

SOLID LIQUID SEPARATION BY FILTRATION

- Module VI
- Filtration: Theory of solid-liquid filtration, principle of filtration, constant pressure and constant rate filtration, compressible and incompressible cakes, Filter aids, Equipment of liquid solid filtration, Batch and continuous pressure filters. Theory of centrifugal filtration, Equipment for centrifugal filtration.

FILTRATION

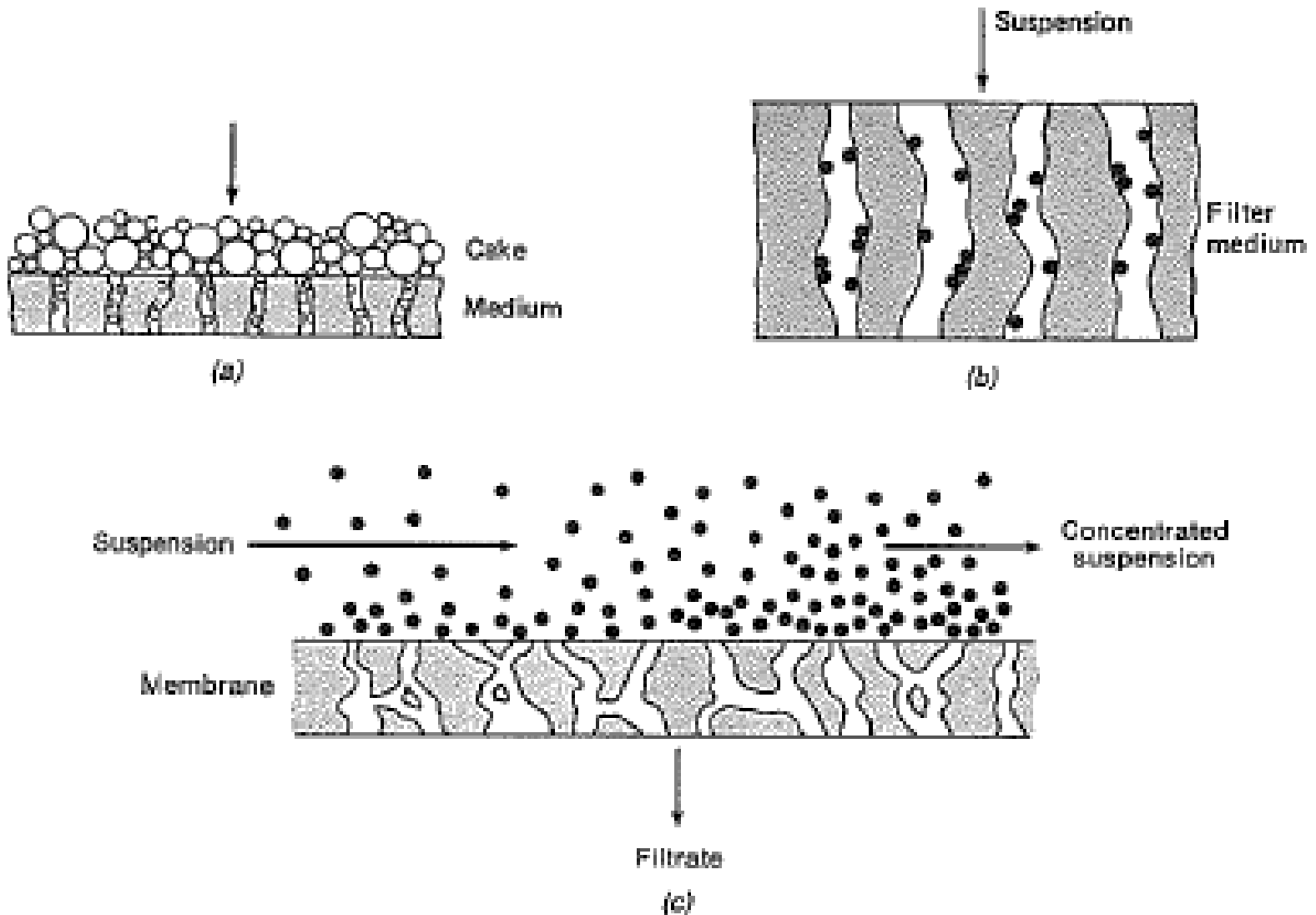
- Process of removal of solid particles from a fluid by passing through a filtering medium or septum.
- Requirement of the filter medium:
 1. It must retain the solids to be filtered
 2. It must not plug or blind
 3. Chemically resistant, physically strong to withstand process conditions
 4. Permit clean and complete discharge of cake
 5. Not prohibitively expensive
- Examples: Canvas cloth either duck or twill , woolen cloth, metal cloth of monel or SS, glass cloth, paper, nylon, Polypropylene, polyesters

FILTER AIDS

- Filter aids plugging of filter medium by slimy or very fine solids.
- Examples of filter aids: diatomaceous silica, perlite, purified wood cellulose any inert porous solid
- Methods of application
 1. Mixed with slurry
 2. Precoating the filter medium

MECHANISM OF FILTRATION:

(a) Cake (b) Clarifying (c) cross flow



TYPES OF FILTERS

- Bed / Indepth filter
- Plate and frame filter
press
- Shell and Leaf filter
- Rotary Drum filter

BED FILTER

- In depth FILTER

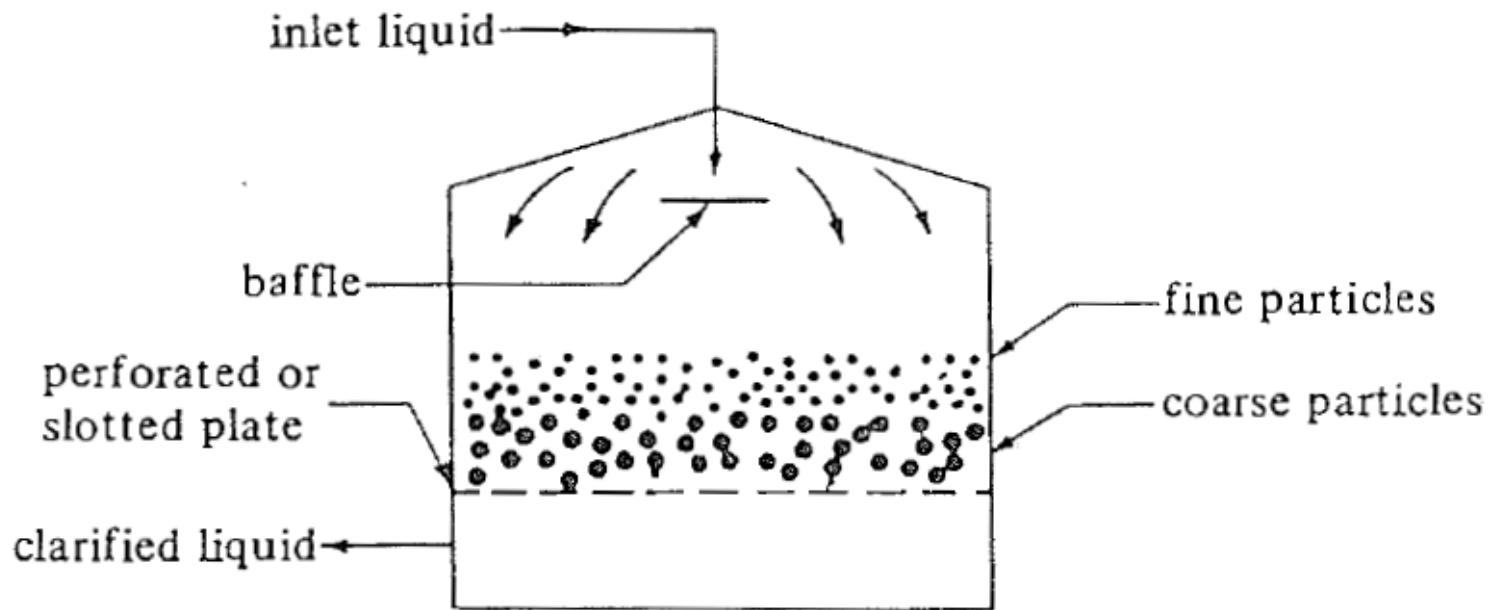


PLATE AND FRAME FILTER PRESS

- A discontinuous pressure filter
- Plates can be square or circular, vertical or horizontal
- 15cm to 2m on a side
- Plates 6 to 50 mm thick
- Frames 6 to 200 mm thick
- Slurry flows at pressure 3 to 10 atm till the frames are full or “Jammed”
- Washing done by Simple wash or Thorough wash
- Drying with steam or air to remove residual liquids

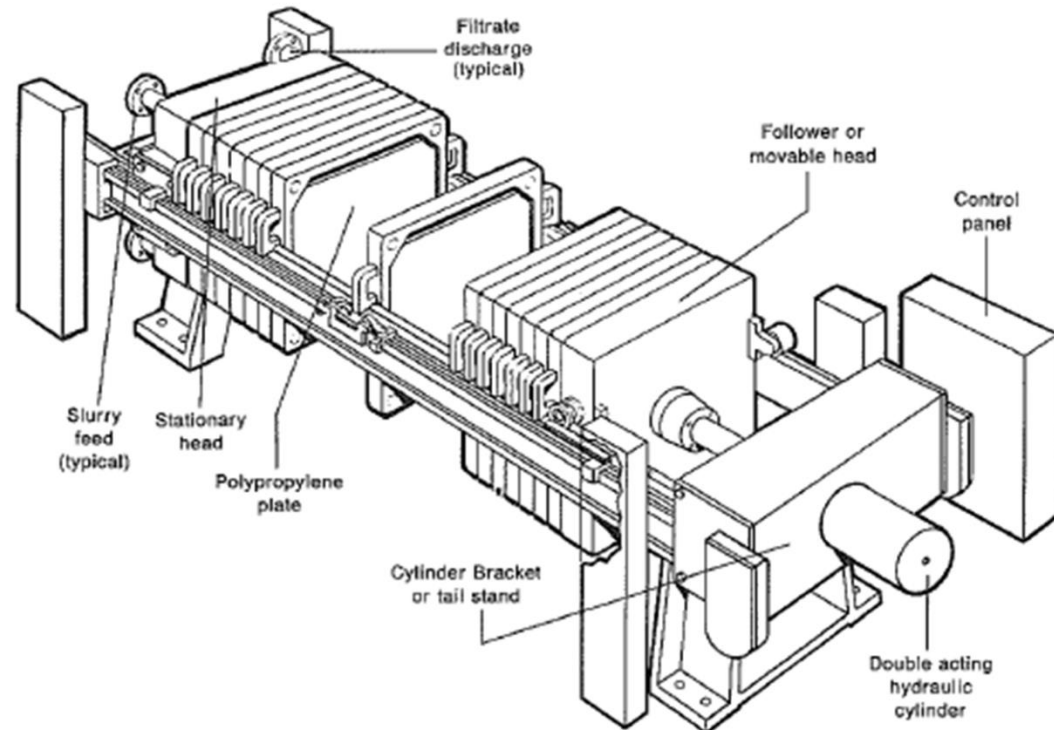
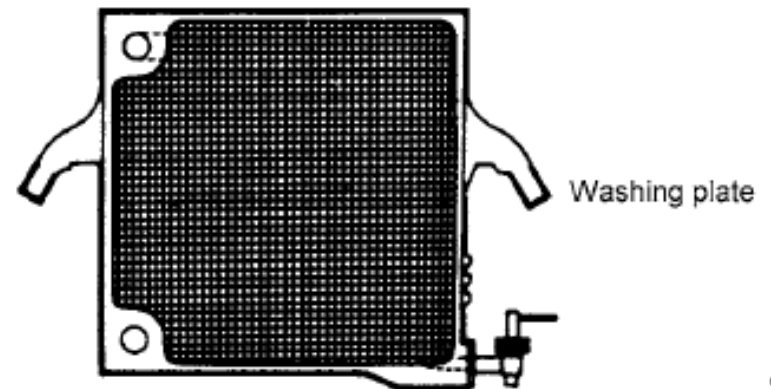
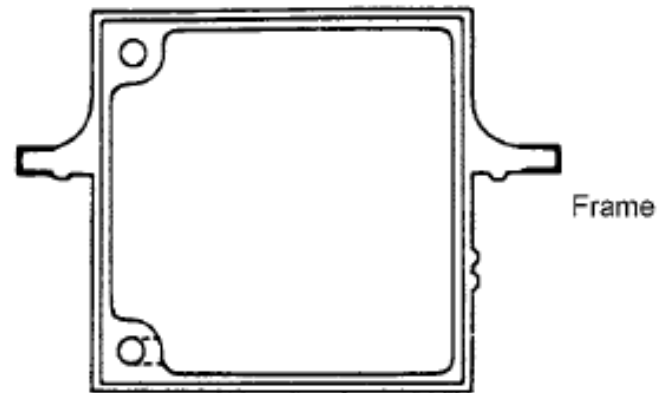
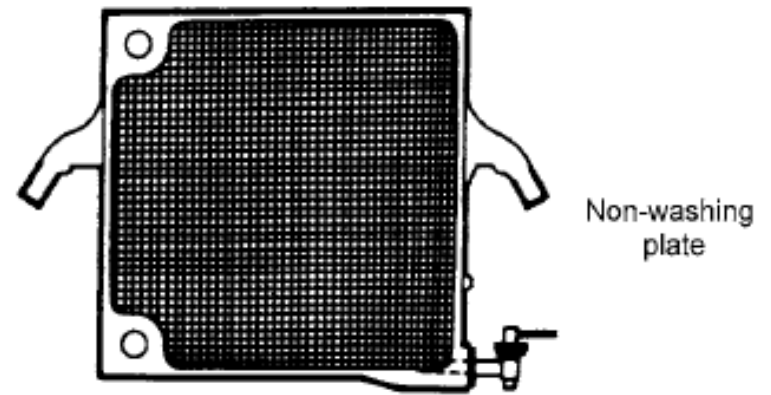
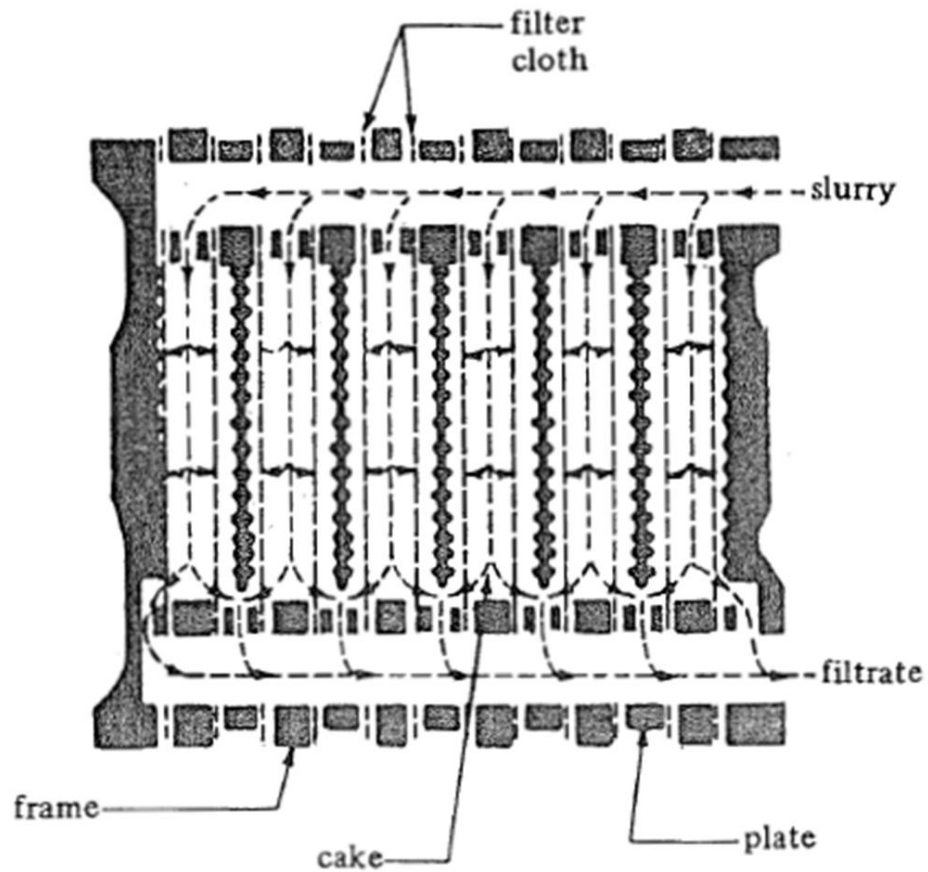
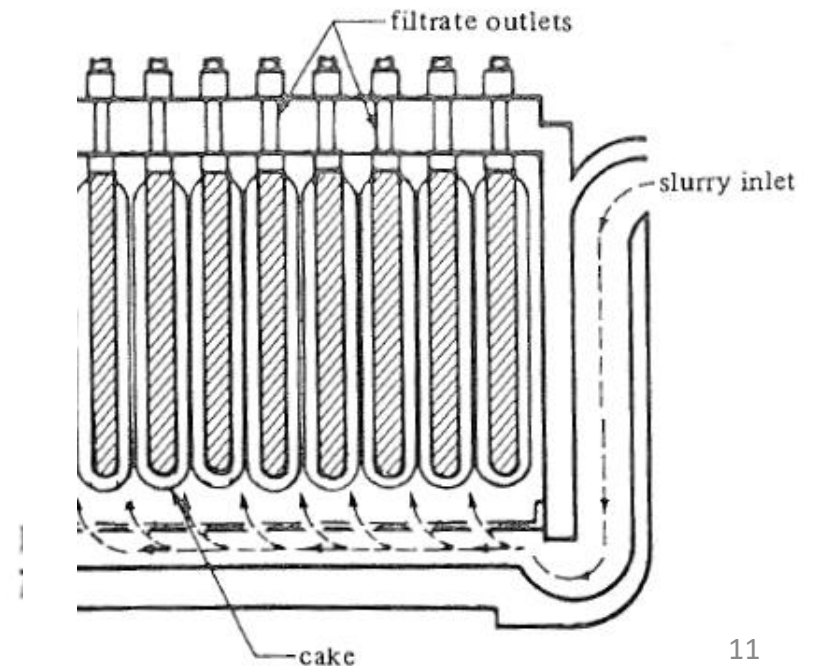
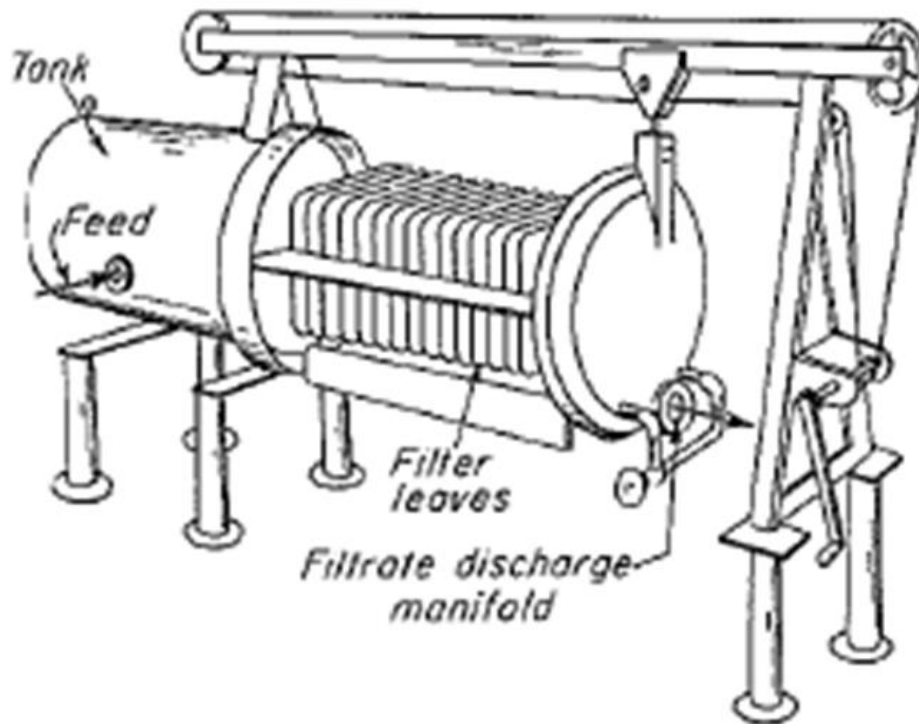


PLATE AND FRAME FILTER PRESS



Shell and Leaf filter

- Higher pressure than plate & Frame press



Pressure drop in Cake Filtration

- $\Delta p = p_a - p_b = (p_a - p') + (p' - p_b) = \Delta p_c + \Delta p_m$
- Δp = Overall pressure drop
- Δp_c = Pressure drop across cake
- Δp_m = Pressure drop across filter medium

$$\frac{dp}{dL} = \frac{150\mu u(1 - \varepsilon)^2}{(\varphi_s D_p)^2 \varepsilon^3}$$

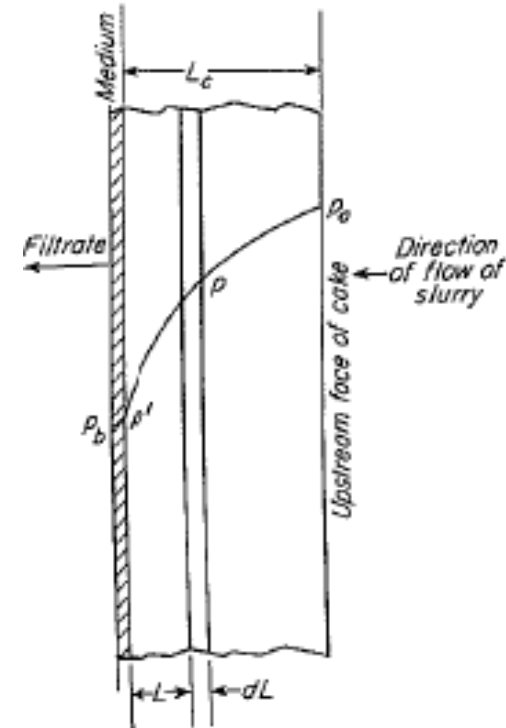
$$\left[\varphi_s = \frac{6v_p}{D_p s_p} \right]$$

$$\frac{dp}{dL} = \frac{4.17\mu u(1 - \varepsilon)^2 (s_p/v_p)^2}{\varepsilon^3}$$

dp/dL = pressure drop across cake

μ = viscosity of filtrate, u = velocity of filtrate

s_p and v_p = surface and volume of a particle, ε = porosity of cake



Hagen Poiseuille equation

$$\frac{\Delta p}{L} = \frac{32V\mu}{D^2}$$

Equivalent channel diameter D_{eq}

Let, Cross sectional area = S_o , depth of bed = L , number of Channels = n

- Surface area of n channels

$$= \frac{\text{Surface of particles}}{\text{Volume of particles}} \text{Volume of particles in bed}$$
- Surface of channels :

$$\pi D_{eq} n L = \frac{6}{\phi_s D_p} S_o L (1 - \epsilon) \quad \text{and}$$

Volume of channels:

$$S_o L \epsilon = \frac{1}{4} \pi D_{eq}^2 n L$$

$$\pi D_{eq} n L = \frac{6}{\phi_s D_p} \frac{1}{4 \epsilon} \pi D_{eq}^2 n L (1 - \epsilon)$$

$$D_{eq} = \frac{2}{3} \phi_s D_p \frac{\epsilon}{(1 - \epsilon)}$$

Velocity of the fluid through channel, $V = \frac{V_o}{\epsilon}$

$V_o = \text{vel. in empty column}$

$$\frac{\Delta p}{L} = \frac{32 V_o \mu}{\epsilon} \frac{9 (1 - \epsilon)^2}{4 \phi_s^2 D_p^2 \epsilon^2} = \frac{72 \lambda_1 V_o \mu (1 - \epsilon)^2}{\phi_s^2 D_p^2 \epsilon^3} \cong \frac{150 \mu u (1 - \epsilon)^2}{(\phi_s D_p)^2 \epsilon^3}$$

$$\frac{dp}{dL} = \frac{150\mu u(1 - \varepsilon)^2}{(\varphi_s D_p)^2 \varepsilon^3}$$

150 is empirical considering a correction factor for tortuous path. $\left[\varphi_s = \frac{6v_p}{D_p s_p} \right]$

$$\frac{dp}{dL} = \frac{4.17\mu u(1 - \varepsilon)^2 (s_p/v_p)^2}{\varepsilon^3}$$

dp/dL = pressure drop across cake, μ = *viscosity of filtrate*, u = *velocity of filtrate*

s_p and v_p = surface and volume of a particle, ε = *porosity of cake*

PRESSURE DROP IN CAKE FILTRATION

$$\Delta p = p_a - p_b = (p_a - p') + (p' - p_b)$$

$$= \Delta p_c + \Delta p_m$$

- Δp = Overall pressure drop
- Δp_c = Pressure drop across cake
- Δp_m = Pressure drop across filter medium

Pressure Drop through cake:

$$\frac{dp}{dL} = \frac{4.17\mu u(1 - \varepsilon)^2 (s_p/v_p)^2}{\varepsilon^3}$$

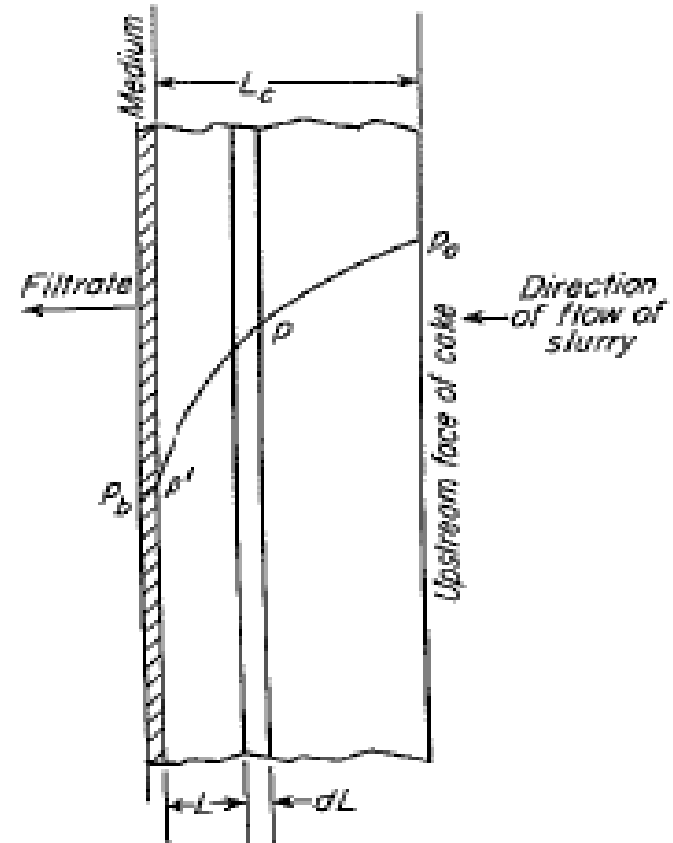
dp/dL = pressure drop across cake

μ = viscosity of filtrate,

u = velocity of filtrate

s_p and v_p = surface and volume of a particle

ε = porosity of cake



Pressure Drop through cake

- $$\frac{dp}{dL} = \frac{4.17\mu u(1-\varepsilon)^2 (s_p/v_p)^2}{\varepsilon^3}$$

- **Linear velocity, u :**
$$u = \frac{dV/dt}{A}$$

- V = volume of liquid collected in time t

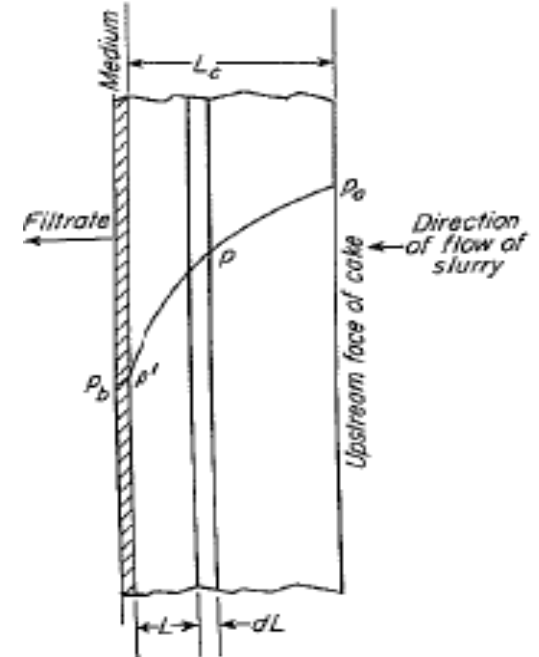
- A = area of filter

- **Mass dm in the cake of thickness dL**

- $$dm = \rho_p(1-\varepsilon)AdL \text{ or } dL = \frac{dm}{\rho_p(1-\varepsilon)A}$$

- $$dp = \frac{4.17\mu u(1-\varepsilon)^2 (s_p/v_p)^2}{\varepsilon^3} dL = \frac{4.17\mu u(1-\varepsilon)^2 (s_p/v_p)^2}{\varepsilon^3} \frac{dm}{\rho_p(1-\varepsilon)A}$$

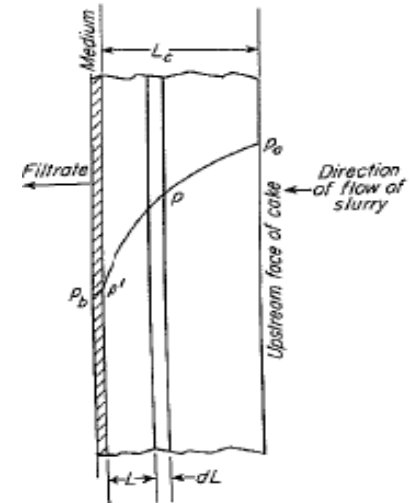
- $$dp = \frac{k_1 \mu u(1-\varepsilon) (s_p/v_p)^2}{\varepsilon^3 \rho_p A} dm$$



Pressure drop for Incompressible Cake: Specific Cake Resistance

- $$dp = \frac{k_1 \mu u (1-\varepsilon) (s_p/v_p)^2}{\varepsilon^3 \rho_p A} dm$$
- $$\int_{p'}^{p_a} dp = dp = \frac{k_1 \mu u (1-\varepsilon) (s_p/v_p)^2}{\varepsilon^3 \rho_p A} \int_0^{m_c} dm$$
- $$p_a - p' = \frac{k_1 \mu u (1-\varepsilon) (s_p/v_p)^2 m_c}{\varepsilon^3 \rho_p A} = \Delta p_c$$
- $$\Delta p_c = \frac{k_1 \mu u (1-\varepsilon) (s_p/v_p)^2 m_c}{\varepsilon^3 \rho_p A} = \frac{k_1 (1-\varepsilon) (s_p/v_p)^2}{\varepsilon^3 \rho_p} \frac{\mu u m_c}{A}$$

$$\Delta p_c = \frac{\alpha \mu u m_c}{A} \quad \text{and} \quad \alpha = \frac{A \Delta p_c}{\mu u m_c}$$
- The **specific cake resistance**, $\alpha = \frac{k_1 (1-\varepsilon) (s_p/v_p)^2}{\varepsilon^3 \rho_p}$
- Dimension of α [LM^{-1}], unit : m/kg



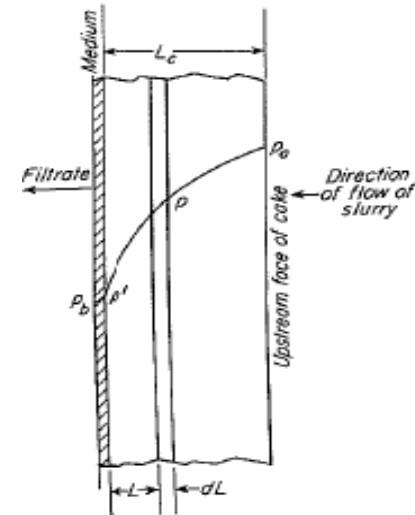
Filter medium resistance and overall resistance

$$\Delta p = (p_a - p') + (p' - p_b) = \Delta p_c + \Delta p_m$$

By analogy, $(p' - p_b) = R_m \mu u$,

Filter medium resistance, $R_m = \frac{\Delta p_m}{\mu u}$,

Dimension and unit of : L^{-1} and m^{-1}



$$\Delta p = \Delta p_c + \Delta p_m = \frac{\alpha \mu u m_c}{A} + R_m \mu u = \mu u \left(\frac{\alpha m_c}{A} + R_m \right)$$

If c is the mass of particles deposited in filter per unit volume of filtrate,

V = volume of filtrate collected in time t

m_c = mass of solids in the filter at time t is Vc and

$$m_c = Vc, \text{ and velocity of filtrate: } u = \frac{dV/dt}{A}$$

$$\Delta p = \mu u \left(\frac{\alpha m_c}{A} + R_m \right) = \mu \frac{dV/dt}{A} \left(\frac{\alpha m_c}{A} + R_m \right)$$

$$\frac{dt}{dV} = \frac{\mu}{\Delta p A} \left(\frac{\alpha m_c}{A} + R_m \right)$$

Mass of particles deposited in the filter per unit volume of filtrate, as function of dry and wet cake weight

c = mass of particles deposited in the filter per unit volume of filtrate

c_F = mass of solid in the feed per unit volume of liquid fed,

V = volume of filtrate collected in time t , V_F = volume of liquid fed in time t

m_c = weight of dry cake, m_F = weight of wet cake, ρ = density of filtrate

- $m_c = V c = V_F c_F$
- $m_F = m_c + (V_F - V)\rho = m_c + \left(V \frac{c}{c_F} - V\right)\rho$
- $m_F = m_c + \left(\frac{c}{c_F} - 1\right) \frac{m_c}{c} \rho$
- $\frac{m_F}{m_c} = 1 + \frac{\rho}{c_F} - \frac{\rho}{c}$ or $\frac{m_F}{m_c} - 1 = \rho \left(\frac{1}{c_F} - \frac{1}{c}\right)$
- $\left(\frac{m_F}{m_c} - 1\right) \frac{1}{\rho} = \frac{1}{c_F} - \frac{1}{c}$ or $\frac{1}{c} = \frac{1}{c_F} - \left(\frac{m_F}{m_c} - 1\right) \frac{1}{\rho}$

$$c = \frac{1}{\frac{1}{c_F} - \left(\frac{m_F}{m_c} - 1\right) \frac{1}{\rho}} = \frac{c_F}{1 - \left(\frac{m_F}{m_c} - 1\right) \frac{c_F}{\rho}}$$

CONSTANT PRESSURE FILTRATION

- $\Delta p = \text{constant}$, and V varies with time t
- $m_c = Vc$, and velocity of filtrate: $u = \frac{dV/dt}{A}$
- $\frac{dt}{dV} = \frac{\mu}{\Delta p A} \left(\frac{\alpha m_c}{A} + R_m \right)$ Noting: At $t=0$, $V=0$, $m_c = 0$, and $\Delta p = \Delta p_m$

$$\left(\frac{dt}{dV} \right)_0 = \frac{\mu R_m}{\Delta p A} = \frac{1}{q_0}$$

- $\frac{dt}{dV} = \frac{\mu}{\Delta p A} \left(\frac{\alpha m_c}{A} + R_m \right) = \frac{\mu}{\Delta p A} \left(\frac{\alpha Vc}{A} + R_m \right)$
 $\frac{dt}{dV} = \frac{\mu \alpha Vc}{\Delta p A^2} + \frac{\mu R_m}{\Delta p A}$ or $\frac{dt}{dV} = K_c V + \frac{1}{q_0}$

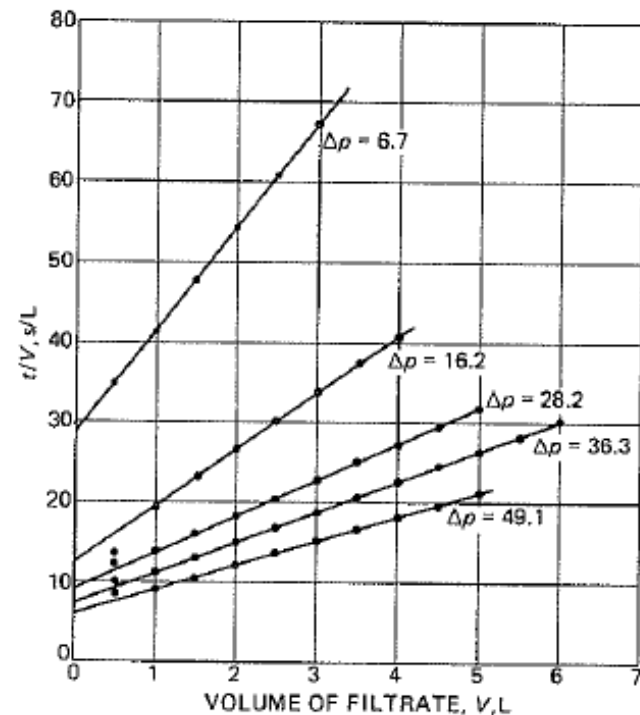
- $K_c = \frac{\mu \alpha c}{\Delta p A^2}$

- $\int_0^t dt = \int_0^V \left(K_c V + \frac{1}{q_0} \right) dV, t = K_c \frac{V^2}{2} + \frac{V}{q_0}$

$$\frac{t}{V} = \frac{K_c}{2} V + \frac{1}{q_0}$$

- $\Delta P =$; $A =$
- $C = \frac{C_F}{1 - \left(\frac{m_F}{m_C} - 1\right) \frac{C_F}{\rho}}$
- $\frac{t}{V} = \frac{K_C}{2} V + \frac{1}{q_0}$
- Plot t/V vs V
- Slope = $\frac{K_C}{2}$ where
- $K_C = \frac{\mu \alpha c}{\Delta p A^2}$
- Determine α
- Intercept = $\frac{1}{q_0} = \frac{\mu R_m}{\Delta p A}$
- Determine R_m

Mass Filtrate	Volume Filtrate, V	Time, t	t/V



COMPRESSIBLE CAKE

- For compressible cakes, the specific cake resistance, α , is not constant and varies with Δp .
- The resistance, α increases with applied Δp
- Most cakes are compressible to some extent
- Empirical equation:

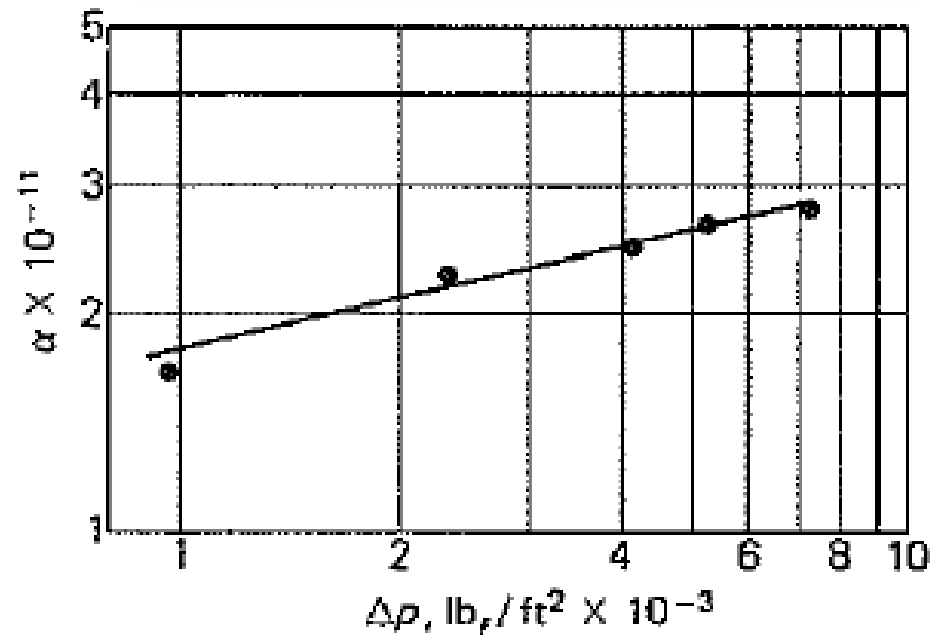
$$\alpha = \alpha_0 \Delta p^s$$

- α_0 and s are empirical constant.
- Constant “ s ” is called **compressibility coefficient** of cake
- Compressibility coefficient generally varies between 0.2 to 0.8, and is 0 for incompressible cakes

Empirical equation to determine compressibility coefficient

- $\alpha = \alpha_0 \Delta p^s$
- $\ln \alpha = \ln \alpha_0 + s \ln \Delta p$
- Prepare log log plot of
- α vs Δp
- Slope = s
- Intercept = α_0

α	Δp



Example 30.2. Laboratory filtrations conducted at constant pressure drop on a slurry of CaCO_3 in H_2O gave the data shown in Table 30.2. The filter area was 440 cm^2 , the mass of solid per unit volume of filtrate was 23.5 g/L , and the temperature was 25°C . Evaluate the quantities α and R_m as a function of pressure drop, and fit an empirical equation to the results for α .

TABLE 30.2
Volume-time data³³ for Example 30.2†

Filtrate volume $V, \text{ L}$	Test I		Test II		Test III		Test IV		Test V	
	t, s	t/V	t, s	t/V	t, s	t/V	t, s	t/V	t, s	t/V
0.5	17.3	34.6	6.8	13.6	6.3	12.6	5.0	10.0	4.4	8.8
1.0	41.3	41.3	19.0	19.0	14.0	14.0	11.5	11.5	9.5	9.5
1.5	72.0	48.0	34.6	23.1	24.2	16.13	19.8	13.2	16.3	10.87
2.0	108.3	54.15	53.4	26.7	37.0	18.5	30.1	15.05	24.6	12.3
2.5	152.1	60.84	76.0	30.4	51.7	20.68	42.5	17.0	34.7	13.88
3.0	201.7	67.23	102.0	34.0	69.0	23.0	56.8	18.7	46.1	15.0
3.5			131.2	37.49	88.8	25.37	73.0	20.87	59.0	16.86
4.0			163.0	40.75	110.0	27.5	91.2	22.8	73.6	18.4
4.5					134.0	29.78	111.0	24.67	89.4	19.87
5.0					160.0	32.0	133.0	26.6	107.3	21.46
5.5							156.8	28.51		
6.0							182.5	30.42		

31. The data in following Table were taken in a constant pressure filtration of slurry of CaCO_3 in water . The filter was a 15.24cm filter press with an area of 0.0929 m^2 . The mass fraction of the solids in the feed to the press was 0.139 . Calculate the value of α , R_m and cake thickness for each experiments. The temperature is 21.1deg C

Result-I		Result-II		Result - III		Result – IV	
Pressure Drop $35 \times 10^3 \text{ Pa}$		Pressure drop $105 \times 10^3 \text{ Pa}$		Pressure drop $210 \times 10^3 \text{ Pa}$		Pressure drop $350 \times 10^3 \text{ Pa}$	
Mass Ratio of wet cake to dry cake = 1.59		Mass Ratio of wet cake to dry cake = 1.47		Mass Ratio of wet cake to dry cake = 1.47		Mass Ratio of wet cake to dry cake = 1.47	
Dry cake density = 1021 kg/m^3		Dry cake density = 1169 kg/m^3		Dry cake density = 1169 kg/m^3		Dry cake density = 1177 kg/m^3	
Filtrate (Kg)	Time (s)	Filtarate (kg)	Time (s)	Filtrate kg	Time, s	FiltrateKg	Time,s
0	0	0	0	0	0	0	0
0.907	24	2.267	50	2.267	26	2.267	19
1.814	71	4.54	181	4.54	98	4.54	68
2.72	146	6.8	385	6.8	211	6.8	142
3.628	244	9.07	660	9.07	361	9.07	241
4.535	372	11.34	1009	11.34	555	11.34	368
3.443	524	13.6	1443	13.6	788	13.6	524
6.35	690	15.876	2117	15.876	1083	15.876	702
7.257	888						
8.164	1188						

CONSTANT RATE FILTRATION

$$u = \frac{dV/dt}{A} = \text{constant} = \frac{V}{At}$$

$$\Delta p = \mu u \left(\frac{\alpha m_c}{A} + R_m \right), \alpha = \alpha_0 \Delta p_c^s$$

Cake pressure drop

$$\Delta p_c = \frac{\mu u \alpha m_c}{A} = \frac{\mu \frac{V}{At} \alpha_0 \Delta p_c^s V c}{A}$$

$$\Delta p_c^{1-s} = (\Delta p - \Delta p_m)^{1-s} = \mu c \alpha_0 t \left(\frac{V}{At} \right)^2 = K_r t$$

Automatic Belt Filter, Larox

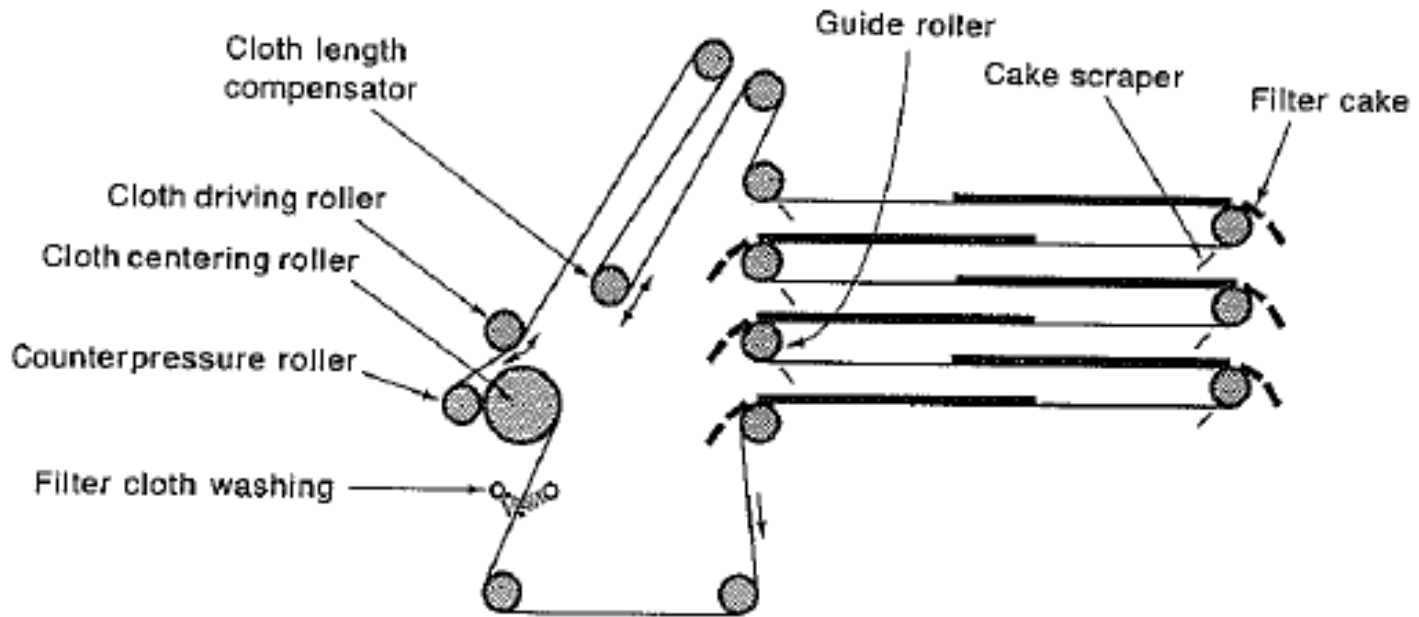
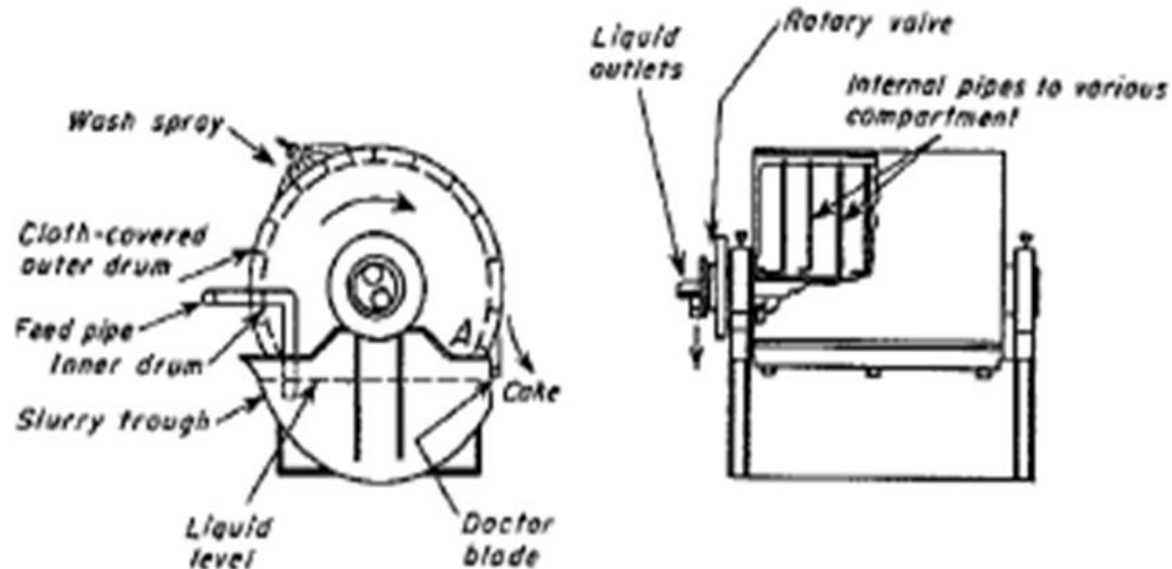


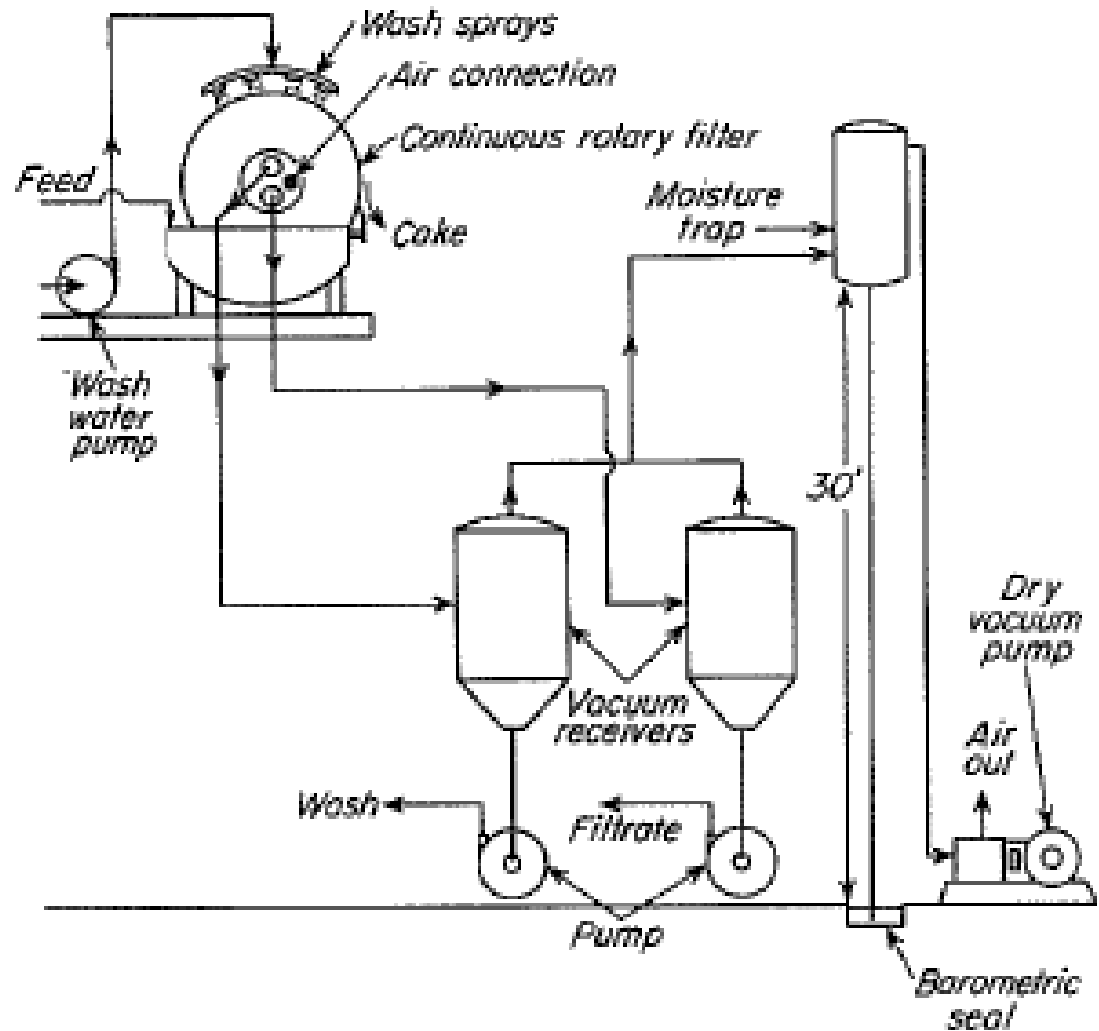
FIGURE 20.5

CONTINUOUS ROTARY VACUUM FILTER



- Horizontal drum with slotted face covered with filter medium, Inside there is a drum with solid surface,
- The annular space divided into several compartments, each with a connecting pipe, through which vacuum, and air alternatively applied.
- 0.1 to 2 rpm partially submerged in slurry drum
- During one rotation – filtering, draining, washing drying, and cake removal by doctor blade, assisted by air blowing.

- Continuous filtration.
- Feed, filtrate and cake production are continuous.
- Pressure drop across the filter constant.



CONTINUOUS CONSTANT PRESSURE FILTRATION

$$t = K_c \frac{V^2}{2} + \frac{V}{q_0} \quad \text{where, } \frac{1}{q_0} = \frac{\mu R_m}{\Delta p A}, \text{ and } K_c = \frac{\mu \alpha c}{\Delta p A^2}$$

$$K_c \frac{V^2}{2} + \frac{V}{q_0} - t = 0$$

$$\bullet \quad V = \frac{-\frac{1}{q_0} + \sqrt{\left(\frac{1}{q_0}\right)^2 - 4 \frac{K_c}{2}(-t)}}{2 \frac{K_c}{2}} = \frac{-\frac{1}{q_0} + \sqrt{\left(\frac{1}{q_0}\right)^2 + 2 K_c t}}{K_c}$$

$$\bullet \quad V = \frac{-\frac{\mu R_m}{\Delta p A} + \sqrt{\left(\frac{\mu R_m}{\Delta p A}\right)^2 + 2 \frac{\mu \alpha c}{\Delta p A^2} t}}{\frac{\mu \alpha c}{\Delta p A^2}}$$

$$\bullet \quad \frac{V}{At} = \frac{-\frac{\mu R_m}{\Delta p A} \frac{1}{t} + \sqrt{\left(\frac{\mu R_m}{\Delta p A}\right)^2 \frac{1}{t^2} + 2 \frac{\mu \alpha c}{\Delta p A^2} \frac{1}{t^2} t}}{\frac{\mu \alpha c}{\Delta p A}} = \frac{-\frac{R_m}{1} \frac{1}{t} + \sqrt{\left(\frac{R_m}{1}\right)^2 \frac{1}{t^2} + 2 \frac{\alpha c \Delta p}{\mu t}}}{\frac{\alpha c}{1}}$$

$$\frac{V}{At} = \frac{-\frac{R_m}{t} + \sqrt{\left(\frac{R_m}{t}\right)^2 + 2 \frac{\alpha c \Delta p}{\mu t}}}{\alpha c}$$

$$\frac{V}{At} = \frac{-\frac{R_m}{t} + \sqrt{\left(\frac{R_m}{t}\right)^2 + 2\frac{\alpha c \Delta p}{\mu t}}}{\alpha c}$$

- Let $\dot{m}_c = \text{rate of solid production}$

- Drum speed = n

- Filter cycle time = $t_c = \frac{1}{n}$,

Total filter area = A_T

- Fraction of drum submerged = f

$$t = f t_c = \frac{f}{n}, \quad \dot{m}_c = c \frac{V}{t}, \quad A = f A_T$$

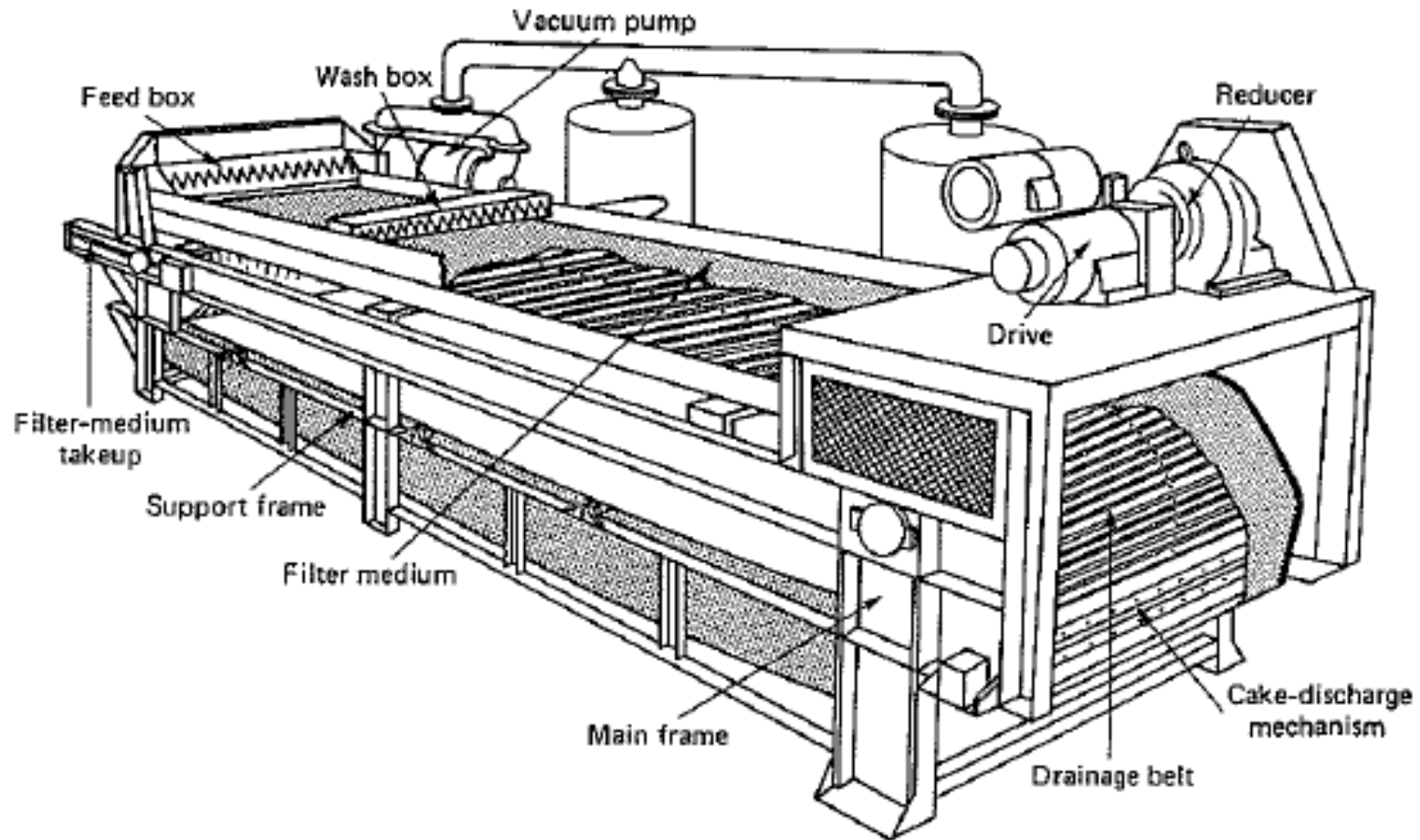
- $$\frac{V}{At} = \frac{\dot{m}_c t}{cAt} = \frac{\dot{m}_c}{cfA_T} = \frac{-\frac{R_m}{t} + \sqrt{\left(\frac{R_m}{t}\right)^2 + 2\frac{\alpha c \Delta p}{\mu t}}}{\alpha c}$$

- $$\frac{\dot{m}_c}{A_T} = \frac{-\frac{R_m f c}{f/n} + \sqrt{\left(\frac{R_m}{(f/n)}\right)^2 f^2 c^2 + 2\frac{\alpha c \Delta p}{\mu f/n} f^2 c^2}}{\alpha c}$$

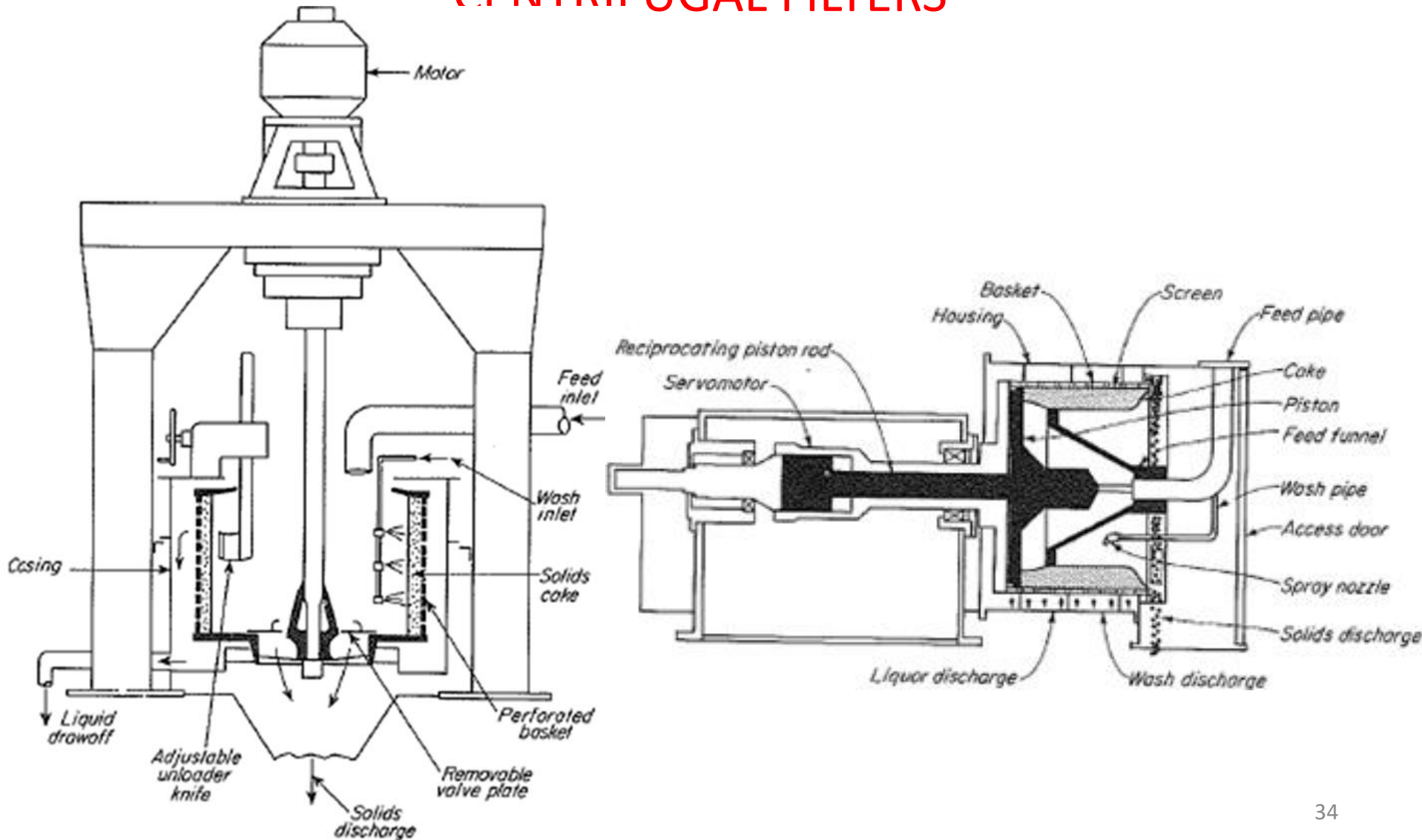
$$\dot{m}_c = \frac{-R_m n + \sqrt{(R_m n)^2 + \frac{2\alpha c \Delta p f n}{\mu}}}{\alpha c}$$

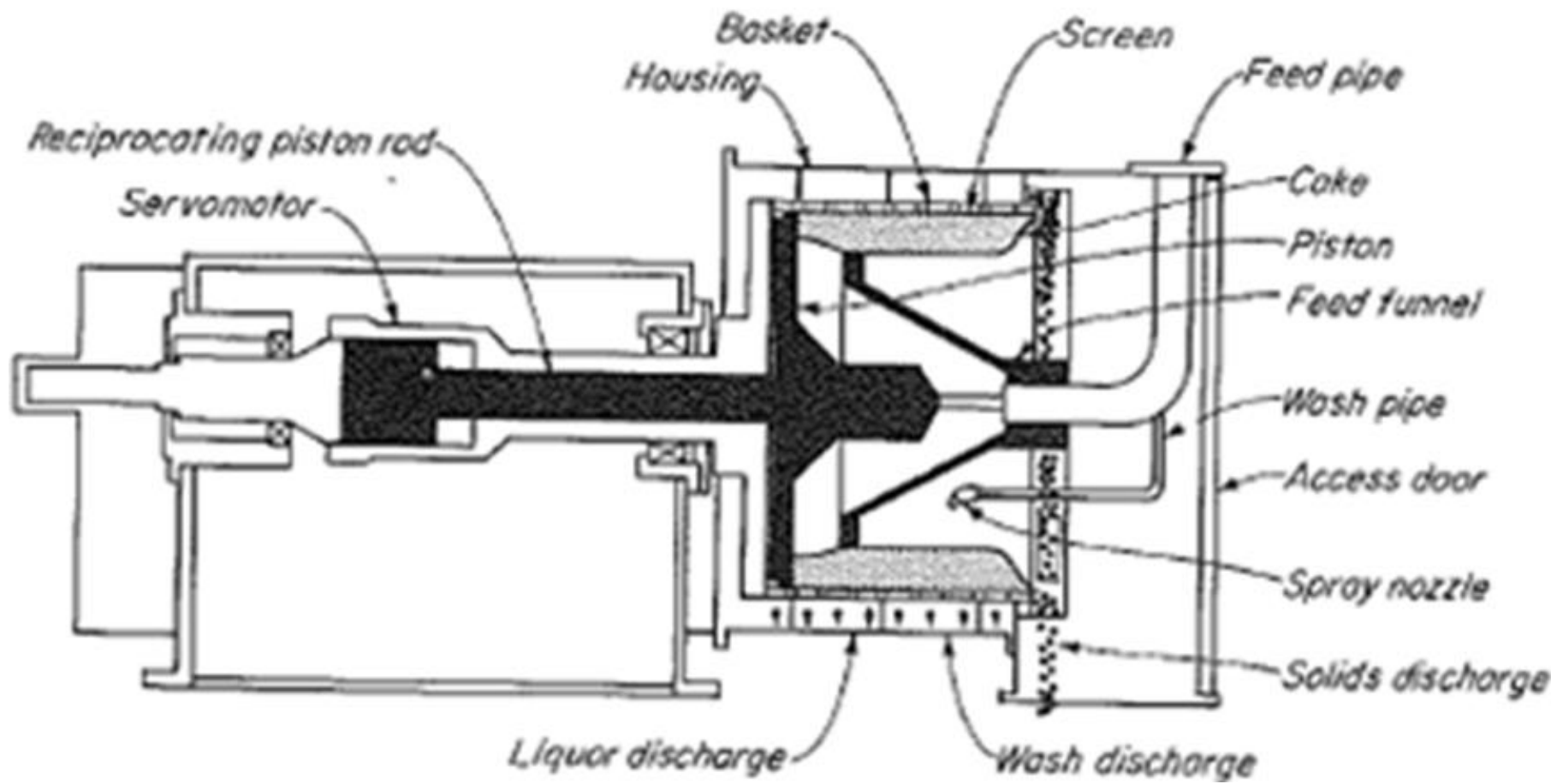
- A rotary drum filter with 30% submergence is to be used to filter a concentrated aqueous slurry of CaCO_3 containing 236 kg of solids per m^3 of water. The pressure drop is to be $6.9 \times 10^4 \text{ N/m}^2$. If the filter cake contains 50% (wet basis) moisture, calculate the filter area required to filter $0.04 \text{ m}^3/\text{min}$ of slurry when the filter cycle time is minute. Assume that the specific cake resistance is $2 \times 10^{10} \text{ m/kg}$ and filter medium resistance is negligible. The temperature is 20°C .
- Viscosity of filtrate = $1 \times 10^{-3} \text{ Pa}\cdot\text{s}$. Density = 1000 kg/m^3

Horizontal Belt filter



BATCH AND RECIPROCATING-CONVEYOR CONTINUOUS CENTRIFUGAL FILTERS





Centrifugal filtration : Constant Pressure, incompressible cake

$$u = \frac{dV/dt}{A} = \frac{q}{A}$$

$$\Delta p = \mu u \left(\frac{\alpha m_c}{A} + R_m \right)$$

$$\Delta p = \mu q \left(\frac{\alpha m_c}{A^2} + \frac{R_m}{A} \right)$$

$$p_2 - p_1 = \Delta p = \frac{\omega^2 \rho (r_2^2 - r_1^2)}{2}$$

$$q = \frac{\omega^2 \rho (r_2^2 - r_1^2)}{2\mu \left(\frac{\alpha m_c}{A^2} + \frac{R_m}{A} \right)}$$

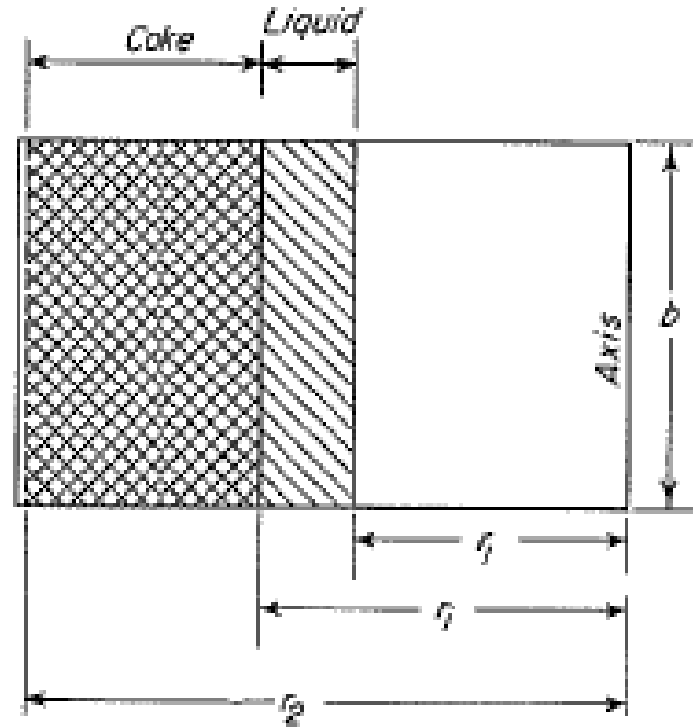
For thick cake:

$$q = \frac{\omega^2 \rho (r_2^2 - r_1^2)}{2\mu \left(\frac{\alpha m_c}{\bar{A}_a \bar{A}_l} + \frac{R_m}{A_2} \right)}$$

Arithmetic mean area,

$$A_a = (r_1 + r_2)\pi b$$

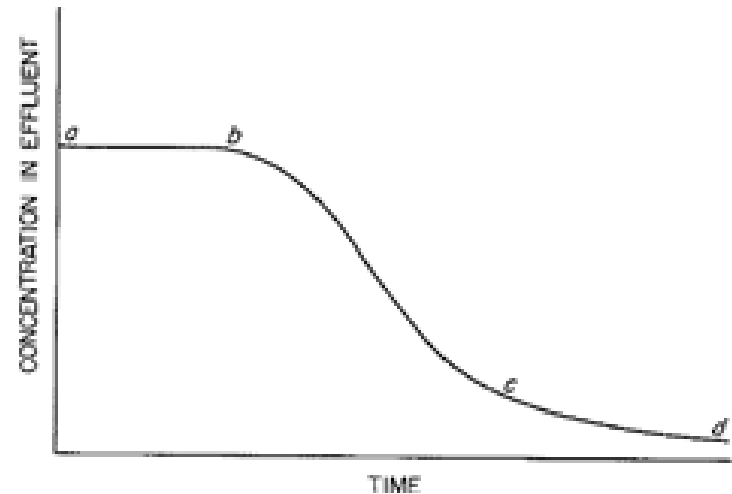
$$\text{Log mean area } A_l = \frac{2\pi b (r_2 - r_1)}{\ln(r_2/r_1)}$$



- $r_1 =$ radius of the inner surface of liquid
- $r_i =$ radius of inner face of cake
- $r_2 =$ inside radius of basket

Washing of filter cake

- If cake is product:
 - To remove solutes
- If cake is waste product
 - To remove valuable solute
- Methods :
- Cake to be uniformly sprayed with water
- The wash water
- Easy for rotary drum filter and Centrifugal filter
- Difficult for Plate and Frame and Leaf filters – simple or thorough wash can be done/ only half filled frames recommended not jamming
- Best washing: cake to be broken up, re-suspended and re-filtered



- a-b – displacement wash: volume of water = $A\varepsilon L$
90% removal
- b – c
Generally volume same as displacement wash
- c-d - very slow process, washing continued till the value of recovered solute more than cost of washing