SETTLING VELOCITY OF PARTICLES

Equation for one-dimensional motion of particle through fluid

- Expression for acceleration of a particle settling in a fluid:

$$m\frac{du}{dt} = F_e - F_b - F_D$$

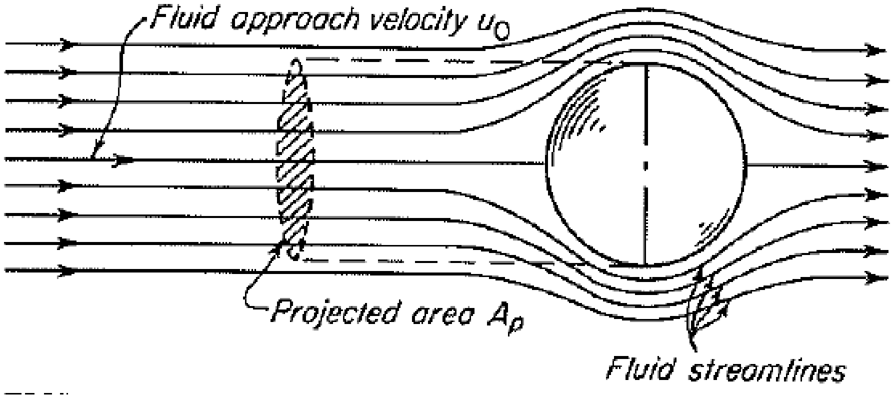
Where , F_e= ma_e

- acceleration in external field = a_e

=g in gravity settling, and $\omega^2 r$in centrifugal field

- F_b =Buoyant force =$\frac{m}{\rho_p}\rho$
- F_D= Drag force, $F_D = \frac{C_D \rho u^2 A_p}{2}$
- CD = Drag coefficient, Ap=Projected area
- F_D always increases with velocity, and soon acceleration becomes 0.
- Terminal velocity is the constant velocity the particle attains when acceleration becomes 0.

DRAG COEFFICIENT & TERMINAL VELOCITY



- $C_D = \dfrac{F_D/A_p}{\rho {u_0}^2/2}$
- $m\dfrac{du}{dt} = F_e - F_b - F_D = ma_e - \dfrac{m}{\rho_p}\rho a_e - \dfrac{C_D\rho u^2 A_p}{2}$
- $\dfrac{du}{dt} = a_e - \dfrac{1}{\rho_p}\rho a_e - \dfrac{C_D\rho u^2 A_p}{2m}$
- $a_e\left(1 - \dfrac{\rho}{\rho_p}\right) = \dfrac{C_D\rho u^2 A_p}{2m}$
- $u^2 = \dfrac{2mg(\rho_p-\rho)}{C_D\rho\rho_p A_p} = \dfrac{2\left(\frac{\pi}{6}{D_p}^3\rho_p\right)g(\rho_p-\rho)}{C_D\rho\rho_p\left(\frac{\pi {D_p}^2}{4}\right)} = \dfrac{4D_p g(\rho_p-\rho)}{3C_D\rho}$
- $u = \sqrt{\dfrac{4D_p g(\rho_p-\rho)}{3C_D\rho}}$

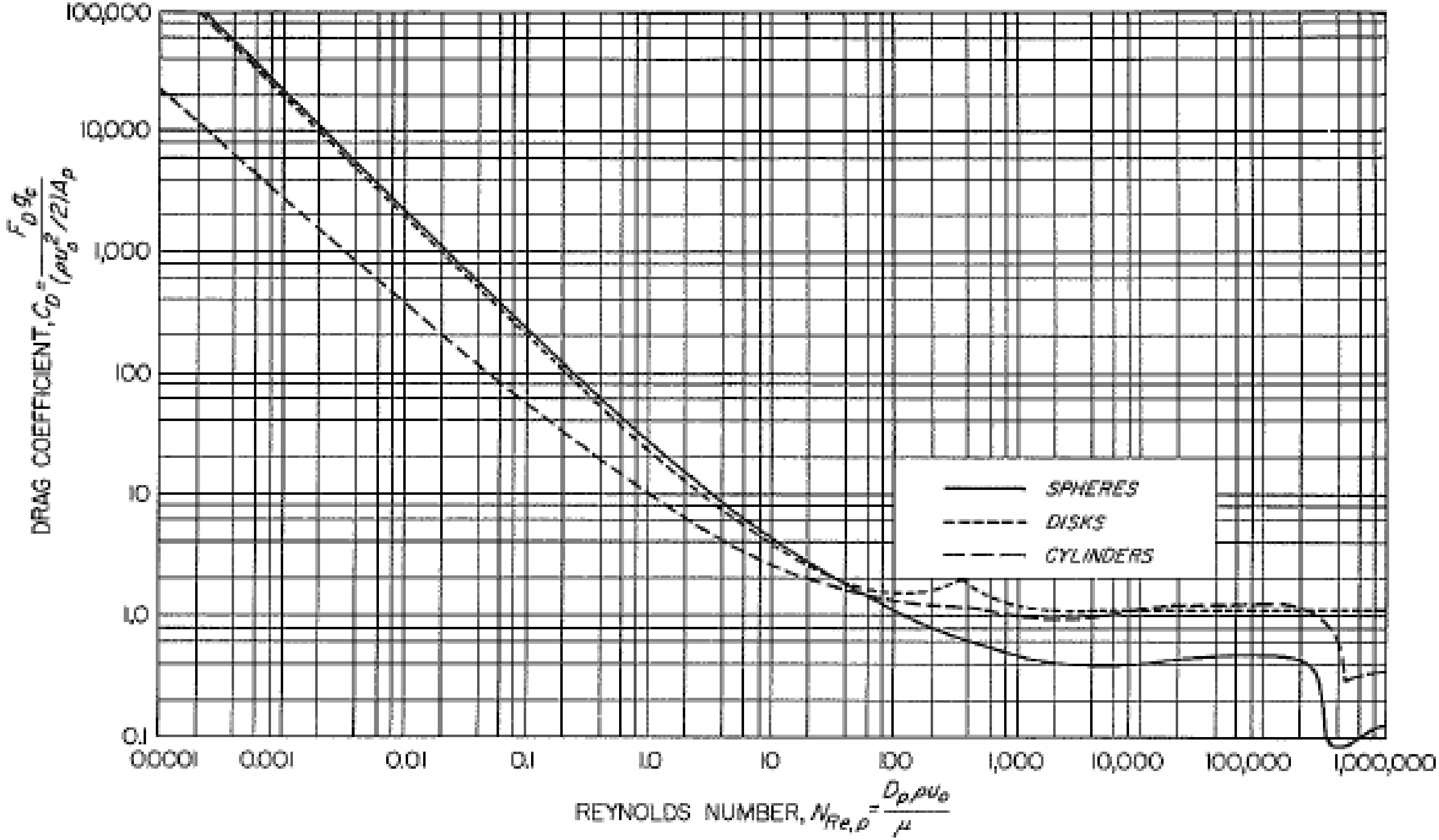


FIGURE 7.3

Drag coefficients for spheres, disks, and cylinders. [*By permission from J. H. Perry (ed.), Chemical Engineers' Handbook, 6th ed., p. 5-64. Copyright, © 1984, McGraw-Hill Book Company.*]

Drag Coefficients for spheres

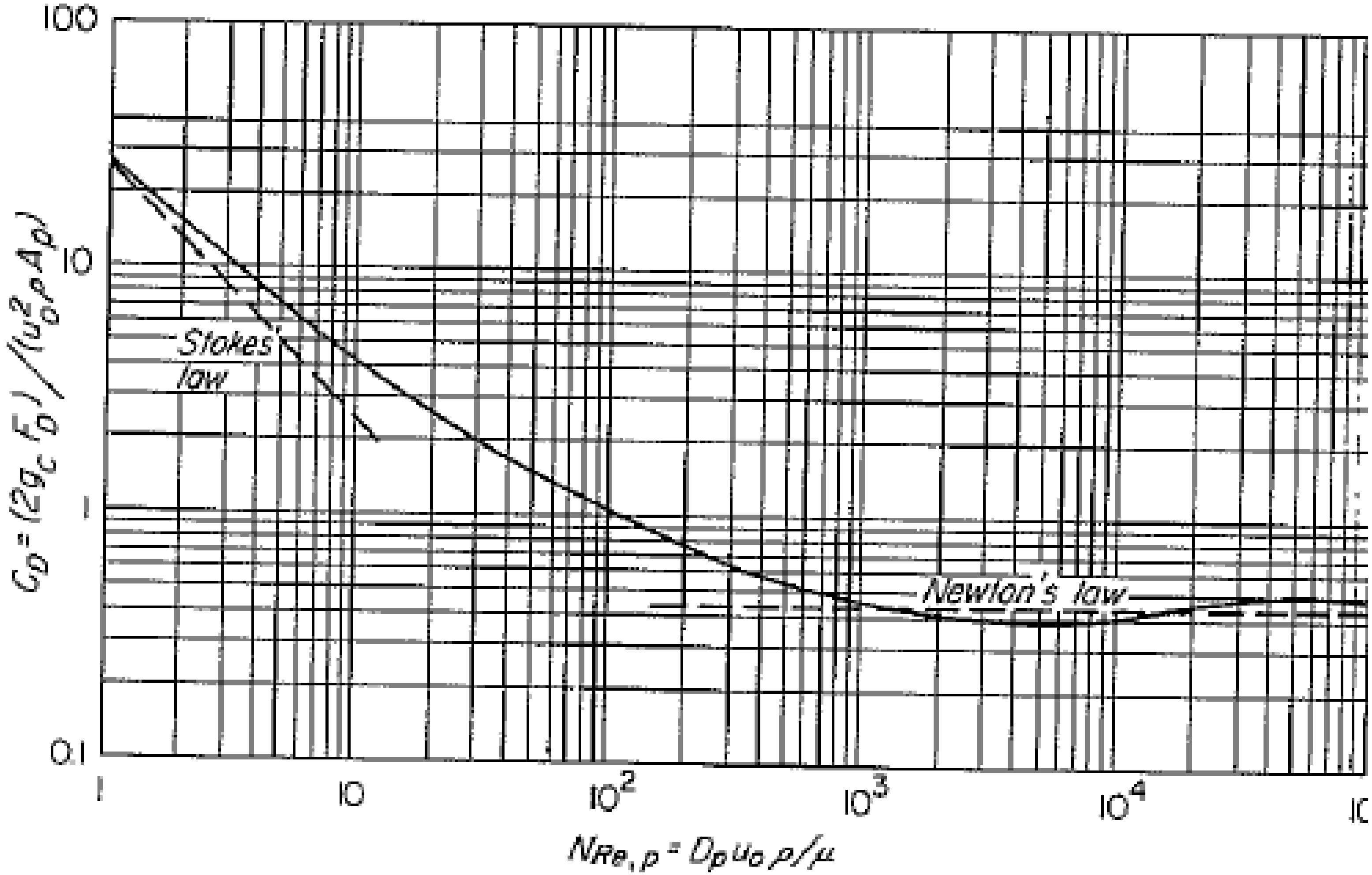


FIGURE 7.6

Trial and error method for determination of terminal velocity

$$\boldsymbol{u = \sqrt{\frac{4D_p g(\rho_p - \rho)}{3C_D\rho}}} \qquad \dots.(\mathbf{1})$$

1. Assume a value of u
2. Calculate Reynolds number of particle, $N_{Re,p}$
3. $N_{Re,p} = \frac{D_p u \rho}{\mu}$
   - $D_p = Diameter\ of\ particle, u = velocity\ of\ particle,$
   - ρ = density of fluid, μ= viscosity of fluid
4. Determine $\boldsymbol{C_D}$ from chart of $\boldsymbol{C_D}\ vs\ N_{Re,p}$
5. Calculate u from equation 1
6. Compare Calculated u with Assumed u, if error is not within limit restart from step 1 for second trial

Terminal velocity in Stokes Law range and Newtons law range

$$u_t = \sqrt{\frac{4g(\rho_p - \rho)D_p}{3C_D\rho}}$$

- Stokes Law range, particle Reynolds number less than 1.0

$$C_D = \frac{24}{N_{Re,p}}, \quad \text{where} N_{Re,p} = \frac{D_p u_o \rho}{\mu}$$

$$u_t = \frac{g{D_p}^2(\rho_p - \rho)}{18\mu}$$

- Newtons Law Range: 1000<$N_{Re,p}$<200,000,

$$C_D = 0.44$$

$$u_t = 1.75\sqrt{\frac{gD_p(\rho_p - \rho)}{\rho}}$$

- $u_t = \sqrt{\frac{4g\left(\rho_p - \rho\right)D_p}{3C_D\rho}}$
- $u_t = \sqrt{\frac{4g\left(\rho_p - \rho\right)D_p}{3\frac{24}{\frac{D_p u_o \rho}{\mu}}\rho}} = \sqrt{\frac{{D_p}^2 g\left(\rho_p - \rho\right)u_t}{18\mu}}$
- $u_t = \frac{{D_p}^2 g\left(\rho_p - \rho\right)}{18\mu}$

Determination of Range of settling

- Determine the value of K

$$K = D_p \left[\frac{g\rho\left(\rho_p - \rho\right)}{\mu^2} \right]^{1/3}$$

- Stokes Law range K<2.6
- Newtons Law range K>68.9
- Intermediate range K greater than 2.6 and less than 68.9

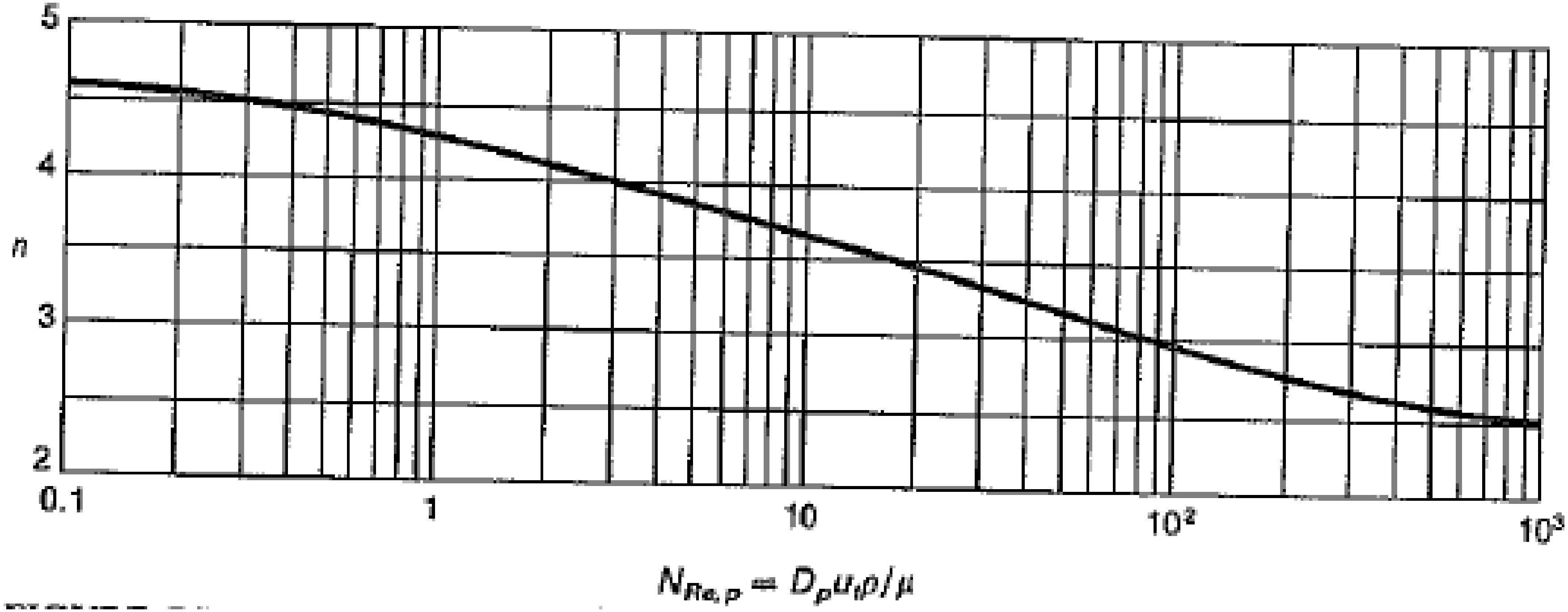
- *For Stokes Law range:*
- $Re_p = \frac{D_p u_t \rho}{\mu} = \frac{D_p \rho}{\mu} \frac{{D_p}^2 g(\rho_p - \rho)}{18\mu} = \frac{{D_p}^3 \rho g(\rho_p - \rho)}{18\mu^2} < 1$
- Let $K = D_p \left[\frac{\rho g(\rho_p - \rho)}{\mu^2}\right]^{1/3}$
- $\frac{1}{18} K^3 < 1, \quad or\ K < 18^{1/3} = 2.6$
- For Newtons law range
- $Re_p = \frac{D_p u_t \rho}{\mu} = \frac{D_p \rho}{\mu} 1.75 \sqrt{\frac{g D_p(\rho_p - \rho)}{\rho}} = 1.75 {D_p}^{1.5} \left[\frac{\rho g(\rho_p - \rho)}{\mu^2}\right]^{\frac{3}{2} \cdot \frac{1}{3}} = 1.75 K^{1.5}$
- $1.75 K^{1.5} > 1000$
- $K > 68.9$

- Estimate the terminal velocity of 80/100 mesh particles of limestone (density 2800kg/m3) settling in water at 30oC. Viscosity =0.801cP

- Determine K, to find range of settling,
  - If K is not much larger than 2.6 Stokes law can be assumed and rechecked
  - If K is not much less than 68.9 Newtons law can be assumed
  - If K is in Intermediate range, trial and erro method to be followed

Hindered settling velocity

- $u_s = u_t(\varepsilon)^n$ where $\varepsilon = porosity$



- Particles of sphalerite (sp. Gr. 4.00) are settling under the force of gravity in the carbon tetrachloride (CCl_4) at 20oC(sp.gr. 1.594). The diameter of the sphalerite particles is 0.1 mm. The free settling terminal velocity is 0.015m/s. The volume fraction of sphalerite in carbon tetrachloride is 0.2. What is the settling velocity?

Solid Liquid Separation
Settling – Gravity and Centrifugal

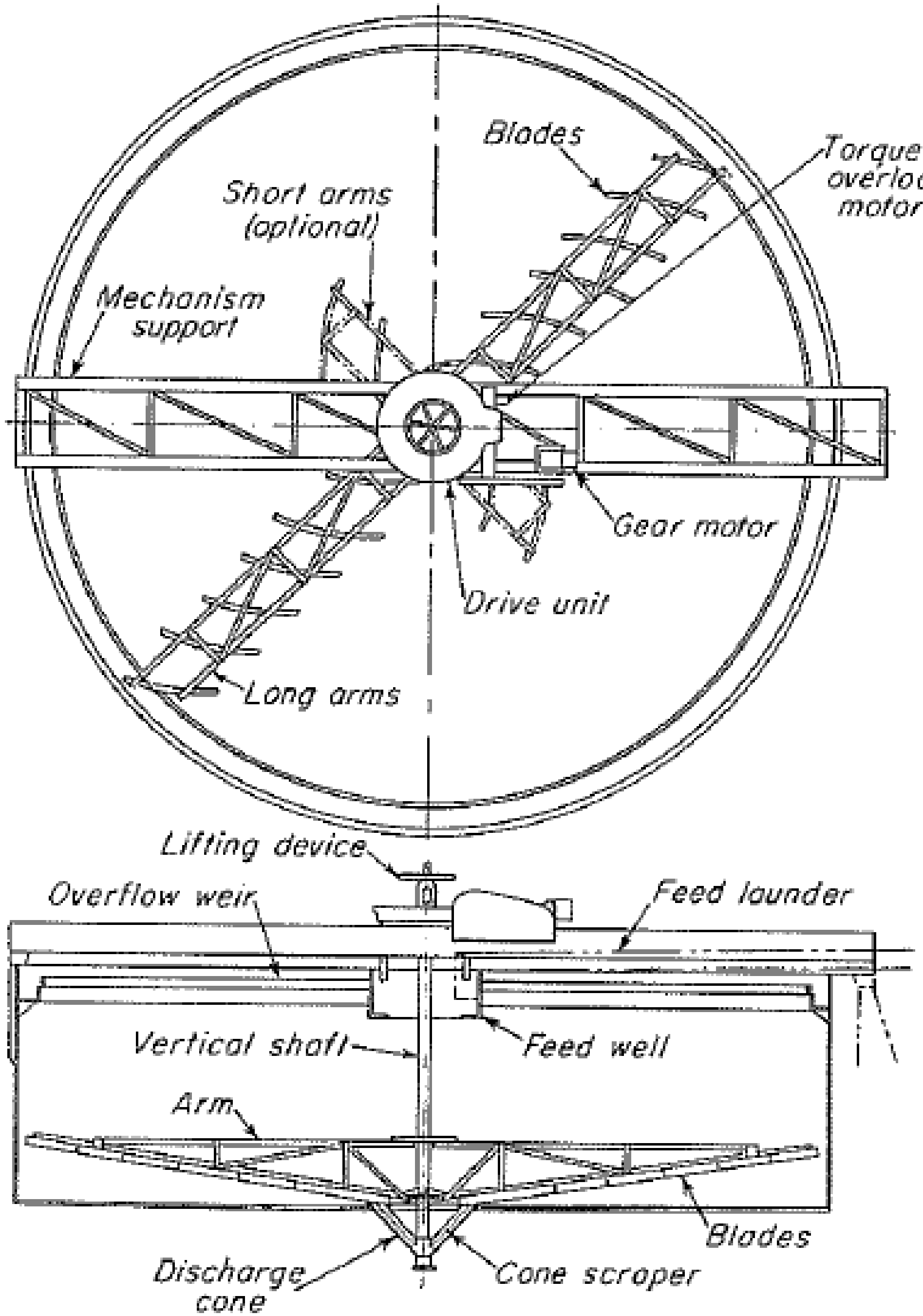
Reference McCabe Smith

GRAVITY SETTLING

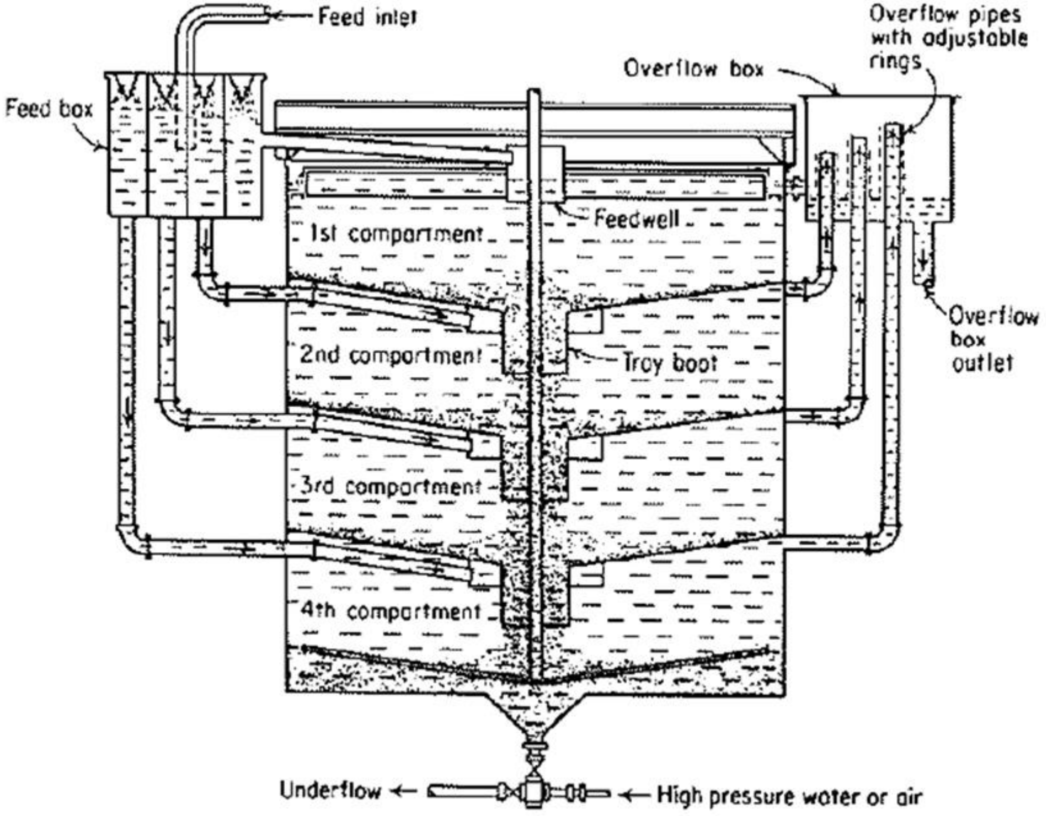
Types of sedimentation tanks

- Circular Radial flow
- Conical Vertical flow
- Rectangular horizontal flow

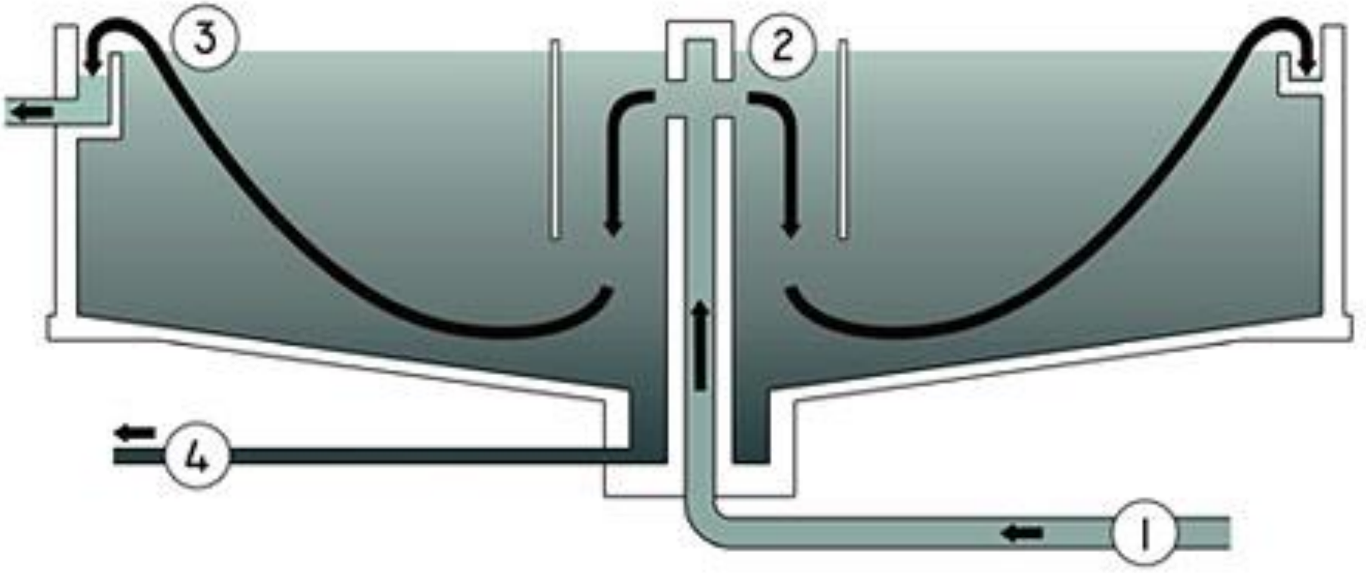
Circular
Radial Flow
Gravity Thickener



DORR THICKENER



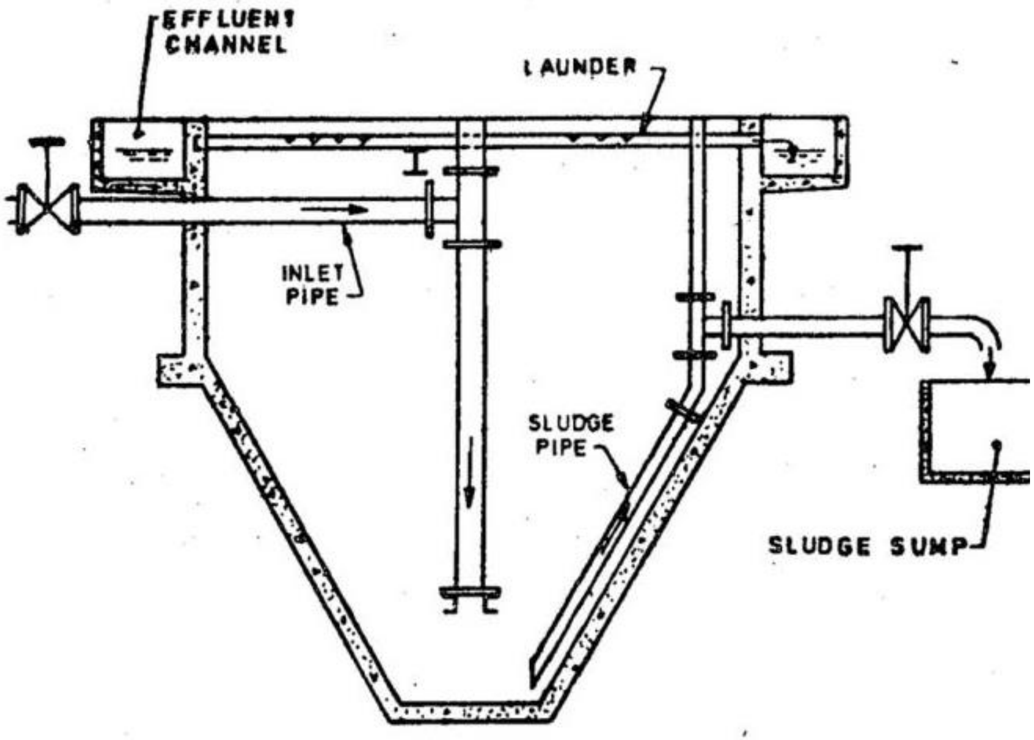
Circular Radial Flow Gravity Thickener



- 1 = Raw wastewater inlet pipe
  2 = Inlet stilling well (baffle)
  3 = Clarified water overflow weir
  4 = Primary sludge outlet pipe
-

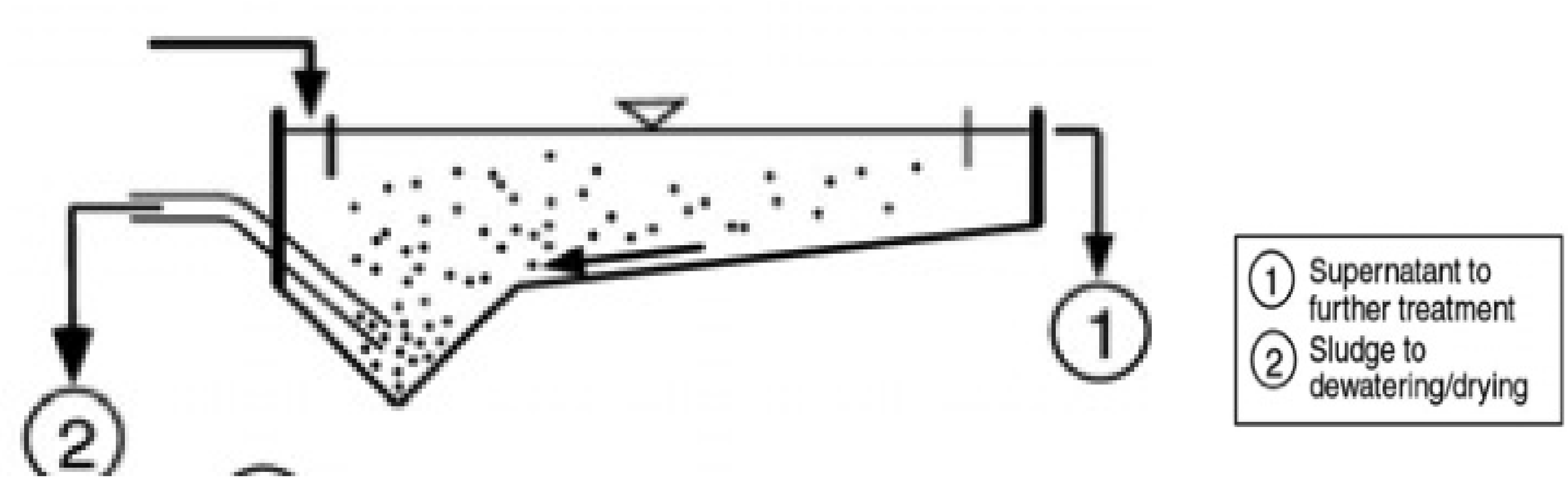
- After the removal of inorganic screenings and grit, the next step in wastewater treatment is the removal of the grosser suspended solids from the raw sewage. This is achieved via the process of primary settling or primary sedimentation. The wastewater flow is directed into one or more settling tanks, also known as primary clarifiers. Large mechanically-raked circular tanks are generally used at the larger works,.
- The overall volume of the primary settling tank is designed to ensure the incoming wastewater will take a certain amount of time to flow completely through the structure. This is known as the retention time, and must be sufficiently long enough to allow suitable settling of the solids to take place, yet short enough to prevent the anaerobic breakdown of the settled solids from occurring (i.e. the settled sludge turning septic).
- The retention time can vary from 1 to 3 hours, depending on the plant loading and incoming wastewater characteristics. In a tank with a retention time of 2 hours, 50 to 70% of the suspended solids may be removed by settling and flotation (scum removal). Removal of these solids will also reduce the BOD by 30 to 35%.
- Of equal importance in the design of the primary settling tank is the surface loading rate. This will dictate the overall diameter of a circular settling tank. This is effectively the upward velocity of the incoming flow from the base of the tank to the top of the overflow weirs. Upflow velocities of around 1.0m per hour are generally desired.

VERTICAL SETTLING TANK Dortmund Type



- For small plants conical Dortmund-type settling tanks are preferred. Occasionally rectangular structures are used, with complex sludge and scum scraper mechanisms.
- Upward flow tank suited small treatment plants
- Steep floor slope, large lower hopper volume enable large sludge storage
- No need for scarping.
- Depth of hopper at least equal to the top dimension which is less than 6m

RECTANGULAR HORIZONTAL FLOW



- Rectangular are for large primary settling tank
- Sensitive depth to width /length ratio
- Occasionally rectangular structures are used, with complex sludge and scum scraper mechanisms.

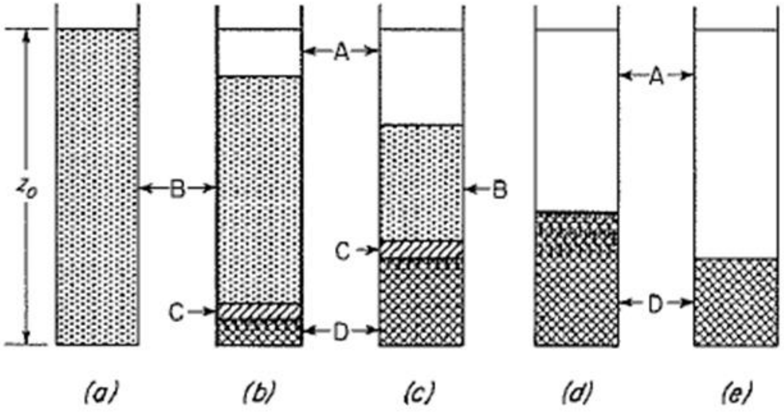
- Rectangular are for large primary settling tank
- Sensitive depth to width /length ratio

THICKENERS AND CLARIFIERS SETTLING OF FLOCCULATED PARTICLES

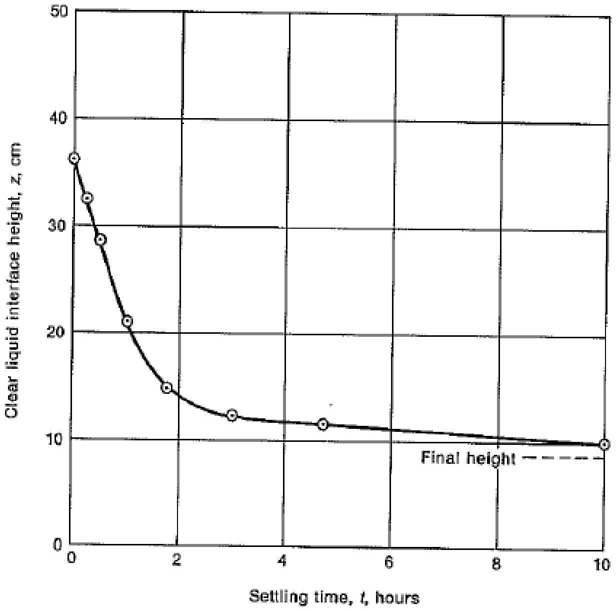
- Fine particles form agglomerates entrapping water within
- Flocculating agents – strong electrolytes, polymeric - polyacrylamide
- Inexpensive water treatment materials lime alumina, sodium silicates, ferrous sulphate, ferric chloride etc. form loose agglomerates and removes fine particles.

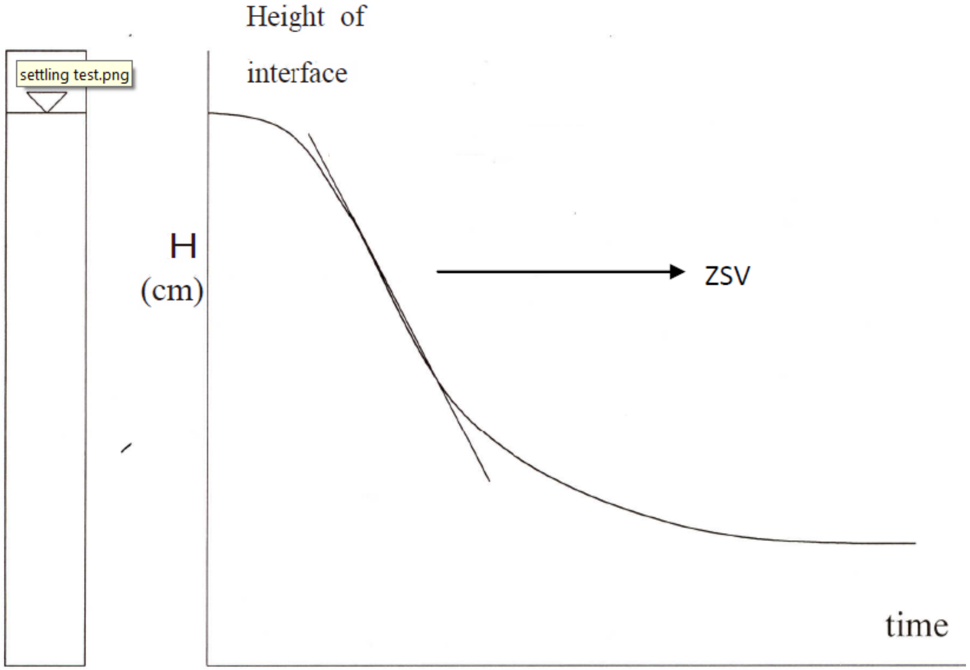
Batch Sedimentation test

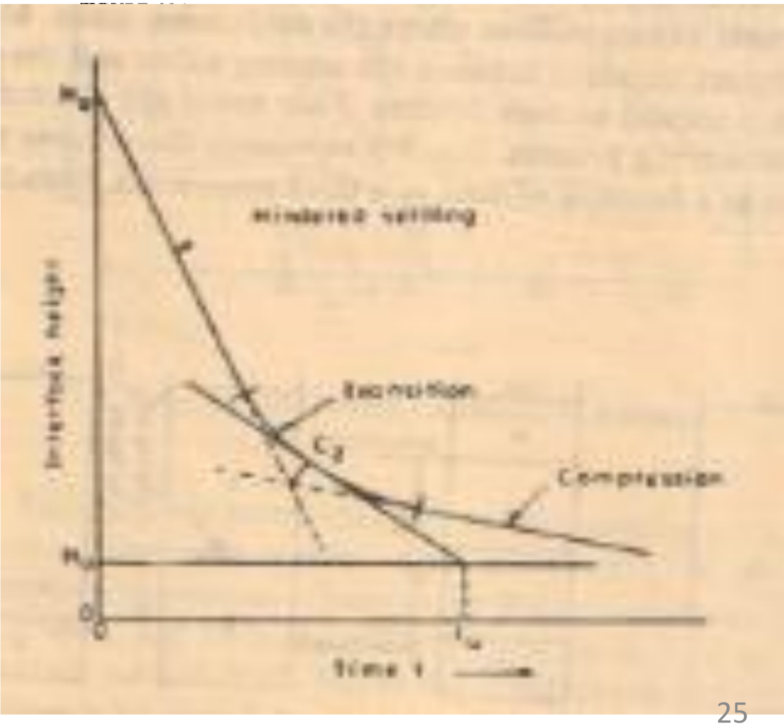
- A= Clear liquid
- B = Uniform initial conc.
- C=Transition zone
- D= Settled solid



Time	Height of clear liquid interface
0	Ho







gffd

- Plot settling rate curve from the following settling experimental data, and report (a)Hindered settling velocity (b) settling in compression zone (c) critical composition.

Time min	Interface height m
0	2.5
20	1.56
40	0.97
60	0.75
80	0.63
100	0.53
120	0.44
140	0.39
160	0.36

SETTLING TANK DESIGN
Ref: McCabe Smith

- There are two ways the solids can move to the bottom:

1. Under the influence of their settling velocity, downward

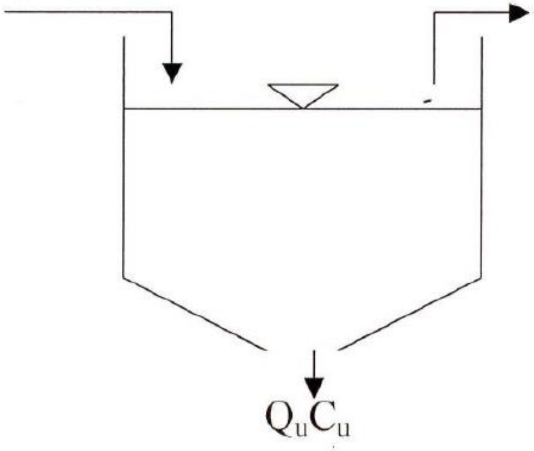
2. Due to the continuous removal of sludge at the bottom as underflow

Total downward flux of solids, Kg/m^2hr, consists of two parts the

1. Transport Flux, G_t
2. Settling flux G_s

Feed Q_i, C_o

overflow Q_o



Underflow

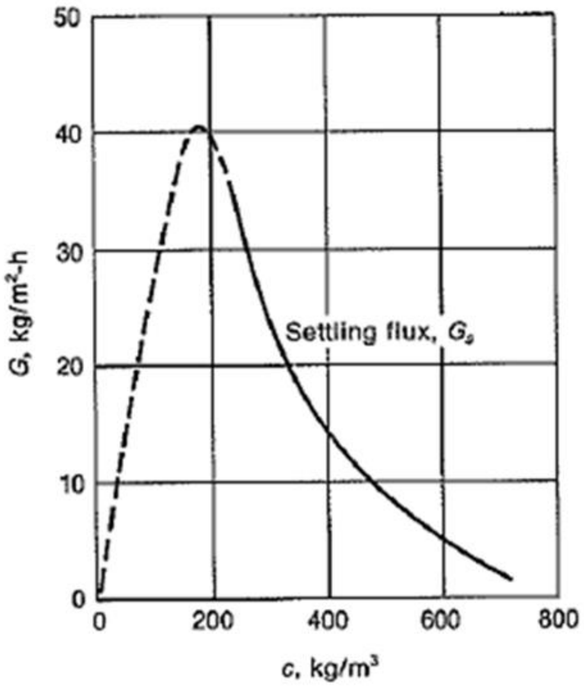
Q_i, Q_o, Q_u=Flowrate of inlet, overflow and underflow

C_o, C_u=Concentration of solids in inlet and underflow

- **Settling flux, Gs,** also called the batch settling flux, this is due to the settlement of solids and as expected is a function of solids concentration and the settling velocity of particles.

$$G_s = \left(\frac{dH}{dt}\right)_i C_i$$

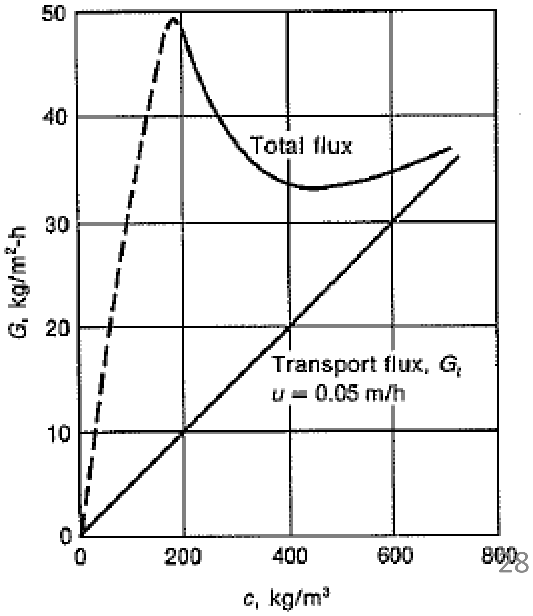
$\left(\frac{dH}{dt}\right)_i$, C_i= velocity and concentration at a layer "i" in the thickener. It passes through a maxima, as at very low concentration settling rate is constant, and at high concentration the rate decreases rapidly.



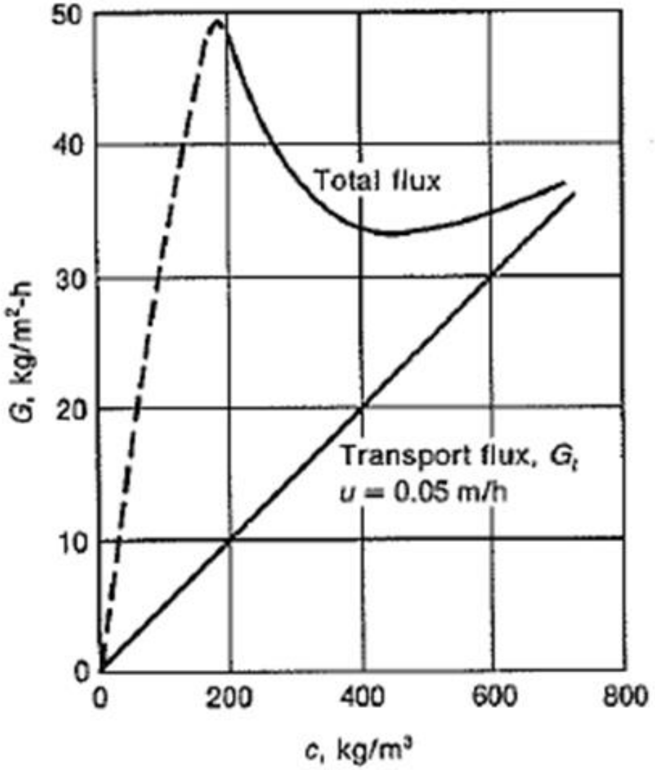
- **Transport flux, Gt,** Fluxof solids carried by liquid, is independent of the solids settling in the thickener, whether the solids settle or not, there pumping of the sludge from the bottom of the tank.

$$G_t = uC_i$$

where, u= the velocity created by the underflow



SETTLING TANK AREA



- Total flux curve has a minimum value between the influent solids concentration (Co) and underflow solids concentration (Cu). This minimum flux is the maximum allowable solids loading for the thickener to work successfully. When flux is at minimum, we calculate for the maximum area required. If this limit is exceeded, the solids will overflow in the effluent. So the design of a thickener is thus reduced to the point of determining this flux.
- $G = G_t + G_s$

$$= uC_i + \left(\frac{dH}{dt}\right)_i C_i$$

$$As\ Solid\ introduced = solid\ removed$$

$$QC_o = Total\ flux\ x\ area$$

$$A = \frac{QC_o}{(G_t + G_S)_{min}}$$

KYNCH Method to obtain settling flux from one batch settling data

- Locate any time, ti
- Draw tangent at the point to cut y axis t Hi’ and x axis at ti’
- Determine Ci: $H_0C_0 = H_i{}'C_i \;\; and \; C_i = \frac{H_0C_0}{H_{i'}}$ [conc. at top of settling zone]
- Determine settling velocity, $\left(\frac{dH}{dt}\right)_i = \frac{H_{i'}}{t_{i'}}$
- Determine settling Flux : $G_{s_i} = \left(\frac{dH}{dt}\right)_i C_i$
- Plot Gsi vs Ci

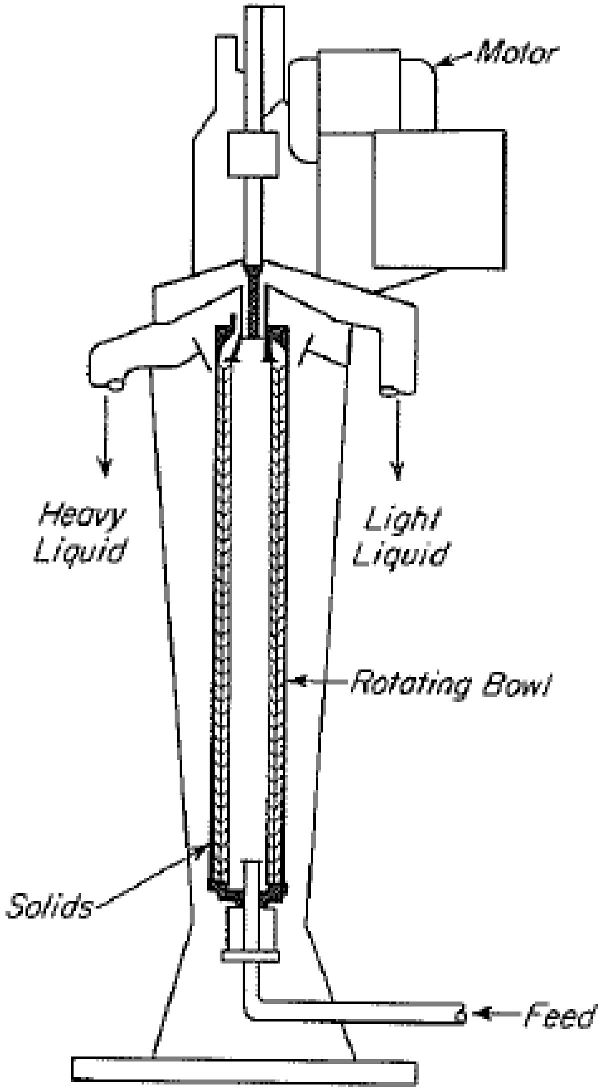
t_i	$t_i{}'$	$H_i{}'$	C_i	$\left(\frac{dH}{dt}\right)_i$	G_{s_i}

- Following is the results of a settling experiment, with initial concentration of 60 g/l of calcium carbonate. Data is to be used to design a settling tank to handle 0.03m3/second of slurry, with an underflow velocity of 0.05m/h.
- Plot (a) Settling flux vs concentration, (b) transport flux vs concentration,(c) total flux vs concentration
- Determine minimum total flux, and cross sectional area of the tank.

Time,min	Height of interface, mm
0	250
10	175
20	123
30	103
40	86
50	75
60	65
70	57
80	52

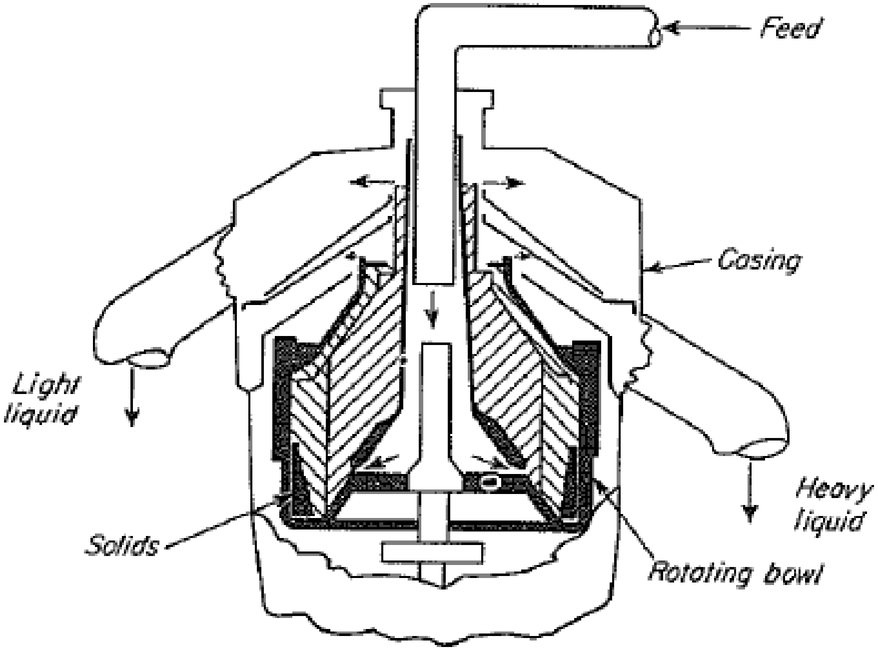
CENTRIFUGAL SETTLING

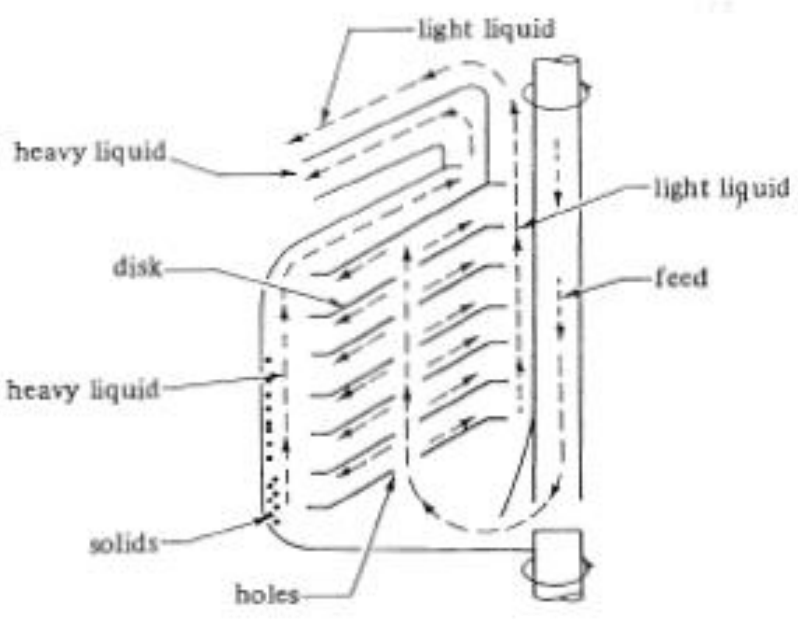
Tubular Centrifuge



- Centrifugal liquid liquid separation
- Removal of solids from lubricating oil, ink, beverages etc.
- 10 to 15 cm dia
- 15,000rpm
- Solid/dirt accumulate inside the bow, and periodically removed

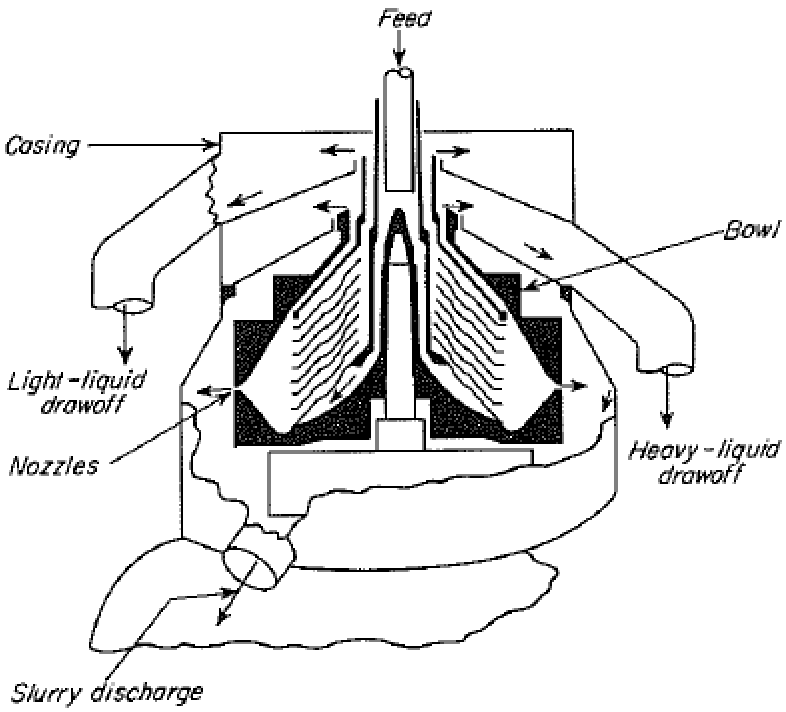
Disk Centrifuge





- Liquid liquid separation
- Removal of solids from lubricating oil, ink., beverages etc.
- Short wide bowl 20to 50cmdia
- Bowl filled with discs- cones of sheet metal set one above the other, with matching holes in the middle, which form the liquid passage
- Heavy liquid moves outwards and light liquid inwards.
- Soon Heavy Liquid touches lower side of a cone and separates out
- Shearing helps break emulsions
- Solid/dirt accumulate inside the bowl, and periodically removed

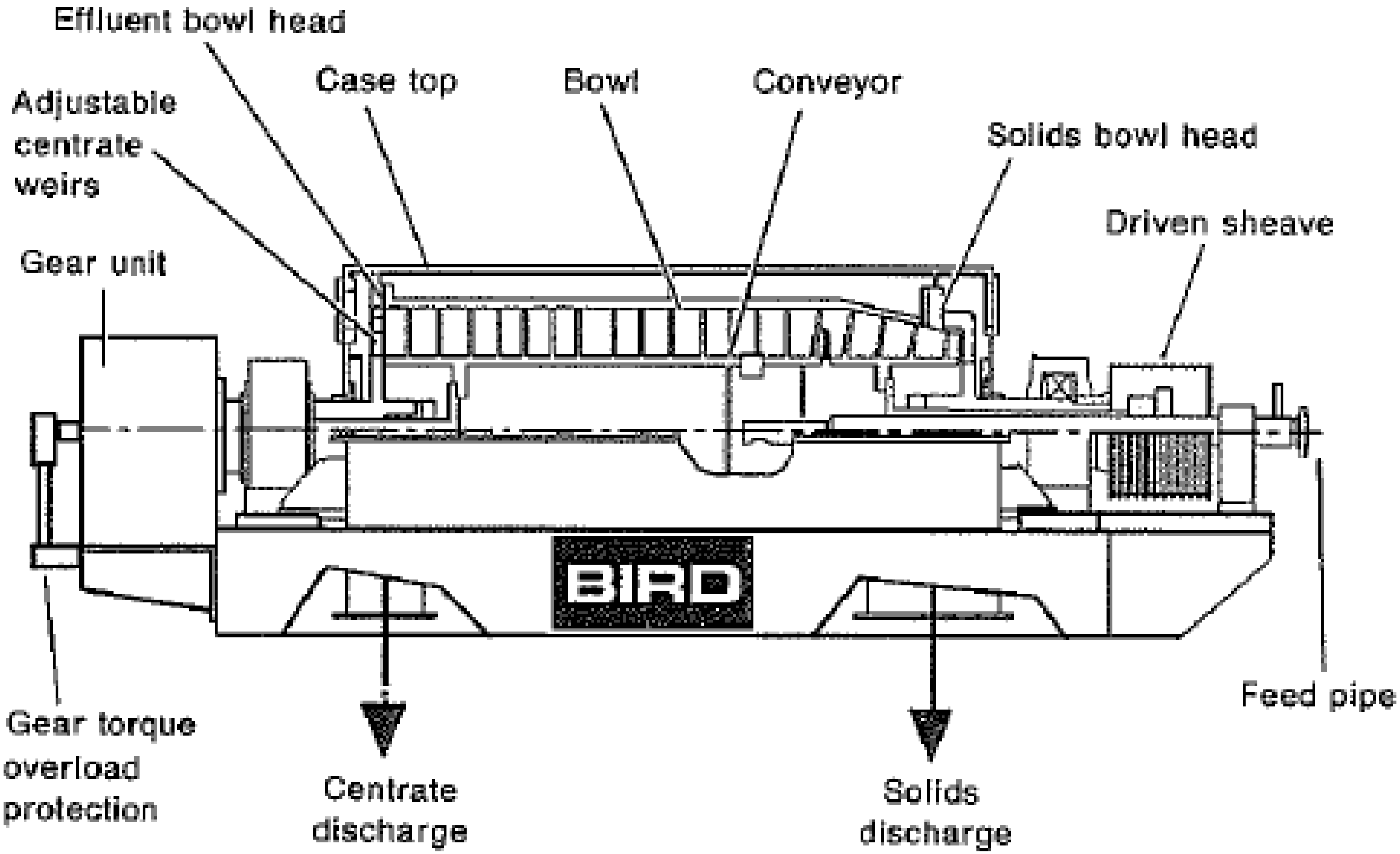
Nozzle discharge Centrifuge



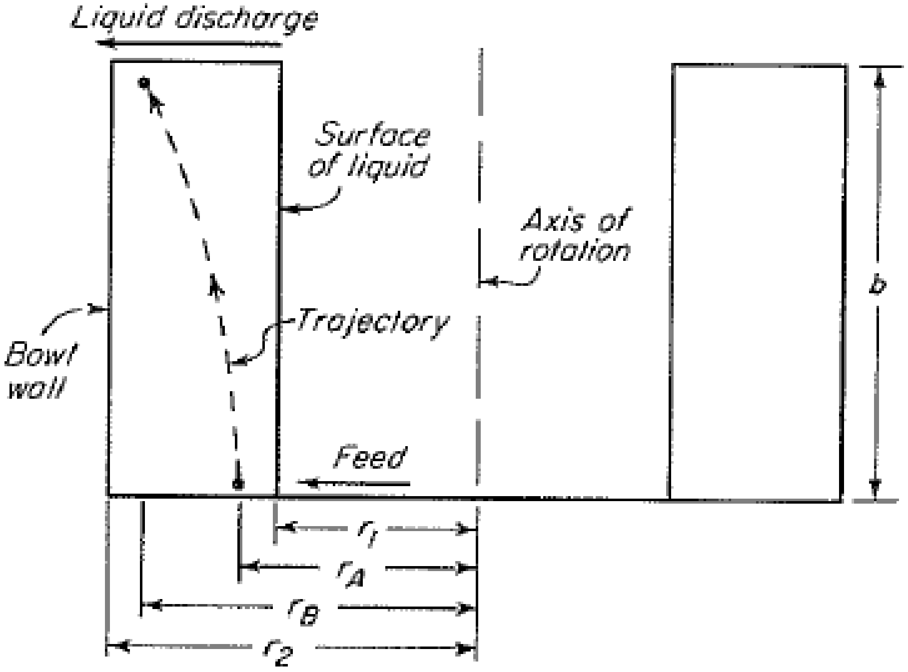
- Modified disk centrifuge
- 3 mm dia holes at the periphery
- Dilute slurry leaves through hole
- Alternatively the holes are plugged most of the time and occasionally opened and thick slurry removed.

HELICAL CONVEYER CENTRIFUGE

- Bowl diameter – 10cm to 140 cm
- Solid removal rate – 1 to 2 tons/h to 50tons/h
- May not clarify 100% may require subsequent clarifier
- Separate free particles from water, like crystals, dewater etc



Sedimenting Centrifuge



- $r_2 =$ radius of the bowl
- $r_1 =$ radius at liquid surface
- $b =$depth of bowl
- $D_p = particle\ size$
- r_A= position of particle at inlet
- r_B = position of particle at outlet

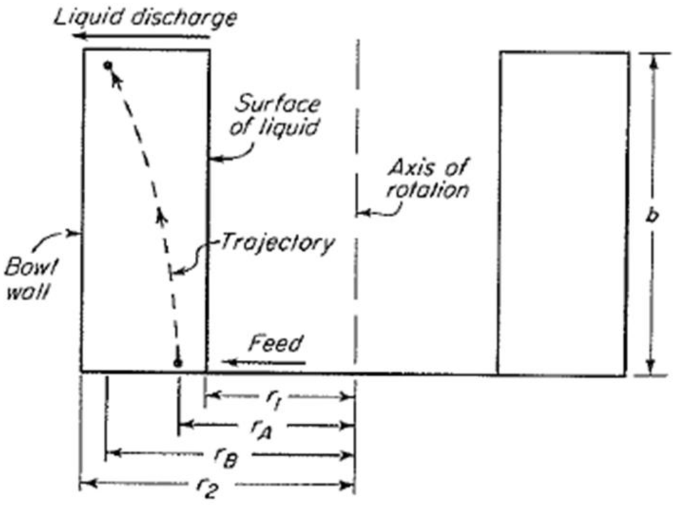
- $u_t = \frac{\omega^2 r D_p{}^2(\rho_p - \rho)}{18\mu}$
- Since $u_t = dr/dt$

Time required for a particle of size Dp to travel from radius rA to rB.

- $\int_0^{t_T} dt = \int_{r_A}^{r_B} \frac{18\mu}{\omega^2(\rho_p - \rho)D_p{}^2} \frac{dr}{r}$
- $t_T = \frac{18\mu}{\omega^2(\rho_p - \rho)D_p{}^2}\ ln \frac{r_B}{r_A}$
- Residence time
- $t_T = \frac{Volume}{q} = \frac{\pi b\left(r_2^2 - r_1{}^2\right)}{q}$
- $q = \frac{\pi b \omega^2(\rho_p - \rho)D_p{}^2}{18\mu} \frac{\left(r_2^2 - r_1{}^2\right)}{ln \frac{r_B}{r_A}}$

CUT DIAMETER

- Cut point diameter particles that travels half the distance between $r_2\ and\ r_1$ in the available time
- For the particles to be removed $r_A = \frac{r_1+r_2}{2}$ and $r_B = r_2$



- $q = \frac{\pi b \omega^2 (\rho_p - \rho) D_p{}^2}{18\mu} \frac{(r_2^2 - r_1{}^2)}{ln\frac{r_B}{r_A}}$
- $q_c = \frac{\pi b \omega^2 (\rho_p - \rho) D_{pc}{}^2}{18\mu} \frac{(r_2^2 - r_1{}^2)}{ln\frac{2r_2}{(r_1+r_2)}}$

Special case: very thin liquid layer

$$r_1 \cong r_2$$

- $u_t = \frac{\omega^2 r_2 {D_p}^2 (\rho_p - \rho)}{18\mu}$
- Let the thickness = s
- If Cut dia is D_{pc} the particles have to travel a distance s/2
- $u_t = \frac{s}{2t_T}$
- Residence time $t_T = \frac{V}{q_c}$

$$q_c = \frac{V}{t_T} = \frac{2Vu_t}{s} = \frac{2V\omega^2 r_2 {D_p}^2 (\rho_p - \rho)}{18\mu s}$$

Sigma Value: a parameter for scale up

$$q_c = \frac{2V\omega^2 r_2 D_p{}^2(\rho_p - \rho)}{18\mu s}$$

Let re and se be average value of radius and liquid layer thickness

$$q_c = \frac{2V\omega^2 r_e D_p{}^2(\rho_p - \rho)}{18\mu s_e}$$

$$= \frac{2V\omega^2 r_e}{g s_e} \frac{D_p{}^2 g(\rho_p - \rho)}{18\mu} = 2\sum U_G$$

$Sigma\ value, \sum = \frac{V\omega^2 r_e}{g s_e}$, is a characteristics of centrifuge .

It is the cross sectional area of a gravity settling tank of the same separation capacity as the centrifuge.

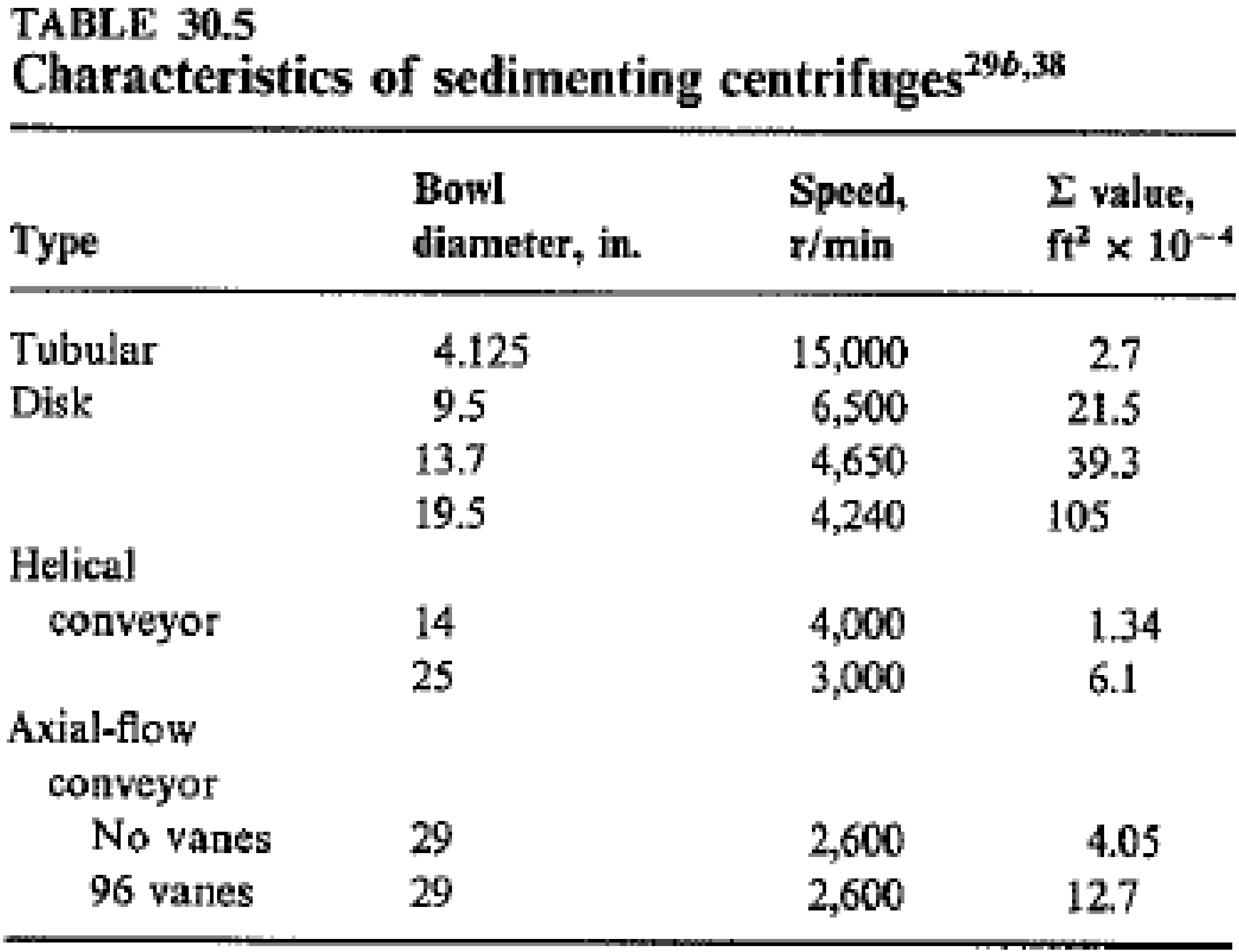


TABLE 30.5

Characteristics of sedimenting centrifuges[29b,38]

Type	Bowl diameter, in.	Speed, r/min	Σ value, $\text{ft}^2 \times 10^{-4}$
Tubular	4.125	15,000	2.7
Disk	9.5	6,500	21.5
	13.7	4,650	39.3
	19.5	4,240	105
Helical conveyor	14	4,000	1.34
	25	3,000	6.1
Axial-flow conveyor			
No vanes	29	2,600	4.05
96 vanes	29	2,600	12.7

- What is the capacity in cubic meters per hour of a clarifying centrifuge operating under the following conditions?
  - Diameter of bowl, 600mm
  - Thickness of liquid layer, 75mm
  - Sp.gr of liquid 1.2 sp.gr of solid 1.6
  - Depth of bow 400 mm Viscosity of liquid 2 cP
  - Speed, 1,200rpm Cut size 30μm