Material balance

It's based of conservation of mass : "Matter is neither created nor destroyed."

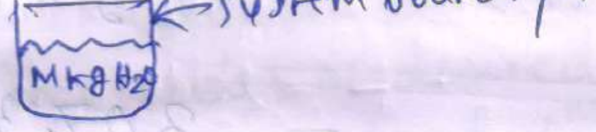
Accumulation of mass in a system = Mass in to the system − Mass out from the system.

Or Rate of accumulation of mass = Mass inflow − Mass outflow

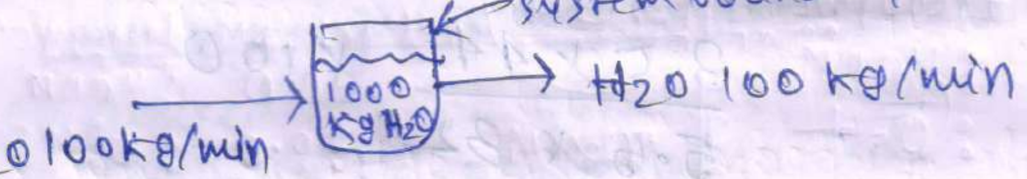
System: An arbitrary portion of a whole process that is considered for analysis.

It has system bundaries.

Closed system: Physical or chemical changes can take place inside the system, but for closed system, no mass exchange occurs with the surrounding.



Open system: It is called a flow-system as material crosses the system boundaries.

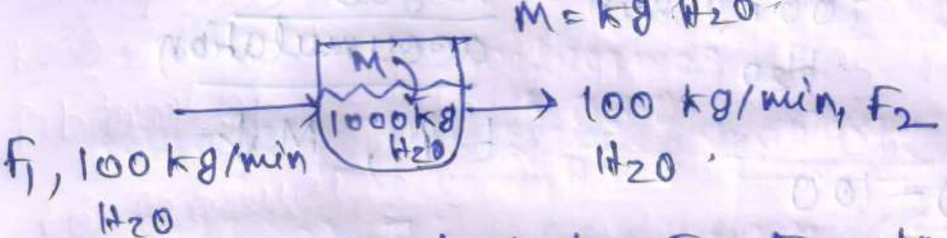


Steady state process: As mass inflow (rate of mass in) is equal to mass outflow (rate of mass out), the amount of mass in the system reamins constant.

Rate of accumulation of mass = 0

⇒ Mass inflow = Mass outflow.

For steady state process :, T, P, mass of material, flow rate remain constant with time.



at steady state $F_1 = F_2 = 100$ kg/min.

$\therefore \frac{dM}{dt} = 0$; integrating, $\boxed{M = C}$

M = Mass of material in the system, kg

t = time, min, sec, hr. etc.

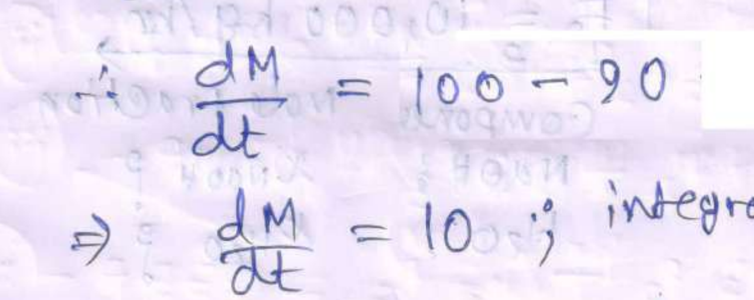
at $t = 0$; $M = M_0$; $\Rightarrow C = M_0$ = initial mass in the system.

$\therefore M = M_0 \Rightarrow$ Mass remains constant at M_0 kg.

Unsteady state (Transient) process.

Mass inflow is not equal to mass out flow.

Like mass; T, P, flow rate can be unsteady or keep changing with time.



Positive accumulation

t ↑ M ↑

$\therefore \frac{dM}{dt} = 100 - 90$

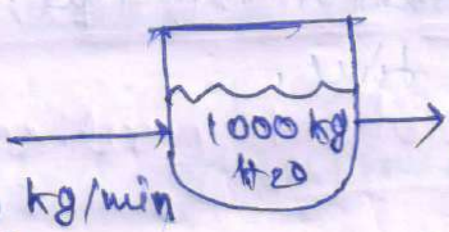
$\Rightarrow \frac{dM}{dt} = 10$; integrating $\boxed{M = 10t + C}$.

at $t = 0$; $M = M_0$; $\Rightarrow C = M_0 = 1000$ kg

$\therefore M = 10t + 1000$

$\therefore$ at $t = 10$ min $M = 10 \times 10 + 1000 = 1100$ kg

$t = 100$ min $M = 10 \times 100 + 1000 = 2000$ kg



90 kg/min H_2O → 1000 kg H_2O → 100 kg/min H_2O

<u>negative accumulation</u>

t↑, M↓

$$\therefore \frac{dM}{dt} = 90 - 100$$

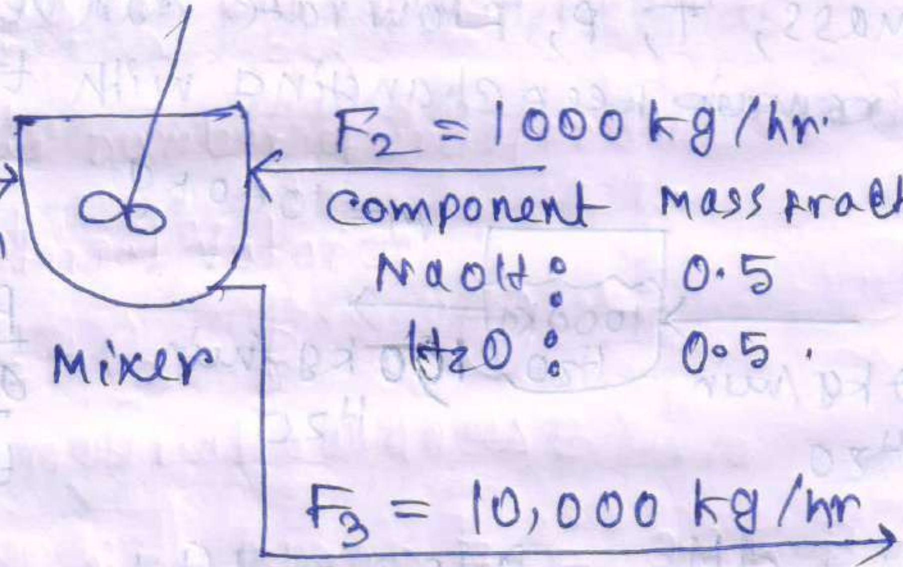
$$\Rightarrow \boxed{M = -10t + C} \quad \text{at } t=0,\ M = M_0 = 1000\text{ kg}$$

$$\Rightarrow M = -10t + 1000 ;$$

∴ at t = 10 min, M = 900 kg

at t = 100 min; M = 0 kg.

<u>Multiple component system</u>



$F_1 = 9000$ kg/hr

Component	Mass fraction
NaOH :	0.05
H_2O :	0.95 .

Mixer

$F_2 = 1000$ kg/hr

Component	Mass fraction
NaOH :	0.5
H_2O :	0.5 .

$F_3 = 10,000$ kg/hr

Component	Mole fraction
NaOH :	x_{NaOH} ?
H_2O :	x_{H_2O} ?

total inflow total outflow

As $F_1 + F_2 = F_3$ overall material balance.

$\Rightarrow 9000 + 1000 = 10,000$

$\therefore$ component balance or species material balance

NaOH: $0.05 \times 9000 + 0.5 \times 1000 = x_{NaOH} \times 10000$

$\Rightarrow 450 + 500 = x_{NaOH} \times 10000$

$\Rightarrow x_{NaOH} = \frac{950}{10,000} = 0.095$.

H_2O: $0.95 \times 9000 + 0.5 \times 1000 = x_{H_2O} \times 10000$

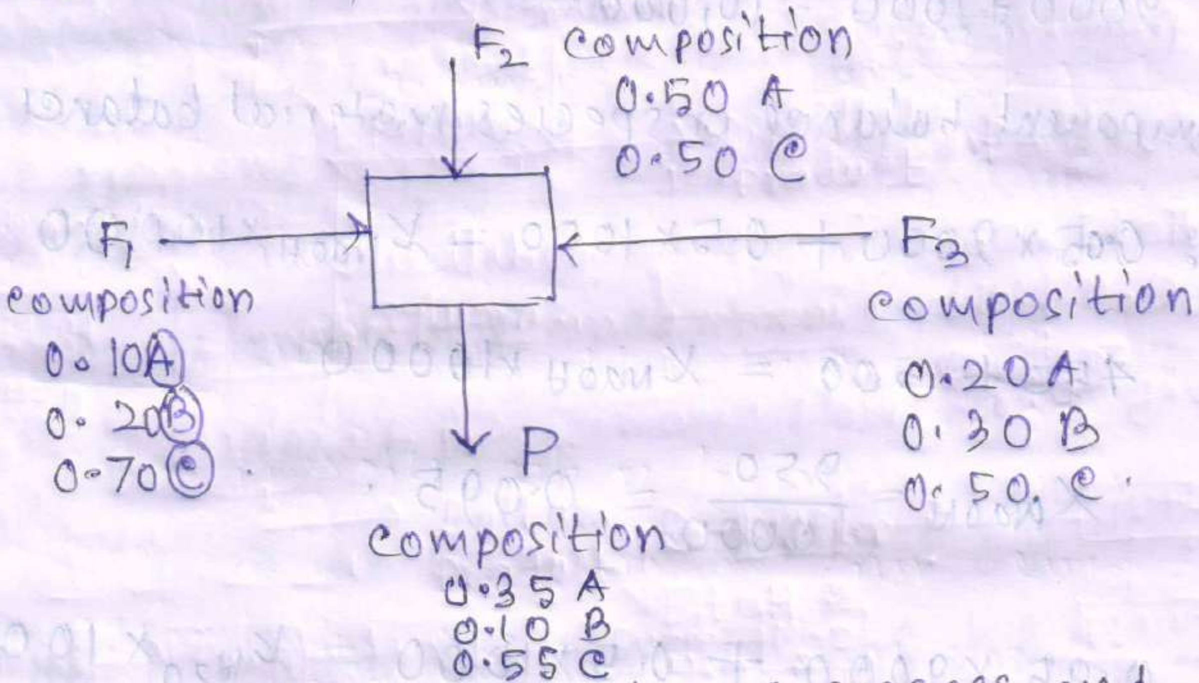
$\Rightarrow 8550 + 500 = x_{H_2O} \times 10000$

$\Rightarrow x_{H_2O} = \frac{9050}{10000} = 0.905$.

$\therefore x_{NaOH} + x_{H_2O} = 0.095 + 0.905 = 1$ (balanced)

Degree of Freedom Analysis

Consider a steady state process:



* After drawing the sketch of a process and specify system boundaries.

1. Determine the number of unknown variables.

∴ F_1, F_2, F_3, F_4, i.e., 4 flowrates in kg/hr.

∴ number of unknown variables = 4.

all compositions are knowns.

2. Determine the number of independent equations.

here. overall material valance

$$F_1 + F_2 + F_3 = P \quad \text{— (i)}$$

components material balances.

$$A: \; 0.10F_1 + 0.5F_2 + 0.2F_3 = 0.35P \quad \text{— (ii)}$$

$$B: \; 0.20F_1 + 0 \times F_2 + 0.3F_3 = 0.10P \quad \text{— (iii)}$$

$$C: \; 0.70F_1 + 0.5F_2 + 0.5F_3 = 0.55P \quad \text{— (iv)}$$

∴ Total 4 number of equations

but 4 equations are not independent equations.

as if (ii), (iii) and (iv) equations are added the equn (i) forms.

As (ii) + (iii) + (iv) = (i)

∴ number of independent equations is (3).

They can be any three of 4 equations.

∴ Degree of freedom, $N_D = N_U - N_E$

N_U = number of unknown variables.

N_E = number of independent equations.

here, $N_D = 4 - 3 = 1$.

Basis or number of basis required to fix the system is, $N_D = 1$;

in the other way if bassis is taken $N_U = 4 - 1 = 3$

Now $N_D = 3 - 3 = 0$.

∴ For fixed system $N_D = 0$.

» solvable. (has a set of solution vector).

Here, take the basis, $P = 100$ kg/hr.

∴ set of independent equations are:

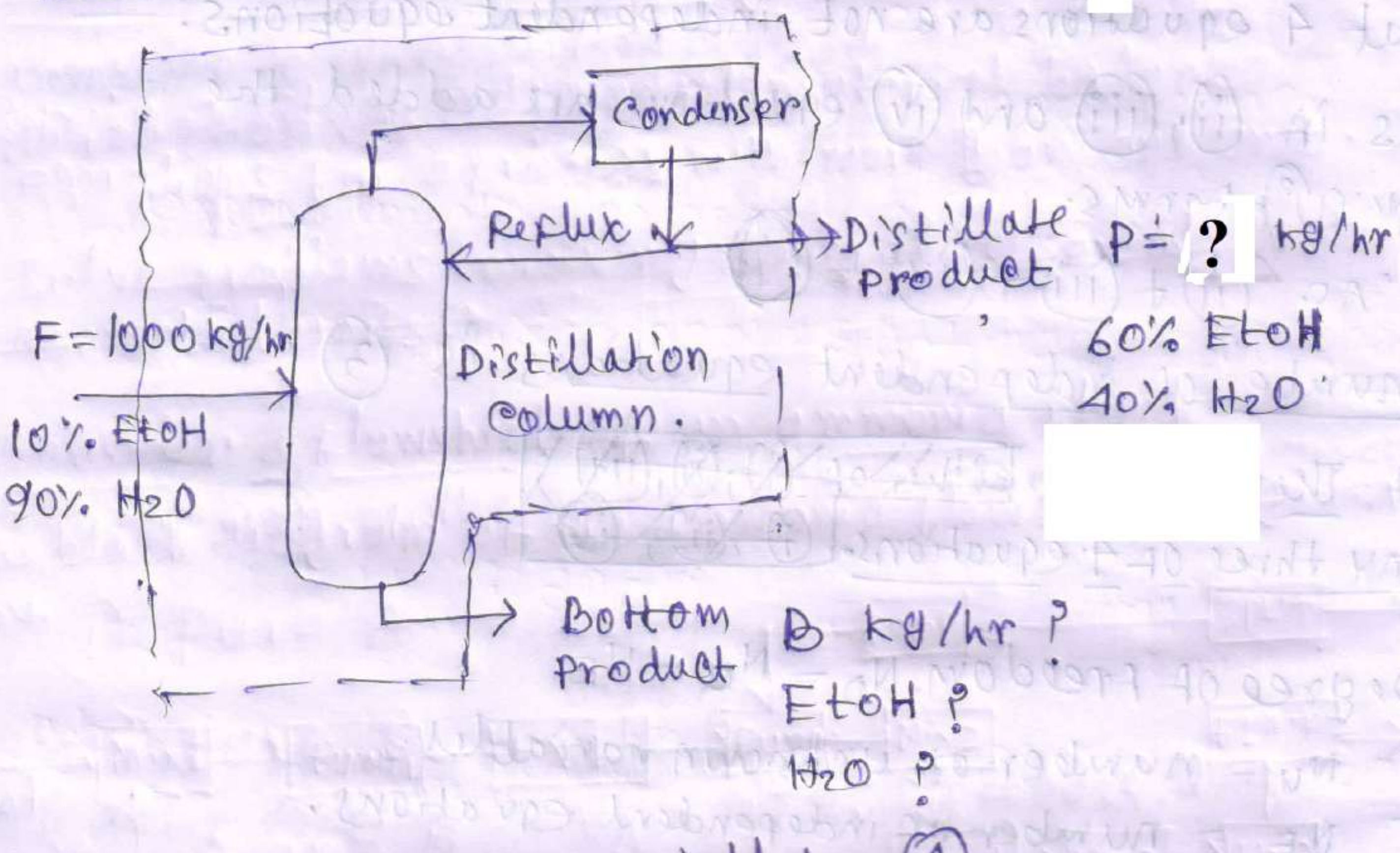
A: $0.10F_1 + 0.5F_2 + 0.2F_3 = 35$ — (ii)

B: $0.20F_1 + 0.3F_3 = 10$ — (iii)

C: $0.70F_1 + 0.5F_2 + 0.5F_3 = 55$ — (iv)

After solving $F_1 = 25$ kg/hr; $F_2 = 58.33$ kg/hr; $F_3 = 16.66$ kg/hr

Continuous distillation column



number of unknown variables = 4.

$P, B, x^B_{EtOH}\ x^B_{H_2O}$.

$\underbrace{P, B}_{\text{2 flowrates}}$, $\underbrace{x^B_{EtOH}\ x^B_{H_2O}}_{\text{2 compositions}}$

∴ Overall material valance.

$$F = P + B.$$

$$\therefore P + B = 1000 \quad \text{— (i)}$$

One comonent balance.

EtOH: $0.10 \times 1000 = P \times 0.60 + B x^B_{EtOH}$. — (ii)

H_2O $0.90 \times 1000 = P \times 0.40 + B x^B_{H_2O}$ — (iii)

not an independent if (i) & (ii) are considered.

Sum of composition for bottom product

$$x^B_{H_2O} + x^B_{EtOH} = 1 \quad \text{— (iv)}$$

$\therefore$ number of independent eqn. (3).

$\therefore N_D = N_U - N_E$
$= 4 - 3 = (1)$

$\therefore$ Number of bassis required to fix the system is (1).

Now assume any one composition or
assume any relation between streams.

Say $P = \frac{1}{10} F$ (Basis equation).

So $N_D = 4 - 4 = 0$. (Fixed).

Now $P = 1000/10 = 100$ kg/hr.

$\therefore P + B = 1000$; $B = 1000 - 100 = 900$ kg/hr.

$\therefore$ (ii) eqn. $B x^{B}_{EtOH} + 100 \times 0.6 = 100$

$\Rightarrow 900 \times x^{B}_{EtOH} = 40$

$\Rightarrow x^{B}_{EtOH} = \frac{4}{90} = 0.044$

$\therefore x^{B}_{H_2O} = 1 - 0.044 \simeq 0.956$

$\therefore P = 100$ kg/hr; $B = 900$ kg/hr,
$x^{B}_{H_2O} = 0.956$; $x^{B}_{EtOH} = 0.044$

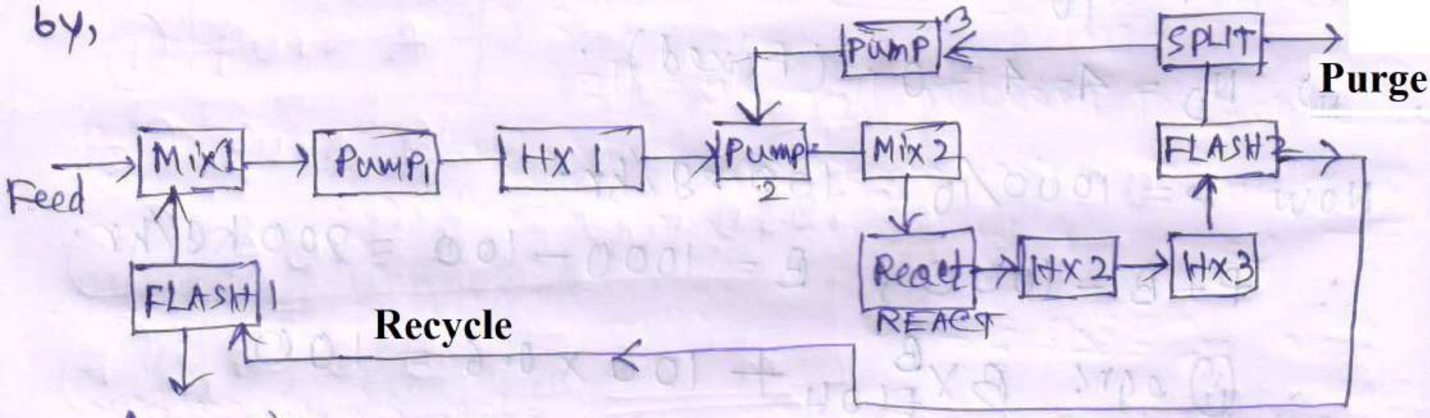
Now Check $\Sigma x^{B}_{i} = 1 \Rightarrow 0.044 + 0.956 = 1$.

Material balance involving multiple units.-

* write a set of independent material balance equations for a process with more than one unit.
* solve problems involving several ~~xx~~ serially connected units.

Consider a flowsheet of an ammonia plant.

The block diagram of the ammonia flowsheet is given by,



MIX → Mixer (unit operation)

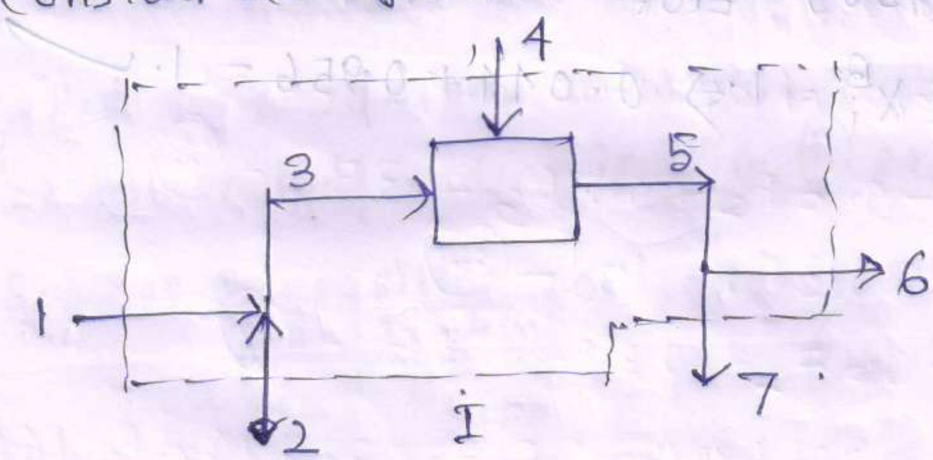
HX → Heat exchanger (unit op.).

SPLIT → Splitter (unit op.).

REACT → Reactor (unit process)

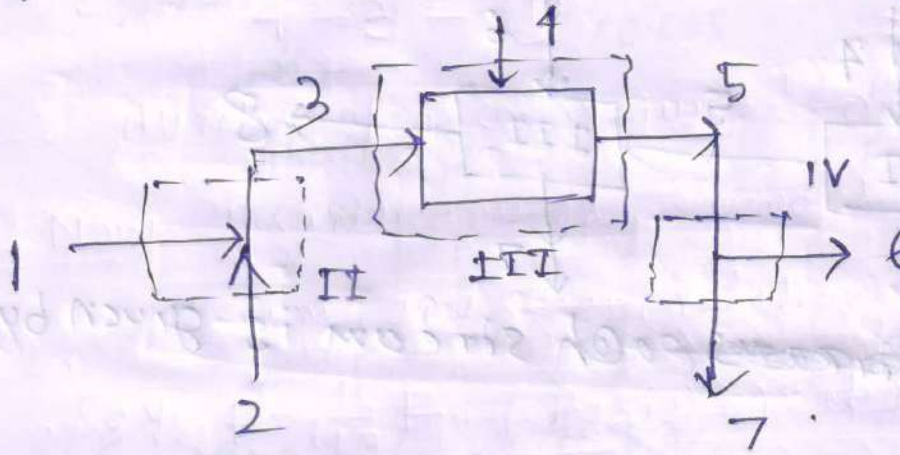
FLASH → Flash chamber. (unit operation

Consider a single unit with its connecting flow streams



The dotted box reprents the overall process (I) where input streams are 1, 2, 4, and output streams are 6 & 7.

One can write overall material balance equation and component balance equations with summation of composition equations, considering the overall process.

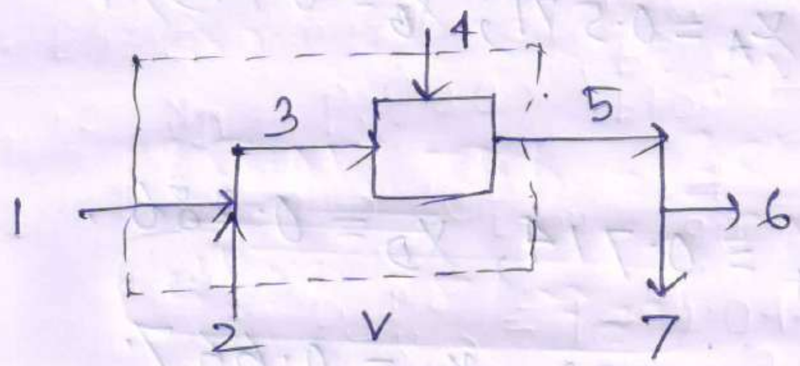


The independent equations of unit II, III, and IV can be written separately and can be solved.

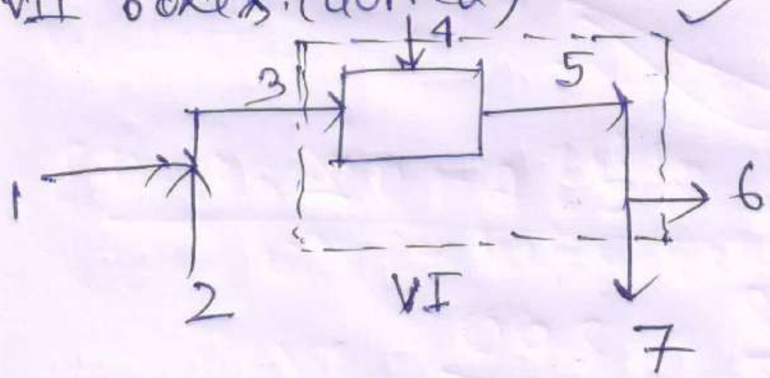
Here II is splitter.

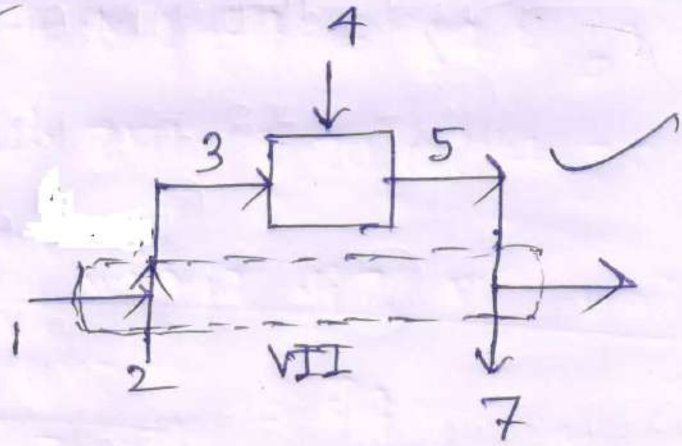
III is an unit.

IV is splitter.



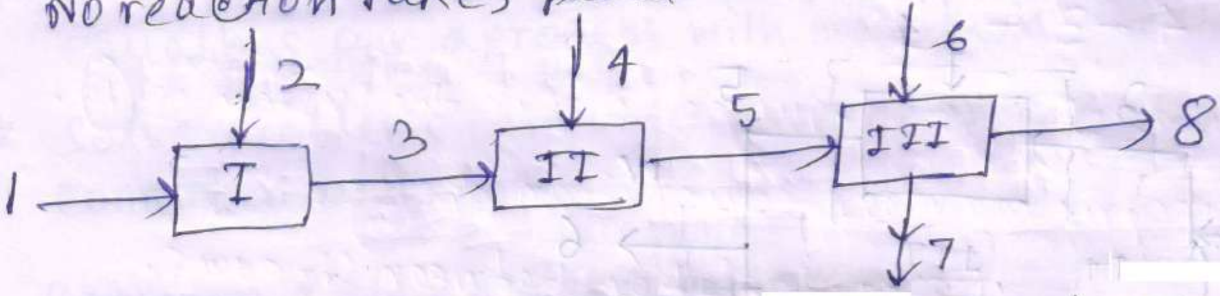
One can write independent material balance equations of combinations of two or more units simultaneously, as indicated by V, VI and VII boxes. (dotted).





Example 1

consider an open system with steady state.
No reaction takes place.



composition of the each stream is given by;

(1) pure A

(2) Pure B

(3) A and B, $x_A = 0.80$; $x_B = 0.20$

(4) pure C.

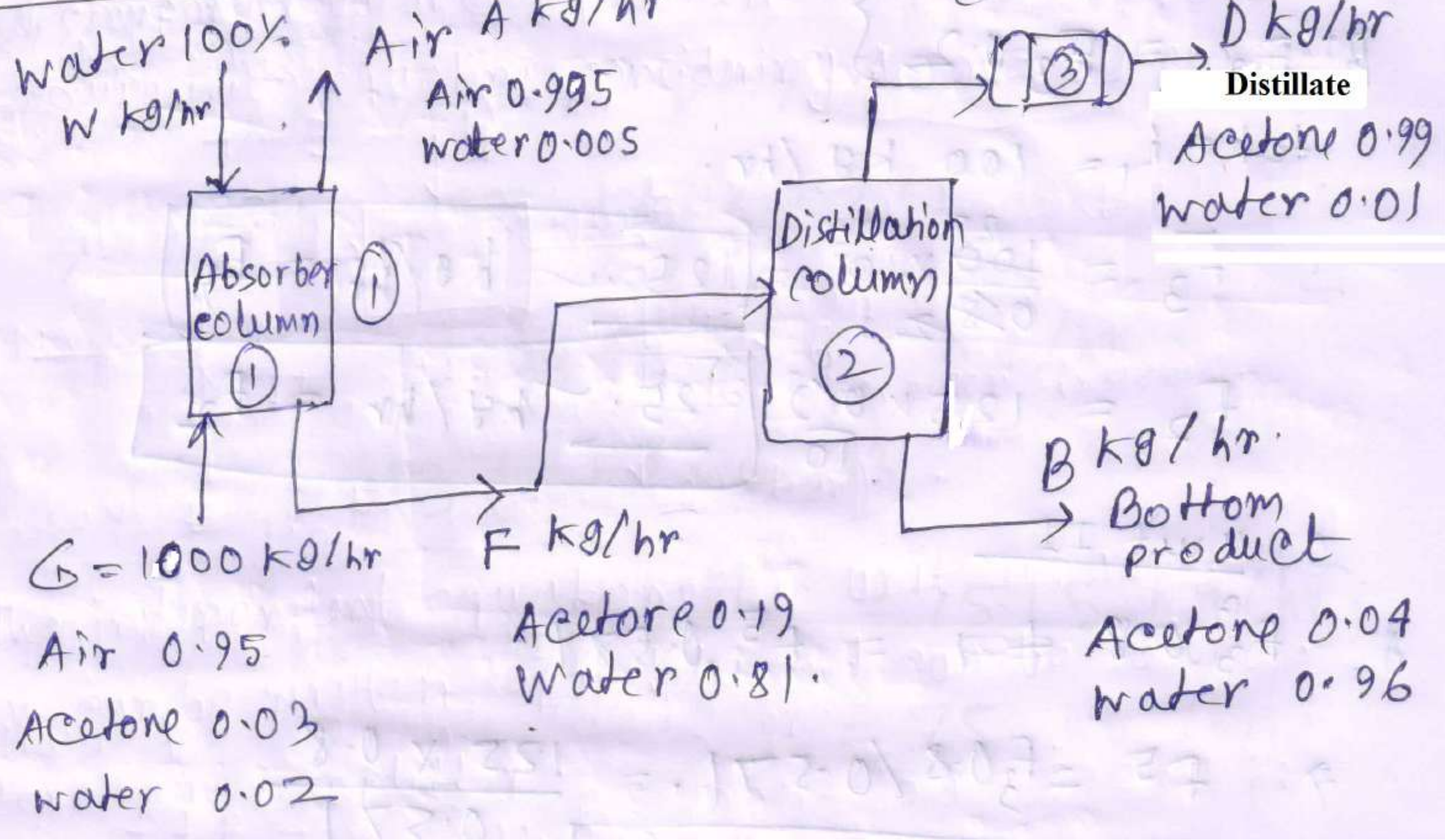
(5) A, B, and C ; $x_A = 0.571$; $x_B = 0.143$; $x_C = 0.286$

(6) Pure D.

(7) A and D, $x_A = 0.714$, $x_D = 0.286$.

(8) B and C, $x_B = 0.333$, $x_C = 0.667$.

Example 2


Water 100%
W kg/hr
Air A kg/hr
Air 0.995
water 0.005
Absorber column ①
①
G = 1000 kg/hr
Air 0.95
Acetone 0.03
water 0.02
F kg/hr
Acetone 0.19
Water 0.81
Condenser
③
D kg/hr
Distillate
Acetone 0.99
water 0.01
Distillation column
②
B kg/hr
Bottom product
Acetone 0.04
water 0.96

Chemical Process Calculations CL204
Module-3
Material Balance Without Chemical Reaction

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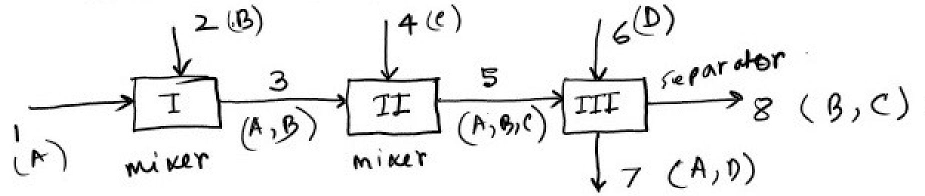
Material balance for multiple units system

#Example-1

Consider an open system with steady state and no reaction takes place.



Composition of the each stream is given as:

1. F_1 : Pure A (100%).
2. F_2 : Pure B (100%).
3. F_3 : A & B ⇒ $x_A^3 = 0.80$; $x_B^3 = 0.20$
4. F_4 : Pure C (100%).
5. F_5 : A, B, and C $x_A^5 = 0.571$; $x_B^5 = 0.143$; $x_C = 0.286$
6. F_6 : Pure D (100%).
7. F_7 : A & D, $x_A^7 = 0.714$; $x_D^7 = 0.286$
8. F_8 : B & C, $x_B^8 = 0.333$; $x_C^8 = 0.667$.

$F_1, F_2, F_3, F_4, F_6, F_7, F_8$ & 8 flowrates in kg/hr
or kmol/hr.

x_A, x_B are mass fraction.

$$x_A^{mol} = \frac{x_A / M.W_A}{x_A/M.W_A + x_B/M.W_B}$$

$$x_A = \frac{x_A^{mol} \times M.W_A}{x_A^{mol} \times M.W_A + x_B^{mol} \times M.W_B}$$

if F_1, is in kg/hr.

$$F_1^{mol} = \frac{F_1}{(Av.MW\ of\ F_1)}$$

$$Av\ mol\ wt = x_A^{mol} M.W_A + x_B^{mol} M.W_B$$

unit I (A, B) 2 independent material balance equations

A: $F_1 = F_3 x_A^3 \Rightarrow F_1 = F_3 \times 0.8$ — (i)

B: $F_2 = F_3 x_B^3 \Rightarrow F_2 = F_3 \times 0.2$ — (ii)

unit II (A, B, C) 3 independent material balance equations.

A: $F_3 x_A^3 = F_5 x_A^5 \Rightarrow F_3 \times 0.8 = F_5 \times 0.571$ — (iii) — find F_5.

B: $F_3 x_B^3 = F_5 x_B^5 \Rightarrow F_3 \times 0.2 = F_5 \times 0.143$ — (iv)

C: $F_4 = F_5 x_C^5 \Rightarrow F_4 = F_5 \times 0.286$ — (v) $\rightarrow F_4$.

unit III (A, B, C, D). 4 independent material balance equations.

A: $F_5 \times 0.571 = F_7 \times 0.714$ — (vi) ×

B: $F_5 \times 0.143 = F_8 \times 0.333$ — (vii)

C: $F_5 \times 0.286 = F_8 \times 0.667$ — (viii)

D: $F_6 = F_7 \times 0.286$ — (ix)

F_8, F_6, F_7 ?

number of independent variable 8 (8 flowrates)

number of independent equations 9.

$N_D = 8 - 9 = -1$ (overall system).

Overspecified system

if unit 1 is considered.

$N_e = 2$. $N_u = 3$ (F_1, F_2, F_3).

$N_D = 3 - 2 = 1$

Take $F_1 = 100$ kg/hr. (basis)

$\therefore N_D = 0$ and

unit I is solvable.

Solve eqn. i & ii find F_2? & F_3?

$F_3 = \frac{F_1}{0.8} = \frac{100}{0.8} = 125$ kg/h

$F_2 = F_3 \times 0.2$

$= 125 \times 0.2$

$= 25$ kg/h.

$F_1 + F_2 = F_3$

$\Rightarrow 100 + 25 = 125$

Unit II

A: $F_3\, 0.8 = F_5\, 0.571$

$$F_5 = \frac{125 \times 0.8}{0.571} = 175.13 \text{ kg/h} \simeq 175 \text{ kg/h}.$$

B: $0.2 F_3 = F_5\, 0.143$

$$F_5 = \frac{0.2 \times 125}{0.143}$$

$$= 174.825 \text{ kg/hr}$$

$$\simeq 175 \text{ kg/hr}.$$

C: $F_4 = F_5\, 0.286$

$\Rightarrow F_4 = 175 \times 0.286 = 50$ kg/hr

$\therefore F_3 + F_4 = F_5$.

$\Rightarrow 125 + 50 = 175$

Say in the 5 th stream F_5 $\left.\begin{array}{l} A = 0.5 \\ B = 0.2 \\ C = 0.3 \end{array}\right\} \times$

$F_3 \times 0.8$

A: $F^5 = \dfrac{125 \times 0.8}{0.5} = 200$ kg/hr

B: $F^5 = \dfrac{0.2 \times 125}{0.2} = 125$ kg/h

$\underline{F^5 \text{ from A} \neq F^5 \text{ from B}}$

So F_5 compositions $\left.\begin{array}{l} x_A^5 = 0.571 \\ x_B^5 = 0.143 \\ x_C^5 = 0.286 \end{array}\right\}$ Fixed

It can not be changed.

unit 3

A: $F_5 \times 0.571 = F_7\ 0.714$

$\Rightarrow F_7 = \dfrac{175 \times 0.571}{0.714} = 139.95\ kg/h$

$= 140\ kg/h$

B: $F_5 \times 0.143 = F_8 \times 0.333$

$\Rightarrow F_8 = \dfrac{175 \times 0.143}{0.333} = 75.15\ kg/h$

$\simeq 75\ kg/h.$

C: $F_8 \times 0.667 = F_5 \times 0.286$

$\Rightarrow F_8 = \dfrac{175 \times 0.286}{0.667} = 75.03\ kg/hr$

$= 75\ kg/hr$

F_8 from C $= F_8$ from B

$= 75$

$F_1 = 100\ kg/hr$

$F_2 = 25$ "

$F_3 = 125$ "

$F_4 = 50$ "

$F_5 = 175$ "

$F_6 = 40$ "

$F_7 = 140$ "

$F_8 = 75$ "

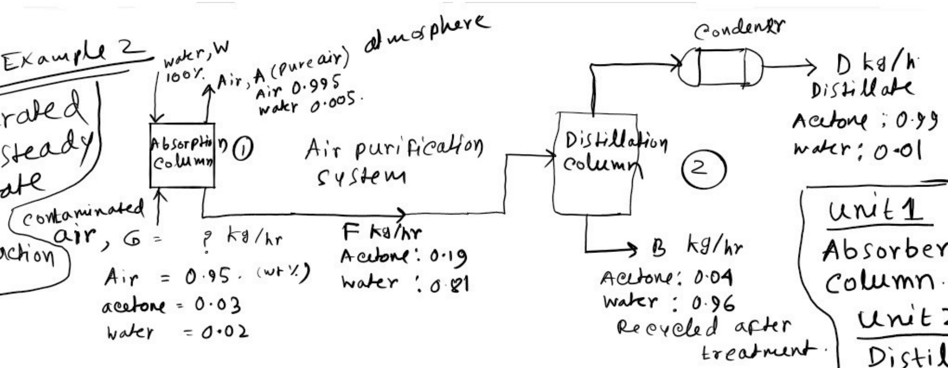
$F_5 + F_6 = F_7 \,\&\, F_8$

$\Rightarrow 175 + 40 = 140 + 75$

$\Rightarrow 215 = 215$

Example 2

Operated at steady state with no reaction



Unit 1
Absorber column.

Unit 2
Distillation column.

Operated at steady state with no reaction

N_u, Number of unknowns.

G, A, W, F, D, B = 6 flowrates.

N_E, number of independent equations

1st unit 3 compositions (air, water, acetone)

3 independent material balance equations.

2nd unit 2 compositions (water, acetone)

2 independent material balance eqns.

N_E; number of independent equations (1 and 2)

$= 3 + 2 = 5$.

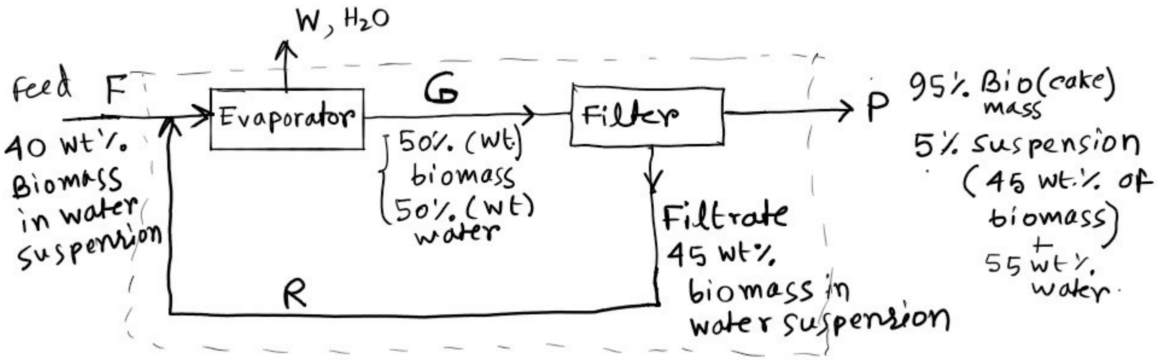
$N_D = N_U - N_E = 6 - 5 = 1$

Take one basis. G = 1000 kg/hr

$N_D = 5 - 5 = 0$ (solvable).

Material balance for multiple units with recycle, purge, and bypass streams

Drying system (dryer), concentration of feed, etc. (evaporator)



Components, 2 — Biomass ①, water ②

Number of unknowns = F, W, G, P, R ⇒ 5

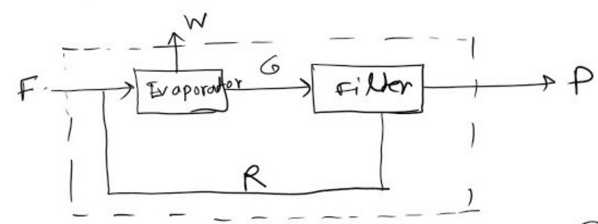
Number of independent equations = 4 (2 evaporator, 2 filter)

or {2 overall, 2 filter}

$N_D = 5 - 4 = 1$

Take basis F = 1000 kg/hr.

Material balances for overall system



$$F = W + P \quad \text{(overall)} \quad \text{— (i)}$$

Bio: $0.4F = 0.95P + 0.05 \times 0.45P$ — (ii)

water: $0.6F = W + 0.05 \times 0.55P$ — (iii)

$N_U = 3$, F, P, W.

$N_E = 2$ (i, ii)

$N_D = 3 - 2 = 1$; Take $F = 1000$ kg/hr.

Put in eqn (ii) $P = \dfrac{400}{0.95 + 0.05 \times 0.45} = 411.31$ kg/hr

$\simeq$ 411.3 kg/hr.

Put $P = 411.3$ kg/hr in eqn. (i)

$W = 1000 - 411.3 = 588.88$ kg/hr

$\simeq$ 588.9 kg/hr

$\simeq$ 589 kg/hr

Filter

$$G = P + R \quad \text{— (iv) (overall).}$$

Bio: $0.5\,G = (0.95 + 0.05 \times 0.45)\,P + 0.45\,R$ — (v)

water $0.5\,G = 0.05 \times 0.55\,P + 0.55\,R$ — (vi)

$N_E = 2$, $N_U = 2$, (G, R).

$N_D = 2 - 2 = 0$; no basis equired.

$$G = 411.3 + R \quad \text{— (vii)}$$

Replace G in v th eqn.

$$(411.3 + R)\,0.5 = \underbrace{399.98}_{\simeq 400} + 0.45\,R$$

$\Rightarrow$ $R = 3886.78$ kg/hr.

$\simeq 3887$ kg/hr.

$$G = P + R = 411.3 + 3887$$

$$= 4298.3 \text{ kg/hr.}$$

Overall material balnce for the evaporator

$$F + R = W + G \quad (\textbf{Overall})$$

$$\Rightarrow \quad 1000 + 3887 = W + 4298.3$$

$$\Rightarrow \quad \underline{W = 588.7 \text{ kg/hr}} \simeq$$

Solution set $\therefore$ $F = 1000$ kg/hr (basis)

$P = 411.3$ kg/hr

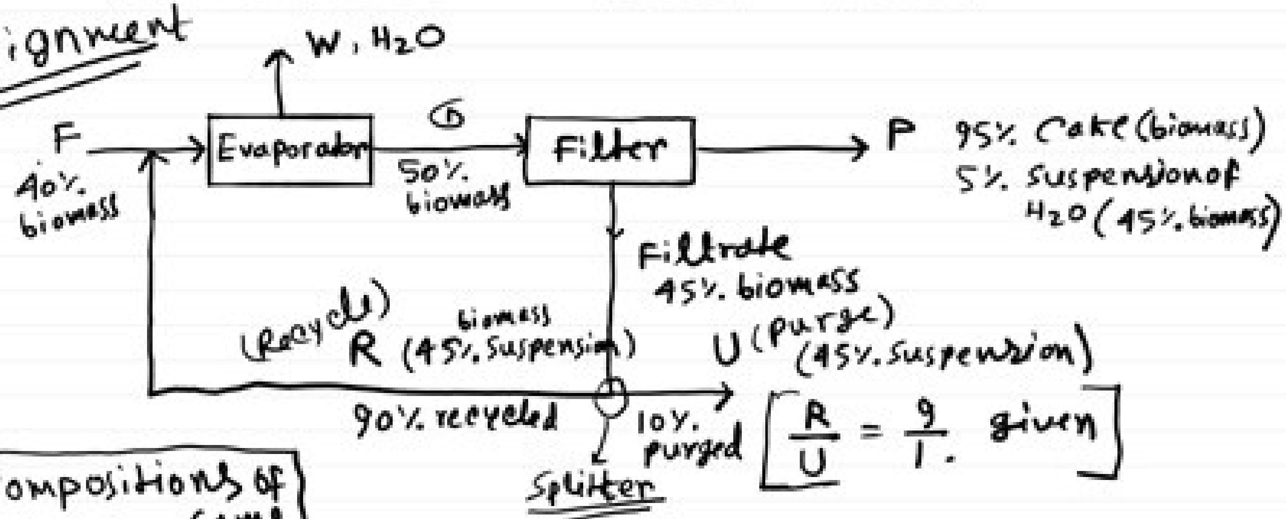
$W = 599$ kg/hr

$R = 3887$ kg/hr

$G = 4298.3$ kg/hr.

Material balance with recycle & purge.

Assignment



Compositions of R & U are same

$N_U = F, W, G, P, R, U \} = 6$

$N_E = 5$ (4 material balance

1 $R/U = 9$)

$N_D = 6 - 5 = 1$ Take basis

$F = 1000\ kg/hr$

overall system

$$F = W + P + U$$

biomass $\quad F \times 0.4 = (0.95 + 0.05 \times 0.45) P + U \times 0.45$

$F, U, W, P = 4$ unknowns.

$N_D = 2$ (equations).

$N_D = 2$, but 2 bases can't be taken.

Solve 5 independent equations simultaneously with $F = 1000$ kg/hr basis.

> Linear algebraic equations.

Material balance of a humidification/dehumidification system

Y = Absolute molar humidity of air stream at specific temperature and pressure.

Y is in kmol of water / kmol of dry air (dry basis)

y = Absolute molar humidity (Total basis or mol fraction of water in air)

$$y = \frac{\text{mol of water}}{\text{mol of water} + \text{mol of dry air}}$$

$$= \frac{\text{mol of water}/\text{mol of dry air}}{\frac{\text{mol of water}}{\text{mol of dry air}} + 1} = \frac{Y}{1+Y}$$

$\therefore$

$$F, Y_F \rightarrow \boxed{AC} \rightarrow P, Y_P \qquad \downarrow D, Y_D$$

overall: $F = P + D$

water: $F \dfrac{Y_F}{1+Y_F} = P \dfrac{Y_P}{1+Y_P} + D \dfrac{Y_D}{1+Y_D}$

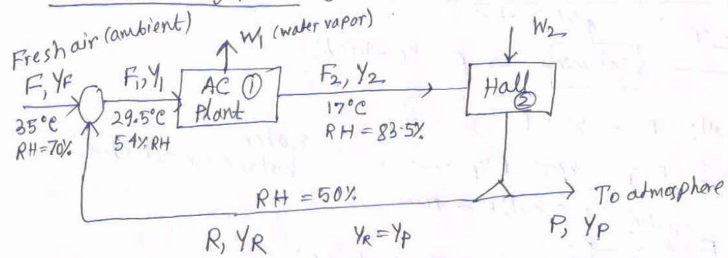
if D is pure water $y_D = 1$ or $\dfrac{Y_D}{1+Y_D} = 1$

overall: $F = P + D$

water: $F y_F = P y_P + D$

$$y_F \downarrow \frac{Y_F}{1+Y_F} \qquad y_P \downarrow \frac{Y_P}{1+Y_P}$$

Air conditioning plant (AC Plant)



RH (Relative humidity).

Y = Absolute molar humidity of air stream at specific T & P.

F = Flowrate of air stream (kmol/hr).

Y is in kmol of water/kmol of dry air (dry basis)

Mixer before AC plant:

overall: $F + R = F_1$ — (i)

water $F\frac{Y_F}{1+Y_F} + R\frac{Y_R}{1+Y_R} = F_1\frac{Y_1}{1+Y_1}$ — (ii)

AC plant

overall: $F_1 = W_1 + F_2$ — (iii)

water: $F_1\frac{Y_1}{1+Y_1} = W_1 + F_2\frac{Y_2}{1+Y_2}$ — (iv)

Hall overall: $F_2 + W_2 = R + P$. — (v)

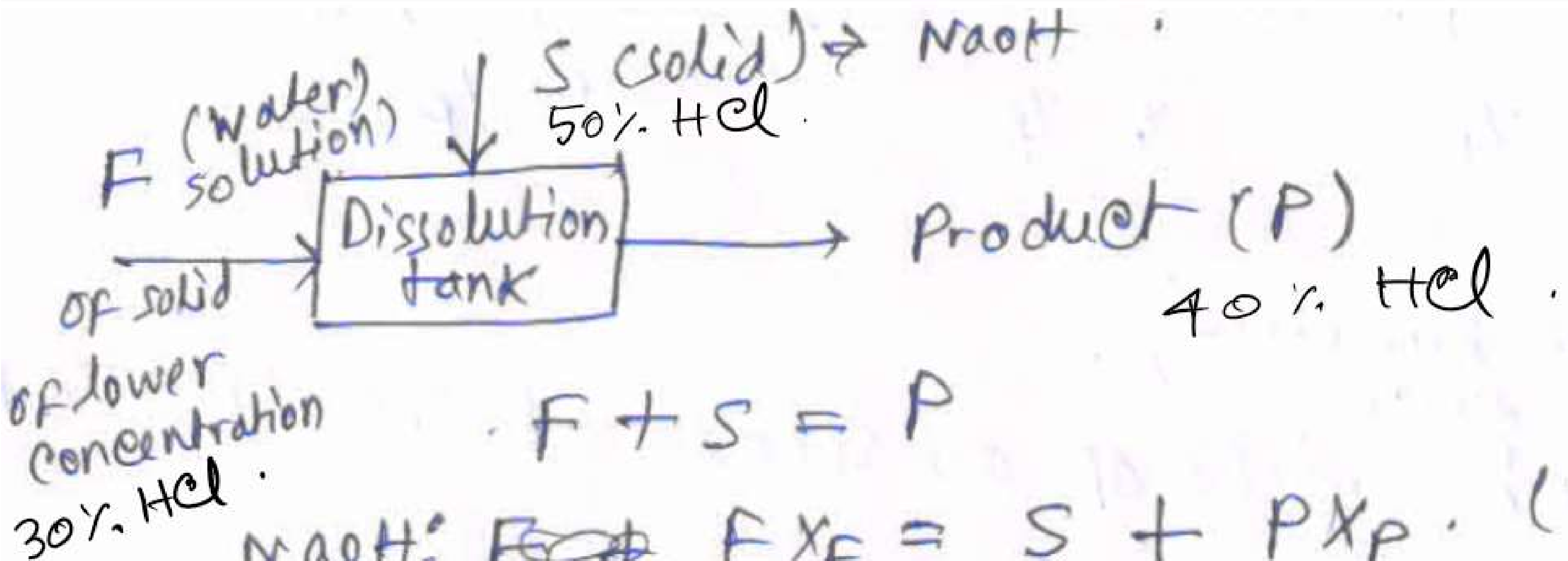
$F_2\frac{Y_2}{1+Y_2} + W_2 = (R+P)\frac{Y_R}{1+Y_R}$. — (vi)

$F, F_1, F_2, W_1, W_2, R, P$ } unknown flowrates (7)

$Y_F, Y_1, Y_2, Y_R (Y_P)$ · are known. as $Y = F(T, RH)$.

known known.

∴ DOF = 7 − 6 = ① Take Basis $F = 50$ kmol/hr.

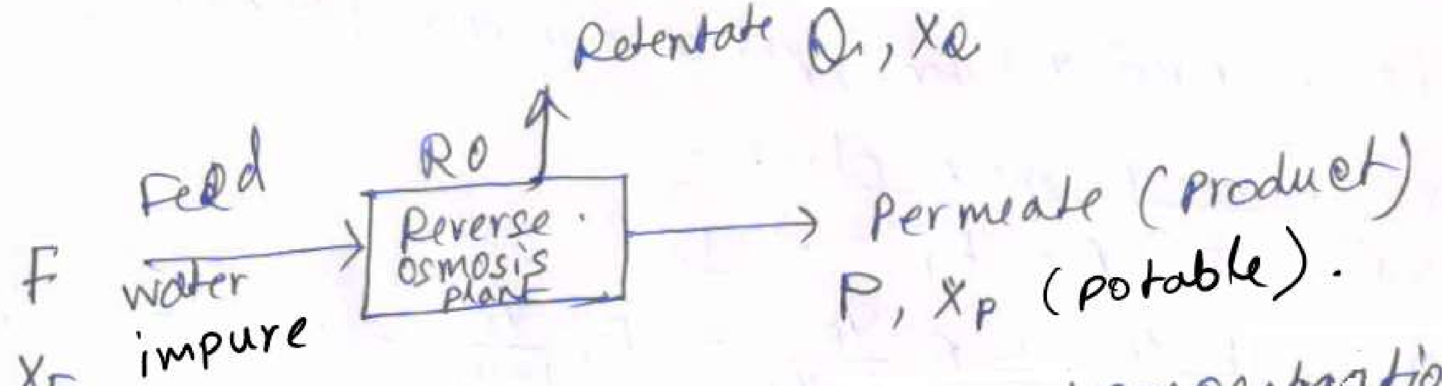


$$F + S = P$$

NaOH: ~~Feed~~ $F x_F = S + P x_P$ (water balance neglected)

/HCl : $x_F = 0.3$ HCl: $S x_S$ HCl: $x_P = 0.4$

$x_S = 0.5$



x_F, x_Q, x_P are dissolved solid concentrations in feed, retentate and perpeate respectively.

Overall: $F = Q + P$

Dissolved solid: $F x_F = Q x_Q + P x_P$.

dissolved solid in feed

x_F = fraction of

x_Q = " " " " in retentate

x_p = " " " " in potable water product.

x_F = 1000 mg/kg water

x_Q = 2000 mg/kg water.

x_P = 50 mg/kg water.

(TDS) total dissolved solid.

take basis F = 2000 kg/hr.

Solve Q and P.

$$x_F F = 1000 \frac{\text{mg}}{\cancel{\text{kg water}}} \frac{\cancel{\text{kg water}}}{\text{hr}} \; 2000$$

$$= 2000 \times 1000 \text{ mg/hr.}$$

References

- Himmelblau, D.M., Riggs, J.B., Basic Principles and Calculation in chemical engineering, Prentice Hall.
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