DEPARTMENT OF MATHEMATICS

BIRLA INSTITUTE OF TECHNOLOGY MESRA, RANCHI

MA107 Mathematics-II, Session: (SP-20)

Tutorial - 1 (Module I)

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1. a) Show that the Wronskian of $e^{ax}\cos(bx)$ and $e^{ax}\sin(bx)$ $(b \neq 0)$ is b^2e^{2ax}.

   b) Prove that the Wronskian of functions $e^{m_1x}, e^{m_2x}, e^{m_3x}$ is equal to $(m_1 - m_2)(m_2 - m_3)(m_3 - m_1)e^{(m_1+m_2+m_3)x}$.

   c) Show that $y_1(x) = \sin x$ and $y_2(x) = \sin x - \cos x$ are linearly independent solutions of $y'' + y = 0$.

   d) Show that the Wronskian of function $x^2, x^2\log(x)$ is non-zero. can these functions be independent solution of an ordinary differential equation; if so determine this equation.

   e) Evaluate the Wronskian of the functions e^x, xe^x. Hence conclude whether or not they are linearly independent. If they are independent, set the differential equation having them as its independent solution.

   f) check whether $e^x, xe^x, \sinh(x)$ are linearly independent or dependent.

2. Solve the following differential equations:

   a) $y'' + 3y' - 10y = 0$.

   b) $y''' - 9y'' + 23y' - 15y = 0$.

   c) $y^{(iv)} + 8y'' + 16y = 0$.

   d) $y'' + 3y' + 4y = x^2 - 2x$.

   e) $y'' - 3y' + 2y = e^{3x}$.

   f) $y'' - 4y' + 4y = e^{2x} + x^3 + \cos 2x$.

   g) $y''' - 12y' + 16y = \left(e^x + e^{-2x}\right)^2$.

   h) $y''' - 5y'' + 7y' - 3y = e^{2x}\cosh(2x)$.

   i) $y'' - 4y' + 4y = 8x^2e^{2x}\sin 2x$.

   j) $y'' - (a+b)y' + aby = e^{ax} + e^{bx}$

   k) $y'' + 9y = \sec 3x$

   l) $y'' + a^2y = \tan ax$

   m) $y^{iv} + 3y'' - 4y = \cos^2 x - \cosh x$

3. Solve the following Cauchy-Euler equations:

   a) $y'' + \frac{1}{x}y' = \frac{12\log x}{x^2}$.

   b) $x^2y'' - 4xy' + 6y = \frac{42}{x^4}$.

4. Solve the following Legendre's linear equations:

   a) $(3x+2)^2y'' + 3(3x+2)y' - 36y = 3x^2 + 4x + 1$.

   b) $(1+x)^2y'' + (1+x)y' + y = 2\sin(\log(1+x))$.

5. Solve by the method of variation of parameters

   a) $y'' + y = x$.

   b) $xy' - y = (x-1)(y'' - x + 1)$.

   c) $y'' + 9y = \sec 3x$.

   d) $y'' - y = \frac{2}{1+e^x}$.