Department of Mathematics

Birla Institute of Technology, Mesra, Ranchi.

IMM 4001 Mathematics-IV-Integral Transform & Partial Differential Equation

2017----

Module I & II:

1. Define
   (a) Laplace Transform of a function.
   (b) Functions of exponential order.
   (c) Functions of Class A.
2. State and prove the theorem on Existence of Laplace Transform of a function
3. Show that the function $F(t) = t^2$ is of exponential order 2.
4. Show that the function $F(t) = e^{t^2}$ is not of exponential order.
5. Find the Laplace Transform of the following functions:
   (a) $F(t) = (1+t^2)^2$ (b) $F(t) = 3t^4 + 4e^{-3t} - 2\sin 5t + 3\cos 2t - 4\sinh 5t + 3\cosh 5t$
   (c) $F(t) = \begin{cases} (t-1)^2, & t > 1 \\ 0, & 0 < t < 1 \end{cases}$ (d) $F(t) = \dfrac{e^{at} - 1}{a}$ (e) $F(t) = \sin\sqrt{t}$
6. State and prove the following properties of Laplace Transform
   (i) First Shifting property (ii) 2nd Shifting property
   (iii) Change of Scale property
7. Find the Laplace Transform of the following functions:
   (a) $F(t) = e^t(2+t)^2$ (b) $F(t) = t^4 e^{-3t}$ (c) $F(t) = e^{2t}(3\sin 5t + 5\cos 5t)$
   (d) $F(t) = \begin{cases} \cos\left(t - \dfrac{2\pi}{3}\right), & t > \dfrac{2\pi}{3} \\ 0, & t < \dfrac{2\pi}{3} \end{cases}$ (e) $F(t) = e^{-t}(3\sinh 2t - 5\cosh 2t)$
8. State and prove the following theorem:
   (a) The theorem on Laplace Transform of the derivative of a function.
   (b) The theorem on Laplace Transform of nth derivative of a function.
   (c) Initial value theorem
   (d) Final Value theorem
9. Prove the following theorems:
   (a) If $F(t)$ is piecewise continuous for all $t \geq 0$ and is of exponential order a as $t \to \infty$ and if $L\{F(t)\} = f(s)$, then prove that $L\left\{\int_0^t F(x)dx\right\} = \dfrac{f(s)}{s}$, $s > 0, s > a.$

(b) If $F(t)$ is a function of class A and if $L\{F(t)\} = f(s)$, then prove that

(i) $L\{tF(t)\} = -\frac{d}{ds}\left(f(s)\right).$

(ii) $L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n}\left(f(s)\right), \; n = 1,2,3,.........$

(iii) $L\left\{\frac{F(t)}{t}\right\} = \int_s^{\infty} f(x)dx$ provided $\lim_{t \to \infty}\left\{\frac{F(t)}{t}\right\}$ exists.

(c) If $F(t)$ is a periodic function with period $T > 0$, then prove that $L\{F(t)\} = \frac{\int_0^T e^{-st}F(t)dt}{1 - e^{-sT}}$.

10. Find the Laplace Transform of the following functions:

(a) $F(t) = \frac{\cos(\sqrt{t})}{\sqrt{t}}$ (b) $F(t) = t^3 e^{-3t}$ (c) $F(t) = (3t^3 \sin 5t + 5t \cos 5t)$

11. Prove that

(a) $L\left\{\int_0^t \frac{\sin x}{x} dx\right\} = \frac{\cot^{-1}(s)}{s}, \; s > 0.$ (b) $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}(1/s)$

12. Prove that

(a) $L\{J_0(t)\} = \frac{1}{\sqrt{1+s^2}}$ (b) $L\{J_0(at)\} = \frac{1}{\sqrt{a^2+s^2}}$ (c) $L\{tJ_0(at)\} = \frac{s}{(a^2+s^2)^{3/2}}$

13. Evaluate the following integrals using Laplace transform:

(a) $\int_0^{\infty} \frac{\sin x}{x} dx$ (b) $\int_0^{\infty} \frac{e^{-at} - e^{-bt}}{t} dt$ (c) $\int_0^{\infty} t e^{-3t} \sin t \; dt$ (d) $\int_0^{\infty} t^3 e^{-3t} \sin t \; dt$

14. Evaluate $L\{F(t)\}$

(a) $F(t) = (t^2 - 1)u_3(t)$ (b) $F(t) = (e^{(3-t)}u_3(t)$ (c) $F(t) = \cos(t-3)u_3(t)$

(d) $F(t) = \sin(t)u_2(t)$ (e) $F(t) = \cos(t)\; u_1(t)$

15. Evaluate $L\{F(t)\}$

(a) $F(t) = \begin{cases} 3t, & 0 < t < 2 \\ 6, & 2 < t < 4 \end{cases}$ and $F(t)$ is a periodic function with period 4.

(b) $F(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$ and $F(t)$ is a periodic function with period 2π .

(c) $F(t) = t^2, \quad 0 < t < 2$ and $F(t)$ is a periodic function with period 2.

16. Evaluate :

(a) $L^{-1}\left\{\frac{1}{s^{7/2}}\right\}$ (a1) $L^{-1}\left\{\frac{s}{s^2+2} + \frac{6s}{s^2-16} + \frac{3}{s-3}\right\}$

(b) $L^{-1}\left\{\frac{s}{(s+1)^5}+\frac{3s-2}{s^2-4s+20}\right\}$ (b1) $L^{-1}\left\{\frac{e^{-5s}}{(s-2)^4}\right\}$

(c) $L^{-1}\left\{\frac{1}{(s^2+1)(s+1)^2}\right\}$ (c1) $L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\}$

(d) $L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$ (d1) $L^{-1}\left\{\frac{s}{(s^2-a^2)^2}\right\}$

(e) $L^{-1}\left\{\frac{s+1}{(s^2+2s+2)^2}\right\}$ (e1) $L^{-1}\left\{\log\left(\frac{s+3}{s+2}\right)\right\}$

(f) $L^{-1}\left\{\log\left(1+\frac{1}{s^2}\right)\right\}$ (f1) $L^{-1}\left\{\frac{1}{s}\log\left(\frac{s+3}{s+2}\right)\right\}$

17. Using Convolution theorem evaluate :

(a) $L^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$ (a1) $L^{-1}\left\{\frac{1}{(s+2)(s-1)}\right\}$

(b) $L^{-1}\left\{\frac{1}{(s-1)^5(s+2)}\right\}$ (b1) $L^{-1}\left\{\frac{1}{\sqrt{s}(s-a)}\right\}$

(c) $L^{-1}\left\{\frac{s^2-6}{s^3+4s^2+3s}\right\}$ (c1) $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$

18. Solve the following differential equations using Laplace transform:

(i) $\frac{d^2y}{dt^2}+y=0,\ t>0$,under the conditions that $\frac{dy}{dx}=0$ and $y=1$ at $t=0$.

(ii) $(D+2)^2y=4e^{-2t},\ t>0$, under the conditions that $y'(0)=4$ and $y(0)=-1$.

(iii) $y''+9y=\cos 2t,\ t>0,$ under the conditions that $y(0)=1$ and $y(\pi/2)=-1$.

(iv) $y''+y=t\cos 2t,\ t>0,$ under the conditions that $y'(0)=0$ and $y(0)=0$.

(v) $y'''+y=1,\ t>0,$ under the conditions that $y''(0)=y'(0)=y(0)=0$.

(vi) $y'''-2y''+5y=0,\ t>0,$ under the conditions that $y'(0)=1, y(0)=0$ and $y(\pi/8)=1$

(vii) $ty''+y'+4ty=0$, under the conditions that $y'(0)=0, y(0)=3$.

(viii) $ty''+(1-2t)y'-2y=0$ under the conditions that $y'(0)=2, y(0)=1$.

19. Solve the following system of differential equations using Laplace transform:

(a) $(D^2-3)x-4y=0,$

$x+(D^2+1)y=0,\quad t>0,$

subject to the condtions: $x(0)=y(0)=y'(0)=0$ and $x'(0)=2$

(b) $(D-2)x-(D+1)y=6e^{3t},$

$(2D-3)x+(D+3)y=6e^{3t},\quad t>0,$ subject to the condtions: $x(0)=3, y(0)=0.$

20. Solve the following partial differential equations with initial and boundary conditions using Laplace transform:

(a) Solve the initial value problem:

$$x\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=xt,$$

subject to the condtions: $u(x,0)=0, u(0,t)=t.$

(b) The temperature distribution $u(x,t)$ in a semi-infinite, thin, insulated rod is given by the one-dim. Heat equation: $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$

Assume that the left end of the rod is maintained at an arbitrary time dependent temperature $u(0,t)=f(t)$. Initially the rod is at zero temperature, $u(x,0)=0$. Find the distribution of the heat flow, if the temperature is bounded as $x\to\infty$

Module III:

21. Define
    - (a) Fourier complex transform of a function
    - (b) Fourier sine transform of a function
    - (c) Fourier cosine transform of a function
22. State and prove the following properties of Fourier Transform
    - (i) First Shifting property
    - (ii) 2nd Shifting property
    - (iii) Change of Scale property
23. If $F\{f(x)\}=\bar{f}(s)$, then prove that $F\{f(x)\cos ax\}=\frac{1}{2}\left[\bar{f}(s+a)+\bar{f}(s-a)\right].$
24. If $F_s\{f(x)\}=\varphi(s)$ for $s>0$, then prove that $F_s\{f(x)\}=-\varphi(-s)$ for $s<0.$
25. Find the Fourier transform of the function defined by $f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a\end{cases}$

    Hence evaluate the integral (i) $\int_{-\infty}^{\infty}\frac{\cos sx\ \sin as}{s}ds$ (ii). $\int_{0}^{\infty}\frac{\sin s}{s}ds$
26. Find the Fourier transform of the function defined by $f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0, & |x|>1\end{cases}$

    Hence evaluate the integral $\int_{0}^{\infty}\frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}dx$.

27. Find the Fourier transform of the function defined by $f(x)=\begin{cases}1, & 0\le x\le \pi \\ 0, & x>\pi\end{cases}$

Hence show that $\int_0^\infty \frac{1-\cos \pi\lambda}{\lambda} \sin \pi\lambda \, d\lambda = \frac{\pi}{4}$.

28. Find the Fourier transform of $f(x)=e^{-x^2/2}$

29. Find the Fourier transform of $f(x)=e^{-a|x|}$, $\quad -\infty < x < \infty$

30. Find the Fourier transform of $f(x)=\begin{cases}x^2, & |x|<a \\ 0, & |x|>a\end{cases}$

31. Find the Fourier sine and cosine transforms of

(i) $f(x)=\begin{cases}1, & 0<x<a \\ 0, & x>a\end{cases}$ (ii) $f(x)=\begin{cases}x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2\end{cases}$ (iii) $f(x)=\begin{cases}\cos x, & 0<x<a \\ 0, & x>a\end{cases}$

32. Find the Fourier sine transform of (i) $f(x)=e^{-|x|}$,

Hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2}\,dx = \frac{\pi}{2}e^{-m}$.

33. Find the Fourier sine and cosine transform of $f(x)=e^{-bx}$.

Hence evaluate the integrals (i) $\int_0^\infty \frac{s\sin sx}{s^2+b^2}\,ds$ (ii) $\int_0^\infty \frac{\cos(sx)}{s^2+x^2}\,ds$

34. Find the Fourier cosine transform of $f(x)=e^{-bx^2}$.

35. Find $F_c\{f(x)\}$ when $f(x)=\frac{1}{1+x^2}$ and hence find $F_s\{g(x)\}$ where $g(x)=\frac{x}{1+x^2}$.

36. Find the Fourier sine transform of (i) $f(x)=\frac{e^{-ax}}{x}$ (ii) $f(x)=1/x$

37. State and prove the Convolution theorem for Fourier Transform.

38. Verify the Convolution theorem for $f(x) = g(x) = e^{-x^2}$.

39. If the Fourier sine transform of $f(x)$ is $\frac{s}{1+s^2}$, find $f(x)$.

40. Derive the relation between the Fourier and Laplace .

41. If $f(x)$ and its first (n-1) derivatives are continuous and if nth derivative of $f(x)$ is piecewise continuous , then prove that $F\{f^n(x);s\}=(-is)^n\ F\{f(x);s\}\ r=0,1,2,.........$
Provided the function and its derivatives are absolutely integrable such that $f(x)$ and its first (n-1) derivatives vanishes as $x\to\pm\infty$

42. If $f(x)$ and $f'(x)$ be continuous on the interval $[0, \infty)$ and if $f''(x)$ is piecewise continuous on every finite subinterval $[0, l]$, then prove that

$$\mathrm{F_c}\{f''(x)\} = -\omega^2 \ F_c\{f(x)\} - f'(0)$$

and

$$\mathrm{F_s}\{f''(x)\} = -\omega^2 \ F_s\{f(x)\} + \omega f(0)$$

Provided the function and its derivatives are absolutely integrable such that $f(x)$ and $f'(x)$ vanishes as $x \to \infty$

43. Find the solution of the differential equation:

$$y'(t) - 2y(t) = H(t)e^{-2t}, \quad -\infty < t < \infty.$$

Using Fourier transform, where $H(t) = u_0(t)$ is the unit step function.

44. The temperature distribution $u(x,t)$ in a thin, homogeneous, infinite bar can be modelled by the initial boundary value problem :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad t > 0$$

$\mathrm{u}(x,0) = f(x)$ and $u(x,t)$ ix finite as $x \to \pm\infty$. Find the distribution of the heat flow $u(x,t)$.

45. Solve the initial boundary value problem :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \quad t > 0$$

$$\mathrm{u}(x,0) = \begin{cases} 1, & 0 < x < l \\ 0, & x > l \end{cases} \quad \text{and} \quad u(0,t) = 0, \ t > 0.$$

Module: IV & V

46. Define :
   - (i) Integral Equation
   - (ii) The Abel's Integral Equation
   - (iii) The Fredholm Integral Equation of 1st first kind and 2nd kind
   - (iv) The Volterra Integral Equation of 1st first kind and 2nd kind.
   - (v) Integral Equation of Convolution type

47. Define:
   - (i) Symmetric Kernels
   - (ii) Separable Kernels
   - (iii) Resolvant Kernels
   - (iv) Degenerate Kernels.

48. Prove that: $\int_a^x \int_a^x \int_a^x ....\int_a^x f(x)dx\,dx\,dx.....dx = \frac{1}{(n-1)!}\int_a^x (x-t)^{n-1} f(t)dt$

49. Established the relationship between the Linear Differential equation and the Volterra Integral equation

50. Convert the following Differential equations into the corresponding Integral equations:

(a) $y''(x) - 3y'(x) + 2y(x) = 5\sin x$, subject to the conditions $y'(0) = -2,\ y(0) = 1$

(b) $y''(x) + \lambda\, y(x) = 0$ subject to the conditions $y(0) = y(1) = 0$.

(c) $y'''(x) + y''(x) - xy(x) = \sin x$

subject to the conditions $y'' = 1/2,\ y' = -1,\ y = 1$ at $x = 0$.

(d) $y''(x) + (1-x)y'(x) + e^{-x}y(x) = x^3 - 3x$

subject to the conditions $y' = 4,\ y = -3$ at $x = 0$.

51. Convert the following integral equations into the corresponding Differential equations:

(a) $y(x) = \int_0^x (x+t)y(t)dt - 1$ (b) $y(x) = \int_0^x (x-t)y(t)dt + 3\sin x$

(c) $y(x) = \int_0^x t(x-t)y(t)dt + \frac{1}{2}x^2$ (d) $y(x) + \int_0^x [(x-t)^2 + 4(x-t) - 3]y(t)dt = e^{-x}$

52. Show that the function $y(x) = 2 - x$ is a solution of the integral equation

$$\int_0^x e^{(x-t)} y(t)dt = e^x + x + 1$$

53. Show that the function $y(x) = (1+x^2)^{-3/2}$ is a solution of the Volterra integral equation

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)dt = e^x + x + 1$$

54. Solve the following homogeneous Fredholm Integral equations:

(a) $y(x) = \lambda \int_0^1 e^{x+t} y(t)dt$ (b) $y(x) = \lambda \int_0^{2\pi} \sin(x+t)y(t)dt$

55. Find the Eigen values and Eigen functions of the following Integral equations with degenerate kernels:

(a) $y(x) = \lambda \int_0^1 (2tx - 4x^2)y(t)dt$ (b) $y(x) = \frac{1}{e^2 - 1}\int_0^1 e^{(x+t)} y(t)dt$

(c) $y(x)=\lambda\int_0^{\pi}\sin(x-t)y(t)dt+\cos x$ (d) $y(x)=\lambda\int_0^{\pi}(1+\sin x\sin t)y(t)dt+x$

(e) $y(x)=\lambda\int_{-\pi}^{\pi}(x\cos t+t^2\sin x+\cos x\sin t)y(t)dt+x$

56. Find the iterated kernel and hence the resolvent kernel for the IE

$$y(x)=f(x)+\lambda\int_a^b k(x,t)y(t)dt \quad \text{when}$$

(a) $k(x,t)=\sin(x-2t),\quad 0\le x\le 2\pi,\quad 0\le t\le 2\pi$

(b) $k(x,t)=e^x\cos t,\qquad a=0, b=\pi$

(c) $k(x,t)=x+\sin t,\qquad a=-\pi, b=\pi$

(d) $k(x,t)=e^{x+t},\quad a=0, b=1$

(e) $k(x,t)=(1+x)(1-t),\quad a=-1, b=1$

57. Solve the following Fredholm IE by the method of Successive approximations:

(a) $y(x)=x+\int_0^{1/2}y(t)dt$ (b) $y(x)=\sin x-\frac{x}{4}+\frac{1}{4}\int_0^{\pi/2}xt\,y(t)dt$

58. Consider the integral equation $y(x)=1+\lambda\int_0^1(1-3tx)y(t)dt$.

Evaluate the Resolvent kernel. For what values of the solution of the IE does not exist. Obtain the solution the above IE.

59. Solve the following Fredholm IE by the method of Successive approximations to third order:

(a) $y(x)=1+\lambda\int_0^1(t+x)y(t)dt$ (b) $y(x)=2x+\lambda\int_0^1(x+t)y(t)dt$ taking $y_0(x)=1$

60. Find the iterated kernel and hence the resolvent kernel for the IE

$$y(x)=f(x)+\lambda\int_a^x k(x,t)y(t)dt \quad \text{when}$$

(a) $k(x,t)=1$ (b) $k(x,t)=e^{x-t}$ (c) $k(x,t)=e^{x^2-t^2}$ (d) $k(x,t)=\dfrac{1+x^2}{1+t^2}$

61. Evaluating resolvent kernel find the solution of the Voltera IE

(a) $y(x) = e^{x^2} + \int_0^x e^{x^2 - t^2} y(t)dt$ (b) $y(x) = \sin x + 2\int_0^x e^{x-t} y(t)dt$

(c) $y(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2} y(t)dt$ (d) $y(x) = x + \int_0^x (t-x)y(t)dt$

62. Solve the following Voltera IE by the method of Successive approximations:

(a) $y(x) = x - \int_0^x (x-t)y(t)dt$ taking $y_0(x) = 0$

(b) $y(x) = \frac{x^3}{2} - 2x - \int_0^x y(t)dt$ taking $y_0(x) = x^2$

63. Solve the following Voltera IE by using Laplace transform:

(a) $y(x) = \int_0^x \sin(x-t)y(t)dt + 3x^2$ (b) $y(x) = 2\int_0^x \cos(x-t)y(t)dt + x$

(c) $\int_0^x y(x-t)y(t)dt = 4\sin 9x$ (d) $t(1+t) = \int_0^t \frac{y(x)}{(t-x)^{1/3}} dx$

64. Solve the following Integro-Differential equations:

(a) $y'(x) + 3y(x) + 2\int_0^x y(t)dt = x,$ given that $y(0) = 1$

(b) $y'(x) = 3\int_0^x \cos 2(x-t)y(t)dt + 2,$ given that $y(0) = 1$

(c) $y'(x) + 4y(x) + 5\int_0^x y(t)dt = e^{-x},$ given that $y(0) = 0$

Module VI and VII:

65. Form the partial differential equations for the following:

(i) $f(x^2 + y^2 + z^2, x + y + z) = 0$, (ii) $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$, (iii) $z = f(x^2 + y) + g(x^2 - y)$.

66. Solve the following first order linear partial differential equations:

(ii) $xy^2 p + xzq = y^2$, (ii) $(p-q)z = z^2 + (x+y)^2$, (iii) $(x^2-yz)p + (y^2-zx)q = z^2-xy$,

(iv) $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$, (v) $(y-z)p/yz + (z-x)q/zx = (x-y)/xy$,

(vi) $(y+z)p + (z+x)q = x+y$, (vii) $x^2(y^2-z^2)p + y^2(z^2-x^2)q = z^2(x^2-y^2)$

67. Solve the following first order non-linear partial differential equations::

(i) $z^2(p^2+q^2+1)=a^2$, (ii) $x^2p^2+y^2q^2=z^2$, (iii) $q=xyp^2$,

(iv) $q(p-\cos x)=\cos y$, (v) $4xyz=pq+2px^2y+2qxy^2$, (vi) $yp=2xy+\log z$.

68. Solve $\frac{\partial^2 z}{\partial x \partial y}=\sin x \sin y$, given that $\frac{\partial z}{\partial y}=-2\sin y$ when y = 0 and z = 0 when y is an odd multiple of $\pi/2$.

69. Solve the following higher order linear partial differential equations with constant coefficients:

(ii) $(D^2-2DD'-15D'^2)z=12xy$

(iii) $(D^2-2DD'-D'^2)z=\sin(2x+3y)$

(iv) $(D^2-2DD'+D'^2)z=\sin x$

(v) $(D^2-2DD')z=2e^{2x}-3x^2y$

(vi) $(D^2-DD'+2D'^2)z=(y-1)e^x$

(vii) $(D^2+2DD'+D'^2)z=2\cos y-x\sin y$

(viii) $(D^2+2DD'+D'^2-2D-2D')z=\sin(x+2y)$

(ix) $(D^2-DD'+D'-1)z=\cos(x+2y)$

(x) $(D+D'-1)(D+2D'-3)z=4+3x+6y$

70. Solve the following higher order linear partial differential equations with variable coefficients:

(a) $(x^2D^2+2xyDD'+2y^2D'^2)z=x^my^n$ (b) $xyu_{xy}-yu_y=x^2$

(c) $yu_{xy}+u_x=\cos(x+y)-y\sin(x+y)$ (d) $xu_{xx}+yu_{xy}+u_x=10xy^3$

71. Solve the following partial differential equation by Monge's method: where the notations *p, q, r, s, t,* used having their usual meanings as:

(a) $r+(a+b)s+abt=xy$ (b) $t-r\sec^4 y=2q\tan y$

(c) $2x^2r-5xys+2y^2t+2(px+qy)=0$ (d) $x^2r+2xys+y^2t=0$

(e) $(x-y)[xr-(x+y)s-yt]=(x+y)(p-q)$ (f) $y^2r-2ys+t=p+6y$

72. Classify, reduce the following Partial Differential Equations with variable coefficients to a canonical form and hence solve it, if possible:

(g) $y^2u_{xx} - x^2u_{yy} = 0, \quad x > 0, \; y > 0.$ (b) $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} - xu_x + 3yu_y = 8\dfrac{y}{x}$

(c) $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0,$ (d) $3u_{xx} + 10u_{xy} + 3u_{yy} = 0,$

(e) $u_{xx} - 2\sin xu_{xy} - \cos^2 xu_{yy} - \cos xu_y = 0$ (f) $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \dfrac{y^2}{x}u_x + \dfrac{x^2}{y}u_y$

73. Solve by the method of separation of variables:

(i) $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial y} + 2u$

subject to the condition u = 0 and $\dfrac{\partial z}{\partial x} = 1 + e^{-3y}$ when x = 0 for all values of y.

(ii) $\nabla^2 u = 0, \quad 0 \le x \le a, \; 0 \le y \le b$

subject to the condition $u(0, y) = u(x,0) = u(x,b) = 0$ & $\dfrac{\partial u}{\partial x}(a, y) = T\sin^3\left(\dfrac{\pi y}{a}\right).$

74. A string is stretched and fastened to two point's l cm. apart. Motion is started by displacing the string in the form y = a sin(πx/l) from which it is released at time t = 0. Show that the displacement of any distance x from one end at time t is given by

y(x,t) = a sin(πx/l)cos(πt/ l).

75. If a string of length l is initially at rest in equilibrium position and each of its points is given by the velocity $\left(\dfrac{\partial y}{\partial t}\right)_{t=0} = b\sin^3(\pi x / l)$ .Find the displacement y.

76. Solve the following boundary value problem:

$\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$, 0< x < l

subject to the condition $\dfrac{\partial u(0,t)}{\partial x} = 0, \; \dfrac{\partial u(l,t)}{\partial x} = 0$ and u (x,0) = x.

77. Solve the following problem:

$\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$, for the conduction of heat along a rod without radiation, subject to the following conditions: (i) if u is not infinite for $t \to \infty$, (ii) $\frac{\partial u}{\partial x} = 0$ for x=0 and x=1 ,

(iii) u = $lx-x^2$ for r = 0, between x = 0 and x = 1.

78. A thin rectangular thin homogeneous thermally conducting plate lies in the xy-plane defined by $0 \le x \le a,\ 0 \le y \le b$. The edge $y = 0$ is held at the temperature $Tx(x-a)$, where T is a constant, while the remaining edges are held at 0^0C. The other faces are insulated and no internal sources and sinks are present, find the steady state temperature inside the plate.

79. Find the steady state temperature distribution in a semi-circular plate of radius a , insulated on both the faces with its curved boundary kept at a constant temperature U_0 and its bounding diameter kept at zero temperature.

80. A rectangular membrane with fastened edges making free transverse vibrations satisfying the PDE: $u_{tt} - c^2\nabla^2 u = 0, \quad 0 \le x \le a,\ 0 \le y \le b$

subject to the BC's: $(i)\ \ u(0,y,t) = \ u(a,y,t) = 0,\ (iii)\ \ u(x,0,t) = \ u(x,b,t) = 0,\ \forall t > 0$

and IC's: $(i)\ \ u(x,y,0) = f(x,y),\ \ (ii)\ \ u_t(x,y,0) = g(x,y).$

Find the vibration functions $u(x,y,t)$ in terms of series.

IMM 4001 Mathematics IV - Integral Transform & Partial Differential Equations (3-1-0-4)

Module I &II

Laplace Transform : Definition of Laplace Transform, Linearity property, condition for existence of Laplace Transform; First & Second Shifting properties, Laplace Transform of derivatives and integrals; Unit step functions, Dirac delta-function. Differentiation and Integration of transforms, Convolution Theorem, Inversion. Periodic functions. Evaluation of integrals by L.T., Solution of boundary value problems. [6]

Module III

Fourier Transform: Fourier Integral formula, Fourier Transform, Fourier sine and cosine transforms. Linearity, Scaling, frequency shifting and time shifting properties. Self reciprocity of Fourier Transform. Convolution theorem. Application to boundary value problems. [6]

Module IV & V

Integral Equations: Integral Equations: Basic concepts, Volterra integral equations, Relationship between linear differential equations and Volterra equations, Resolvent kernel, Method of successive approximations, Convolution type equations, Volterra equation of first kind, Abel's integral equation, Fredholm integral equations, Fredholm equations of the second kind, the method of Fredholm determinants, Iterated kernels, Integral equations with degenerate kernels, Introduction to Singular integral equations. [12]

Module VI & VII

Partial Differential Equations: Formation of P.D.E, Equations solvable by direct integration,Linear and non-linear equations of first order, Lagrange's equations, and Charpit's method, Homogeneous and non-homogeneous linear P.D.E. with constant coefficients, Rules for finding C.F. & P.I. Linear and quasi linear equations, Partial Differential Equations of second order with constant and variable coefficients, Classification and reduction of second order equations to canonical form, Cauchy's, Neumann and Dirichlet's problems, Solution of Laplace and Poisson's equations in two and three dimensions by variable separable method, Solution of wave equation and unsteady heat equation in homogeneous, non-homogeneous cases. [12]

Text Books:

1. The use of integral Transforms -**I.N. Sneddon**, TATA McGraw-Hill
2. Elements of Partial Differential Equations-**I.N. Sneddon** -Dover Publications
3. Simmons G.F., Differential Equations with Applications and Historical Notes, TMH, 2nd ed.,1991.
4. Zill, Differential Equations, Thomson Learning, 5th ed., 2004
5. **F H Miller,** Partial Differential Equations **--** J. Wiley & Sons, Inc.