end{cases}$$

Also, find

a) $P(X \leq 3)$

b) $P(X = 3)$

c) $P(X < 3)$

3. The random variable X has density function

$$f(x) = \begin{cases} (k+1)x^2 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

find the value of constant k .

4. A random variable X has a cumulative distribution function

$$F(x) = \begin{cases} \frac{x}{2} & \text{if } 0 < x \leq 1 \\ x - \frac{1}{2} & \text{if } 1 < x \leq \frac{3}{2} \end{cases}$$

a) $P(X \leq 0.5)$

b) $P(X \geq 0.5)$

c) $P(X \leq 1.25)$

d) $P(X = 1.25)$

5. Let the distribution of X for $x > 0$ be

$$F(x) = 1 - \sum_{k=0}^3 \frac{x^k e^{-x}}{k!}$$

What is the density function of X for $x > 0$

6. Let X is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 7 < x < 9 \\ 0 & \text{otherwise} \end{cases}$$

Find the cumulative distribution function of X .

7. Let X be a random variable with cumulative distribution function

$$F(x) = \begin{cases} 1 - e^{-x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

what is the $P(0 \leq e^X \leq 4)$.

8. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{x}{8} & \text{for } x = 1, 2, 5 \\ 0 & \text{otherwise.} \end{cases}$$

Find

a) $E(X)$

b) $Var(X)$

c) $E(2X + 3)$

d) $Var(2X + 3)$

9. Let X is a random variable with density function

$$f(x) = \begin{cases} \theta x + \frac{3}{2}\theta^{\frac{3}{2}}x^2 & \text{for } 0 < x < \frac{1}{\sqrt{\theta}} \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$. What is the expected value of X .

10. Let X is a random variable with density function

$$f(x) = \begin{cases} 1.4e^{-2x} + 0.9e^{-3x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

then what is the expected value of X .

11. If the moment generating function for the random variable X is $M(t) = k(2 + 3e^t)^4$, what is the value of k .

12. Let X be a random variable with density function

$$f(x) = \begin{cases} ae^{-ax} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $a > 0$. if $M(t)$ denotes the moment generating function of X , what is $M(-3a)$.

13. Let X be a random variable with density function

$$f(x) = \begin{cases} \frac{2x}{k^2} & 0 \leq x \leq k \\ 0 & \text{otherwise} \end{cases}$$

For what value of k is the variance of X equal to 8.

14. What is the probability of getting exactly 3 heads in 5 flips of a fair coin.

15. On six successive flips of a fair coin, what is the probability of observing 3 heads and 3 tails.

16. What is the probability that in 3 rolls of a pair of six-sided dice, exactly one total of 7 is rolled.

17. In a family of 4 children, what is the probability that there will be exactly two boys.

18. A Quiz is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. A person answered randomly all the questions. What is the probability (a) exactly 5 questions are correct (b) exactly 10 questions are correct.
19. A random variable X has a Poisson distribution with a mean of 3. What is the probability that X is bounded by 1 and 3, that is $P(1 \leq X \leq 3)$.
20. If X is any random variable with mean μ and variance $\sigma^2 > 0$, then what are the mean and variance of the random variable $Y = \frac{X - \mu}{\sigma}$.
21. If $X \sim N(\mu, \sigma^2)$, then the random variable $Y = \frac{X - \mu}{\sigma} \sim N(0, 1)$.
22. If X be a normal distribution with mean 2 and variance 4, then find the $P(X > 2)$.

1. $k = \frac{1}{144}$, $F(1) = \frac{1}{144}$, $F(2) = \frac{4}{144}$, $F(3) = \frac{9}{144}$, ..., $F(12) = 1$.

2. $f(-1) = f(1) = f(3) = f(5) = 0.25$, $P(X \leq 3) = 0.75$, $P(X = 3) = 0.25$, $P(X < 3) = 0.5$

3. $k = 2$.

4. $P(X \leq 0.5) = 0.25$, $P(X \geq 0.5) = P(X \leq 1.25) = 0.75$, $P(X = 1.25) = 0$

5. $\frac{1}{6}x^3e^{-x}$

6. $F(x) = \begin{cases} 0 & x \leq 7 \\ \frac{x-7}{2} & 7 < x < 9 \\ 1 & x \geq 9 \end{cases}$

7. $\frac{3}{4}$

8. (a) 3.75, (b) 2.6875, (c) 10.5, (d) 10.75

9. $\frac{17}{24\sqrt{\theta}}$

10. $\frac{9}{20}$

11. $\frac{1}{625}$

12. $\frac{1}{4}$

13. 12 because $k \geq 0$

14. $\frac{5}{16}$

15. $\frac{5}{16}$

16. $\frac{25}{72}$

17. $\frac{3}{8}$

18. (a) 0.2023, (b) 0.0099

19. $12e^{-3}$

20. $E(Y) = 0$, $Var(Y) = 1$

22. 0.5