Process Technology and Economics-1
Module-5

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Profitability Measures or Indexes

- Gross profit(GP)=Sales income(S)−Total product cost (TPC)

- Income tax (IT)=Gross profit(GP)× Fractional tax rate (ϕ)

- Net Profit (NP)= Gross profit(GP) − Income tax (IT)=G(1- ϕ)

- Cash flow (A)=Net Profit (NP)+Depreciation=NP+d

***1) Rate of return on investment** (**ROR**)*$=\frac{\text{Profit (GP or NP)}}{\text{Total captial Investment}} \times 100$

or $=\frac{\text{Cash flow (A)}}{\text{Total captial Investment}} \times 100$

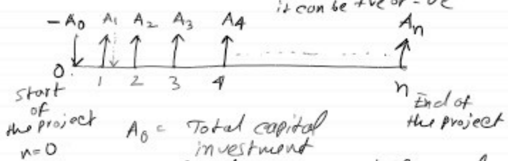
2) Discounted cash flow rate of return based on full-life performance

- **Total present value=$\sum$ Present value of the cash flow**
- $= \frac{A_1}{(1+i)} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3} + \frac{A_4}{(1+i)^4} + \cdots \ldots.. + \frac{A_n}{(1+i)^n} = \sum_0^n \frac{A_n}{(1+i)^n} = 0$
- **Where $i=i_{DCF}$,** Discounted cash flow rate of return
- Higher is $\mathbf{i_{DCF}}$ better is the investment for profitability.

At $i=i_{DCF}$, Total present value=0;

This rate of return represents the after-tax interest rate at which the investment is repaid by proceeds from the project. It is also the maximum after-tax interest rate at which funds could be borrowed for the investment and just *break even* at the end of the service life.

$\pm A_1, \pm A_2, \ldots, A_n$ = cash flow per year
it can be +ve or −ve



A_0 = Total capital investment
initial investment / Borrowed amount
= −20,00000 Rs/-

$$-A_0 + \frac{A_1}{(1+i)^1} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3} + \cdots + \frac{A_n}{(1+i)^n} = 0$$

At discounted cash flow rate of return, i_{DCF},

Total present worth = 0

$$\frac{\pm A_1}{(1+i)^1} + \frac{\pm A_2}{(1+i)^2} + \frac{\pm A_3}{(1+i)^3} + \cdots + \frac{\pm A_n}{(1+i)^n} = A_0$$

nonlinear equation

$i = i_{DCF}$

if $i < i_{DCF}$ (i → available interest rate): LHS $> A_0$ (total capital investment)
(LHS = net present worth of cash flow from 1−n years)
Profitable

if $i > i_{DCF}$: LHS $< A_0$
not profitable.

$i = i_{DCF}$, A_0 will be just break-even or paid by the net cash flow (present value).

$i = i_{DCF}$ A_0 = Total capital investment

$$\frac{A_0}{\text{Total present value } (1-n)} = 1$$

$$A_0 = \sum_1^n \frac{A_n}{(1+i)^n} = \text{total present value}$$

$$NPW = \sum \frac{A_n}{(1+i)^n} - A_0$$

$i = i_{DCF}$, NPW = 0

at i = market interest rate or available interest rate

NPW > 0

→ gives the profit value quantitatively.

$$AE\,\frac{(1+i)^n - 1}{i} = NPW(1+i)^n.$$

AE = Annual equivalent amount.

$$AE = NPW\,\frac{(1+i)^n\, i}{(1+i)^n - 1}$$

PB = pay back period

$$= \frac{F_x + d + TCI(1+i)^n - TCI}{\text{Average cash flow per year}}$$

F_x = Fixed capital investment
d = depreciation.
TCI = Total capital investment

amount of Interest = $TCI(1+i)^n - TCI$

Average cash flow per year

$$= \frac{A_1 + A_2 + A_3 + \cdots + A_n}{n}$$

n = Service Life

Problem

- Consider the case of a proposed project for which the following data apply:
- ***Initial fixed-capital investment*** = $100,000
- ***Working-capital investment*** = $10,000
- ***Service life*** = 5 years
- ***Salvage value*** at end of service life = $10,000

Initial investment=(Fixed + Working) Capital investment =110,000

Year	Predicted after-tax cash flow to project based on total income minus all costs except depreciation, S Year (expressed as end-of-year situation)	Present value i=0.15 (Trial 1)	Present value i=0.175 (Trial 2)
0	-110,000	-110,000	-110,000
1	30,000	26086.96	25531.91
2	31,000	23440.45	22453.6
3	36,000	23670.58	22191.61
4	40,000	22870.13	20984.98
5	43,000	21378.6	19199.02
Total		**117446.7**	**110361.1**

Ratio=Total present value/Initial investment	**=117446.7/110,000 =1.067**	**=110361.1/110,00 =1.003**

		Trial 1	Trial 2
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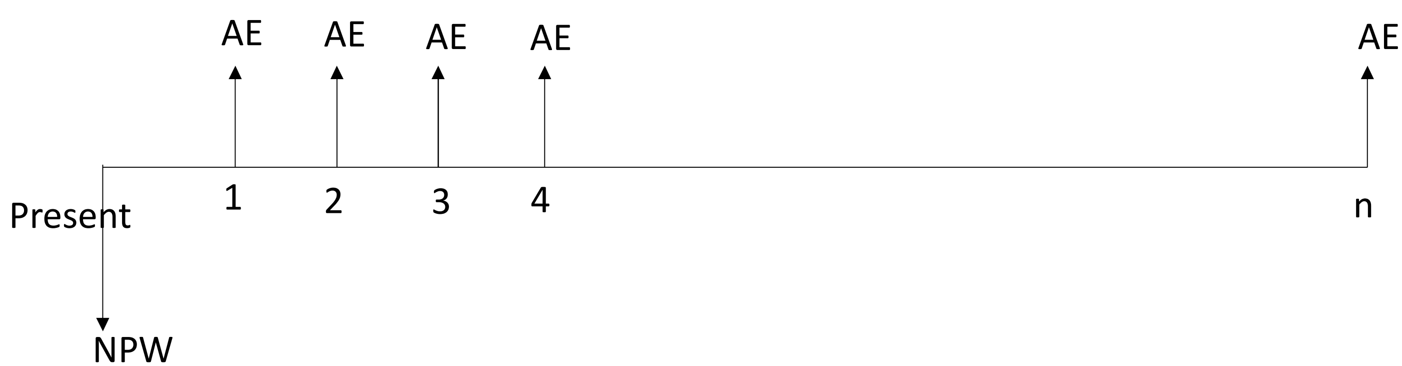
This rate of return represents the after-tax interest rate at which the investment is repaid by proceeds from the project. It is also the maximum after-tax interest rate at which funds could be borrowed for the investment and just *break even* at the end of the service life.

3) Net present worth

- This index gives the rate of return which includes the profit on the project, payoff of the investment, and normal interest on the investment, substitutes the cost of capital at an interest rate *i*for the discounted-cash-flow rate of return
- ***NPW***=$\sum \boldsymbol{Present\ value\ of\ cash\ flow - TCI}$
- **NPW**=$\sum_{0}^{n} \frac{A_j}{(1+i)^j} - TCI$

4) Annual equivalent amount

- It is a hypothetical annuity with uniform annual payment amount equal to AE whose sum of present value is equal to NPW.



- $AE = NPW\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$

4) Payout or payback period (PB)

- $PB = \frac{Depriciable\ fixed\ capital\ investment(FCI)}{Average\ cash\ flow\ per\ year(NP+d)} =$
- $= \frac{Depriciable\ FCI+interest\ rate\ on\ toatal\ capital\ investment(TCI)}{Average\ cash\ flow\ per\ year(NP+d)}$
- $= \frac{F_x+d+TCI(1+i)^n-TCI}{Average\ cash\ flow\ per\ year(NP+d)}$
- $A_{avg} = \left[\sum_1^n \frac{A_j}{(1+i)^j}\right]\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$

5) Capitalized cost K

- $K = K_{FCI} + K_{AOP}$

Capitalized cost of Investment

It is defined as the original cost of the equipment plus the present value of the renewable perpetuity. [It refers to the present worth of cash flow which go on for indefinite period of time.]

→ Perpetuity is an annuity in which periodic payments continue indefinitely. Example operating cost of an equipment.

given:

C_V = current or present value of a piece of equipment.

C_S = Salvage value of the equipment at end of service life.

n = Service life of the equipment.

C_R = Replacement cost.

$\Rightarrow C_R = C_V - C_S$.

If a P ~~sa~~ amount is invested. For n year. ~~The end of~~ At the end of n year C_R. (Replacement cost) will be paid from the interest gained.

i.e. $P(1+i)^n - P = C_R$.

$\Rightarrow P = \dfrac{C_R}{(1+i)^n - 1}$ [This amount is present worth that will be invested].

K = Capitalized cost.

Capitalized cost $K = C_V + \frac{C_R}{(1+i)^n - 1}$.

= Original cost + Present worth of the property.

$= C_V + \frac{C_V - C_S}{(1+i)^n - 1}$.

Problem (1):

For a installed equipment

Current value $C_V = \$12{,}000$.

Salvage value, $C_S = \$2000$

or scrap " ,

service life $n = 10$ years.

$i = 0.06$ (6% compounded yearly).

What is the capitalized cost of the equipment?

$K_{equipment} = 12000 + \frac{12{,}000 - 2000}{(1+0.06)^{10} - 1}$

$K = \$12{,}644.63$.

It is noted that equipent having lower capitalized cost is preferrable.

Equipment A	Equipment B
n_A (service life)	n_B
C_{VA} (current value)	C_{VB}
C_{SA} (salvage value)	C_{SB}
i_A (interest rate)	i_B
OP_A (annual operating expenses)	OP_B

Problem 2

	Reactor A	Reactor B
Initial + installation cost	\$ 25,000	\$ 15,000
Uniform end of year maintenance	\$ 2000	\$ 4,000
Overhaul, end of 3rd year	0	\$ 3,500
Salvage vale	\$ 3000	0
Service life	4 years	6 years.

Intrest rate for the both choices is 8% compounding yearly.

Capitalized cost estimation for reactor A

$K = C_V + P$ (present value of perpetuity)

C_V — present value.

$C_R = C_V - C_{salvage}$.

$$P = \frac{C_R}{(1+i)^n - 1} = \frac{C_V - C_{salvage}}{(1+i)^n - 1}.$$

Reactor A:

$$C_V^A = 25{,}000 + 2{,}000\left[\frac{(1.08)^4 - 1}{0.08\,(1.08)^4}\right]$$

Present value of operating cost [it is same as present value of annuity of amount 2,000.

[Note that $P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$]

$C_V^A = \$31{,}624.25$

$$K_A = 31{,}624.25 + \frac{31{,}624 - 3{,}000}{(1.08)^4 - 1}$$

$= \$111{,}027$

Reactor B:

$$C_V^B = 15{,}000 + 4{,}000\left[\frac{(1.08)^6 - 1}{0.08\,(1.08)^6}\right] + \frac{3{,}500}{(1.08)^3}$$

$$C_V^B = \$36{,}270$$

$$K_B = 36{,}270 + \frac{36{,}270 - 0}{(1.08)^6 - 1}$$

$$= \$98{,}072$$

$K_B < K_A \Rightarrow$ Reactor B is the suitable choice.

Capitalized cost estimation for reactor B

Alternative method

Problem on capitalized cost

	Reactor A	Reactor B
initial + installation cost	→ \$ 25,000	\$ 15,000
Annual maintenance cost	→ \$ 2,000	\$ 4,000
Overhaul, eend of 3rd year cost	→ \$ 0	\$ 3,500
Salvage value	→ \$ 3000	\$ 0
service life	→ 4 years	6 years.
interest rate	8 %.	8 %

which one is more profitable?

Capitalized cost of reactor A ; $K_A = K_{reactor,A} + K_{operation,A}$

Reactor A

$$K_A = \left[C_V + \frac{C_V - C_S}{(1+i)^n - 1}\right]_{reactor,A} + \left[\frac{A}{i}\frac{(1+i)^n - 1}{(1+i)^n}\right]_{operation,A}$$

$$K_A = \left[25{,}000 + \frac{25000-3000}{(1+0.08)^4-1}\right] + \left[\frac{2000}{0.08}\left\{\frac{(1+0.08)^4-1}{(1+0.08)^4}\right\}\right]$$

$$K_A = 86{,}028.22 + 6424.25 = \underline{\$\,92{,}652.47}$$

Reactor B

$$K_B = \left[15{,}000 + \frac{15{,}000-0}{(1+0.08)^6-1}\right] + \left[\frac{4000}{0.08}\left\{\frac{(1+0.08)^6-1}{(1+0.08)^6}\right\}\right]$$

$$+\left[\frac{3{,}500}{(1+0.08)^3}\right]_{overhaul.}$$

$$K_B = 40{,}559.13 + 18{,}491.51 + 2{,}778.41$$

$$K_B = \$61{,}829.05$$

$K_B < K_A$, Thus. reactor B is profitable.

Capitalized cost of → Annual cash flow
→ Annual operating cost.

These are equivalent to annuity sum.

∴ $K_{annuity}$ = Present value or worth of annuity sum

$$= \frac{A}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]$$

$$= \frac{A}{i}\left[1 - (1+i)^{-n}\right]$$

$$K_{\substack{annuity \\ n\to\infty}} = \lim_{n\to\infty} \frac{A}{i}\left[1 - (1+i)^{-n}\right]$$

$$= \frac{A}{i}$$

Problem 3: Capitalized cost estimation for 3 investments

Investment number	Total initial fixed-capital investment, $	Working-capital investment, $	Salvage value at end of service life, $	Service life, years	Annual cash flow to project after taxes,† $	Annual cash expenses‡ (constant for each year), $
1	100,000	10,000	10,000	5	See yearly tabulation §	44,000
2	170,000	10,000	15,000	7	52,000 (constant)	28,000
3	210,000	15,000	20,000	8	59,000 (constant)	21,000

Table (Peters & Timmerhaus)

- Capitalized cost of investment
- *Capitalized cost*= $\boldsymbol{C_R}\frac{(1+i)^n}{(1+i)^n-1}+\boldsymbol{V_s}+\frac{\text{Annual cash expenses}}{i}+\text{Working capital}$
- *Invest no. 1*
- *K*= $\mathbf{90,000}\frac{(1+0.15)^5}{(1+0.15)^n-1}+\mathbf{10,000}+\frac{44,000}{0.15}+10,000=4{,}92{,}000$
- Invest no.2
- *K*= $\mathbf{155,000}\frac{(1+0.15)^7}{(1+0.15)^7-1}+\mathbf{15,000}+\frac{28,000}{0.15}+10,000=460{,}000$
- Invest no.3
- *K*= $\mathbf{190,000}\frac{(1+0.15)^8}{(1+0.15)^8-1}+\mathbf{20,000}+\frac{21,000}{0.15}+15,000=457{,}000$
- Invest no.3 should be recommended.

- Capitalized cost of a plant K=$K_{Machine} + K_{operation}$
- $K_{Machine} = \boldsymbol{C_V} + \frac{\boldsymbol{C_V - C_S}}{\boldsymbol{(1+i)^n - 1}}$
- $K_{operation} = \mathrm{A}\left[\frac{\boldsymbol{(1+i)^n - 1}}{\boldsymbol{i(1+i)^n}}\right]$; A=Annual operating cost

Reference

- Plant Design and Economics for Chemical Engineers, Max S. Peters, K. D. Timmerhaus, 4th Edition, McGraw-Hill Inc.

Thank You

Interest and Investment Cost

a)

Interest and Investment cost

\$1 . \$1 + interest .

Present 1 2 3 4 - - - n-1 n Years.

Interest is the compensation to be paid by the borrower to the lender for using a borrowed capital.

P = Principal at the start of first interest period. or capital sum, \$ or Rs.

n = Total number of interst period.

i = Rate of interest, interest earned by a unit ~~capital~~ of principal in a unit time.

F = Total accumulated sum at the end of n interest periods,

I = Total interest earned after n interest periods.

b)

Interest
- Simple
- Compound
  - Discrete
  - Continuous.

Simple interest: It is paid on the original principal, P.

Simple interest / $I = Pin$; $\Rightarrow F = P + I = P + Pin = P(1 + in)$.

Example: P = \$1000, n = 4 years, simple interest is 16%. (i = 0.16) per annum. calculated interest rate per annum and accumulated sum at the end of 4 years.

$I/\text{annum} = 1000 \times 0.16 \times 1 = \$160$ (in one year).

$I = 1000 \times 0.16 \times 4 = \$640$. (in four years)

$F = P + I = 640 + 1000 = \$\,1640\$$. (accumulated sum).

Compound interest: It is paid based on the accumulated sum.

Same example as earlier → $P = \$1000$; $n = 4$; $i = 0.16$ per annum.

Period	Principal at start of period.	Interest earned	Accumulated Sum
1	1000	\$1000 × 0.16 = \$160	\$1160
2	1160	\$1160 × 0.16 = \$185.6	\$1345.60
3	1345.60	\$1345.60 × 0.16 = \$215.3	\$1560.90
4	1560.90	\$1560.9 × 0.16 = \$249.75	\$1810.64.

$$F = P + I = \underline{P(1+i)^n}.$$

$$= 1000 \times (1 + 0.16)^4 = \$1810.64$$

So $F_{comound} > F_{simple}$ or $I_{compound} > I_{simple}$.

Hence, compound interest is profitable compared to simple interest from lender side.

Problem ①: How much must be invested at present at 16% compounded interest rate annually such that \$20000 can be ~~reco~~ earned after 3 years.

$\therefore \quad F = P(1+i)^n$

$F = 20{,}000$; $n = 3$, $i = 0.16$; $P = ?$

$$P = \frac{F}{(1+i)^n} = \frac{20{,}000}{(1+0.16)^3} = \$12{,}813.15$$

a)

Annuities

An annuity is series of payments (can be equal or unequal) made at equal time interval.

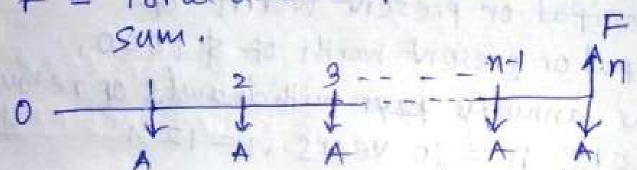
In life insurance plan or in a recurring deposit of a bank for paying debt, a lump sum of capital is accumulated over the periods of installments.

A = uniform periodic payment

n = number of periods in years.

i = interest rate (compounding).

F = Total amount of annuity or accumulated sum.



$$F = A(1+i)^{n-1} + A(1+i)^{n-2} + \cdots + A(1+i) + A$$

Mutipy by $(1+i)$

$$F(1+i) = A(1+i)^{n} + A(1+i)^{n-1} + \cdots + A(1+i)^2 + A(1+i)$$

substructing.

$$Fi = A(1+i)^n - A$$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

b)

Problem: It is required to accumulate \$10,000 by making annuity payments (equal). of 5 years at 12% compounded. interest rate. Find the equal ~~annual~~ annuity payment.

$F = 10{,}000$; $i = 0.12$, $n = 5$, $A = ?$

$F = A[(1+i)^n - 1]\frac{1}{i}$; $A = \frac{Fi}{[(1+i)^n - 1]} = \frac{10{,}000 \times 0.12}{1.12^5 - 1} = \$1574.1$

a)

Capital Recovery

An investor initially deposits an ~~ampe~~ amount P at an annual compound interest rate i.
~~the~~ The investor can withdraw the ~~principal p~~ ~~or~~ accumulated sum in a series of equal year-end amount like annuity withdrawls.

$$\therefore F = P(1+i)^n = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$A = Pi\left[\frac{(1+i)^n}{(1+i)^n - 1}\right]$$

b)

Problem: ① ~~Principal or present worth of~~
For a principal or present worth of \$ 1000, calculate the annuity ~~papp~~ withdrawls of recovery of fund. Take n = 10 years, i = 12%

$$\therefore A = 1000 \times 0.12\left[\frac{(1+0.12)^{10}}{(1+0.12)^{10} - 1}\right]$$

$$= \$176.98$$

Problem: ② calculate the present worth of 8 equal ~~on~~ year-end payments of \$ 223 at 15% annual compound interest rate.

$$P = \left[\frac{(1+i)^n - 1}{(1+i)^n}\right]\frac{A}{i} = \frac{(1.15)^8 - 1}{(1.15)^8} \times \frac{223}{0.15}$$

$$= \$ 1000$$

a)

Nominal and effective interest rate

r = ~~Non~~ Nominal interest rate per year

i = effective / actual interest rate per compounding period.

m = number of compounding periods per year

i_a = effective/actual interest rate per year.

$$F = P\left(1+\frac{r}{m}\right)^{mn} = P(1+i_a)^n$$

$$\Rightarrow \left(1+\frac{r}{m}\right)^{mn} = \cancel{P}(1+i_a)^n$$

$$\Rightarrow i_a = \left(1+\frac{r}{m}\right)^m - 1.$$

b)

Problem 1 ~~Example~~: Find out the most profitable one of ~~12~~ two schemes:

i) 16% compounded annually.

ii) 15% compounded monthly.

iii) 15% compounded semi-annually.

iv) 15% compounded quarterly.

1) The effective interest rate per year are:

i) $i_a = \left(1+\frac{r}{m}\right)^m - 1 \Rightarrow r = 16\%;\ m = 1.$

$= \left(1+\frac{0.16}{1}\right) - 1 = 0.16 = 16\%$

ii) $i_a = \left(1+\frac{0.15}{12}\right)^{12} - 1 = 0.1608 = 16.08\%$

iii) $i_a = \left(1+\frac{0.15}{2}\right)^{2} - 1 = 0.1556 = 15.56\%$

iv) $i_a = \left(1+\frac{0.15}{4}\right)^{4} - 1 = 0.1586 = 15.86\%$

ii) scheme having higher effective interest rate so, it is preferrable.

Problem ②
Example: Find out accumulated sum for a principal amount \$1000 ~~i[illegible]~~ invested for 5 year at a nominal interest rat 18%. compounded semi-annually.

$$F = 1000 \times \left(1 + \frac{0.18}{2}\right)^{5 \times 2}$$

here $r = 0.18$, $m = 2$, $n = 5$ year.

$F = \$2,367.36$

$$i_a = \left(1 + \frac{0.18}{2}\right)^{2} - 1 = 0.1881 = \underline{18.81\%}$$

Continuous interest rate

If number of interest ~~perid~~ periods per year, $m \rightarrow \alpha$, then the scheme is called continuous interest.

For continuous interest effective interest rate is given by the following:

$$i_a = \lim_{m \rightarrow \alpha} \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \lim_{m \rightarrow \alpha} \left[\left(1 + \frac{r}{m}\right)^{\frac{m}{r}}\right]^r - 1$$

$$= e^r - 1 \quad [e = 2.718182\ldots]$$

$$F = P(1 + i_a)^n$$

$$F = P(1 + e^r - 1)^n$$

$$F = Pe^{rn}$$

a)

Effective annual interest rates are compared for different compounding peridos at a nominal interest of 18%.

Compounding Frequency	m	$\frac{r}{m}$	i_a %
annually	1	0.18	18
Semi-annually	2	0.09	18.81
quarterly	4	0.045	19.25
Monthly	12	0.015	19.56
weekly	52	0.00364	19.68
Daily	365	0.000493	19.71
Continuously	∞	0	19.72

Thus continuous interest gives highest amount of effective interest, hence the continuous interest is most preferrable.

b)

Annuity for continuous interest

$$\boxed{F = \bar{A}\left[\frac{e^{rn}-1}{r}\right]}$$

$\bar{A}$ = annuity payments per year.

n = number of years.

r = nominal interest rate per year.

~~Example~~ Problem: ① \$ 8000 is required annually for 25 year. Compare the present worth for i) 6% annual interest rate ii) 6% continuous interest rate.

i) $P = R\,\frac{(1+i)^n-1}{i(1+i)^n} = 8000 \times \frac{1.06^{25}-1}{0.06 \times 1.06^{25}} = \$102{,}264$

ii) $P = R\left[\frac{e^{rn}-1}{r}\right]\frac{1}{e^{rn}} = 8000 \times \frac{e^{0.06\times25}-1}{0.06\; e^{0.06\times25}} = \$108{,}163$

Reference

- Plant Design and Economics for Chemical Engineers, Max S. Peters, K. D. Timmerhaus, 4th Edition, McGraw-Hill Inc.

Thank You

Depreciation

DEPRECIATION

Depreciation is the reduction in value of a physical asset with time due to the following reasons: Physical deterioration, technical advances, economic changes. These factors cause end of the service life of a physical assets like, machinary, equipment, plant etc.

It can be catagorized into three types.

Physical → wear, tear, corrosion, age, etc.

Functional → Obsolescence, decrease in demand, inadequate capacity, closing of enterprise.

Accidents → Accident of plant and its equipment during run time.

The well-designed and well maintained chemical process industries are rarely weared out or declined, until advancement of technology suggests replacement of components with modern-designed counterpart.

Total cost due to depreciation = $\left\{\begin{array}{l}\text{Original value} \\ \text{of property}\end{array}\right.$ − value of property at end of depreciation periods (Salvage value)

$$\boxed{\Rightarrow \text{Original value} = \text{Depreciation value} + \text{Salvage value}}$$

Salvage value

It is the net amount of money obtainable from the sale of used property over and above any charges involved in removal and sale.
It implies that the asset can give some type of further service and is worth more than merely its scrap value or junk value.

→ If the property can not be disposed of as a useful unit, it can often be dismantled and sold as junk to be used again as a manufacturing raw material. The proit obtainable from this type of disposal is known as the scrap or junk.

~~sd~~ Salvage value, scrap value, and service life are usually estimated on the basis of conditions at the time the property is put in use.

Present value

Present value of an asset may be defined as the value of the asset in its condition at the ~~time~~ time of valuation.

Present value are of different types.

a) Book value: The difference between the original cost of a property and all the depreciation charges mad to date.

b) Market value: The price which could be obtained for an asset if it were placed on sale in the open market.

c) Replacement value: The cost necessary to replace an existing property with one at least equally capable of rendering the same service.

<u>Method for Determining ~~D~~ Depreciation:</u>

In general it is determined by two types of method.

1) Arbitrary methods giving no consideration to interest costs. like followings:

   a) Straight-line method.

   b) Declining-balance method.

   c) Sum-of-the-years-digits method.

2) Methds taking into account interest on the investment,

   a) Sinking fund method.

   b) Present-worth method.

1) a) <u>Straight-line method</u>: value of property decreases linearly with time. Equal amounts are charged for depreciation each year throughout the entire service life of the property.

d = annual depriciation, \$/year

V = Original value of the property at start of service life period, \$.

V_s = Salvage value of the property at end of service life, \$.

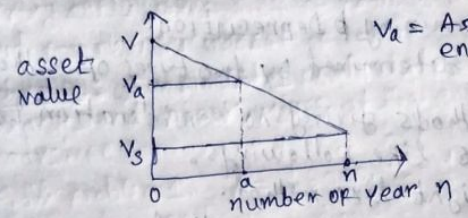
n = service life, year.

$$d = \frac{V - V_s}{n} \Rightarrow V_a = V - da$$

where, V_a = book value, a = number of years in actual use.

~~actual use~~

$$\Rightarrow \boxed{\text{Book value} = V - \left(\frac{V-V_s}{n}\right) a}$$



V_a = Asset value at the end of year a.

End of year	Depreciation charge during year \$	Book value at end of year \$.
0		
1	$d = (V - V_s)/n$	$V - d$
2	do	$V - 2d$
!	do	!
a	do	$V - ad$
!	do	
n	do	$V - nd$

Problem-1: An asset value is \$5000 with salvage value \$1000 and service life 5 year. calculate book value at the end of 4 year and 5 year.

$V = \$5000$, $V_s = \$1000$, $n = 5$.

$$d = \frac{V - V_s}{n} = \frac{5000 - 1000}{5} = \$800$$

Book value at the end of 4 year $V_4 = V - ad$
$= 5000 - 4 \times 800$
$= \$1800$

Book value at the end of 5 year $V_5 = 5000 - 5 \times 800$
$= \$1000$

1) b) Declining- Balance Method (Fixed Percentage Method)

In this method annual depreciation charge (cost) is taken to be a fixed percentage, f of the property value at the beginning of the particular year.

f = fixed percentage factor.

d_a = Depreciation charge during year a.

$= f(1-f)^{a-1}V$

V_a = Book value at end of year $a = V(1-f)^a$

at the end of n years, Book value = Salvage value.

$\Rightarrow V(1-f)^n = V_s$.

$$\boxed{\Rightarrow f = 1-\left(\frac{V_s}{V}\right)^{1/n}}$$

End of year	Depreciation charge	Book value.
0		V
1	$d_1 = fV$	$V_1 = V(1-f)$
2	$d_2 = f(1-f)V$	$V_2 = V(1-f)^2$
3	$d_3 = f(1-f)^2V$	$V_3 = V(1-f)^3$
a	$d_a = f(1-f)^{a-1}V$	$V_a = V(1-f)^a$
n	$d_n = f(1-f)^{n-1}V$	$V_s = V(1-f)^n = V_s$

Note: It is observed that decling-balance of fixed percentage method made larger depreciation charges at the beginning of the years and the depreciation charge reduces with increase in year. It allowes the investment to be paid back more rapidly during early years, i.e., it reduces the income tax load for new business.

Problem 2: For problem 1, calculate depreciation charges and book values at the end of 4 and 5 years.

From problem 1, $V = \$5000$, $V_s = \$1000$, $n = 5$.

$$f = 1 - \left(\frac{V_s}{V}\right)^{1/n} = 1 - \left(\frac{1000}{5000}\right)^{1/5}$$

$$f = 0.2752$$

$$d_4 = f(1-f)^3 V = 0.2752(0.7248)^3 5000$$

$$d_4 = \$523.88$$

$$d_5 = 0.2752(0.7248)^4 \times 5000$$

$$d_5 = \$379.71$$

$$V_4 = V(1-f)^4 = 5000 \times (0.7248)^4$$

$$V_4 = \underline{\$1,379.88}$$

$$V_5 = V(1-f)^5 = \underline{\$1,000.14}$$

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i) c) Sum of Years Digits Method

In this method depreciation charge d_a during year a is given by,

$$d_a = \frac{n-a+1}{\sum_{i=1}^{n} i} (V - V_s)$$

$$d_a = \frac{2(n-a+1)}{n(n+1)} (V - V_s)$$

Hence, book value of property at end of year a, V_a is given by,

$$V_a = V - \frac{2(V-V_s)}{n(n+1)} \sum_{i=n-a+1}^{n} i$$

Now $$\sum_{i=n-a+1}^{n} i = \sum_{i=1}^{n} i - \sum_{i=1}^{n-a} i$$

$$= \frac{n(n+1)}{2} - \frac{(n-a)(n-a+1)}{2}$$

$$= \frac{a(2n+1-a)}{2}$$

$$\therefore V_a = V - \frac{\cancel{2}(V-V_s)}{n(n+1)} \cdot \frac{a(2n+1-a)}{\cancel{2}}$$

$$\Rightarrow V_a = V_s + \frac{(V-V_s)(n-a)(n-a+1)}{n(n+1)}$$

<u>Problem 3</u>: For problem 1, calculate depreciation charges and book values at the end of 4 and 5 years. $V = \$5000$, $V_s = \$1000$, $n = 5$.

$$d_a = \frac{2(n-a+1)}{n(n+1)}(V-V_s)$$

$$\Rightarrow d_4 = \frac{2(5-4+1)}{5(5+1)}(5000-1000) = \underline{\$533}$$

$$\Rightarrow d_5 = \frac{2(5-5+1)}{5(5+1)}(5000-1000) = \underline{\$267.}$$

$$V_a = V_s + \frac{(V-V_s)(n-a)(n-a+1)}{n(n+1)}.$$

$$V_4 = 1000 + \frac{(5000-1000)(5-4)(5-4+1)}{5(5+1)}$$

$$\underline{V_4 = \$1266.66}$$

$$V_5 = 1000 + \frac{(5000-1000)(5-5)(5-5+1)}{5(5+1)}$$

$$\underline{V_5 = \$1000}$$

2) a) <u>Sinking Fund Method</u>

It accounts the effect of interest rate.

The method is applicable for those properties that did not undergo heavy service demands during its early life and having less chance of losing its value.

A hypothetical annuity fund is setup into which a constant amount of money is set aside each year. At the end of service life, total money with its interest in the fund should be equal to the total amount of depriciation $(V-V_s)$.

$\therefore$ d = depreciation per year equivalent to annuity fund.

$$\Rightarrow \left[\frac{(1+i)^n-1}{i}\right]d = V-V_s$$

$$\Rightarrow d = (V-V_s)\left[\frac{i}{(1+i)^n-1}\right]$$

after a year, total amount of depreciation $(V-V_a)$ is realated to d by,

$$d = (V-V_a)\left[\frac{i}{(1+i)^a-1}\right]$$

Equating from above ② equations,

$$V_a = V-(V-V_s)\left[\frac{(1+i)^a-1}{(1+i)^n-1}\right]$$

Problem 4: For problem 1, calculate depreciation charge and book values at the end of 4 and 5 years.

$V = \$5000$, $V_s = \$1000$, $n = 5.$, Assume interest rate 10%., $i = 0.1$.

$$d = (V - V_s)\left[\frac{i}{(1+i)^n - 1}\right]$$

$$d = (5000 - 1000)\frac{0.1}{(1+0.1)^5 - 1} = \$655.$$

Book values $V_4 = 5000 - (5000 - 1000)\left[\frac{(1.1)^4 - 1}{(1.1)^5 - 1}\right]$

$V_4 = \$1959$

$$V_5 = 5000 - (5000 - 1000)\left[\frac{(1.1)^5 - 1}{(1.1)^5 - 1}\right]$$

$V_5 = \$1000$

Depreciation and cash flow

Faderal income tax is charged on gross earning.

S = Total income or revenue.

C = Total annual costs not including depreciation.

d = Annual depreciation charge (cost).

ϕ = Fractional annual tax rate.

$\therefore$ Net cash flow to company after tax

$$= (S - C - d)(1 - \phi) + d.$$

$$= (S - C)(1 - \phi) + \phi d$$

$\therefore$ ϕd is the tax credit due to depreciation.

Reference

- Plant Design and Economics for Chemical Engineers, Max S. Peters, K. D. Timmerhaus, 4th Edition, McGraw-Hill Inc.

Thank You