Direct Search Method (Derivative free Methods)

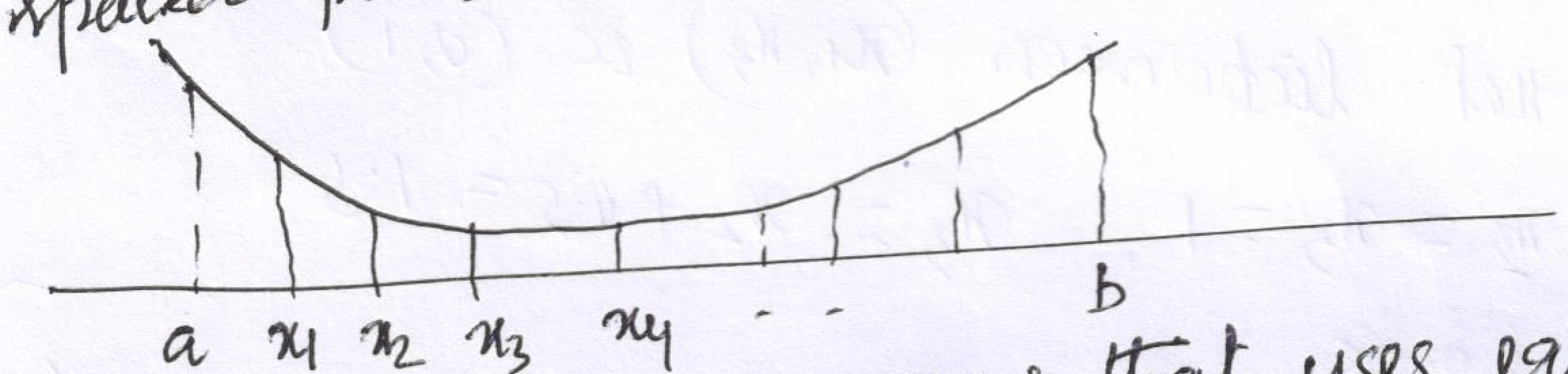
- Exhaustive search method
- Bounding phase method
- Interval halving method
- DChoto mous search
- Fibonacci search
- Gloden section search
- Interval Halving method

Gradient based method (Derivative based method)

- Newton- Raphson Techneque

Exhaustive search method

It is the simplest of all search methods. The optimum of a function is bracketed by calculating the function values at a number of equally spaced points



The exhaustive search method that uses equally spaced points usually search begin from lower bound on the variable and three consecutive function values are compared at a time based on the assumption of unimodality function. Based on the outcome of comparison, the search is either terminated or continued by replacing one of the three points with a new points

Algorithm

Step 1

Set $x_1 = a$, $\Delta x = (b-a)/n$ (n, is number of intermediate points)

$x_2 = x_1 + \Delta x$, $x_3 = x_2 + \Delta x$

Step 2

If $f(x_1) \geq f(x_2) \leq f(x_3)$, the minimum point lies between (x_1, x_3)

Hence terminate

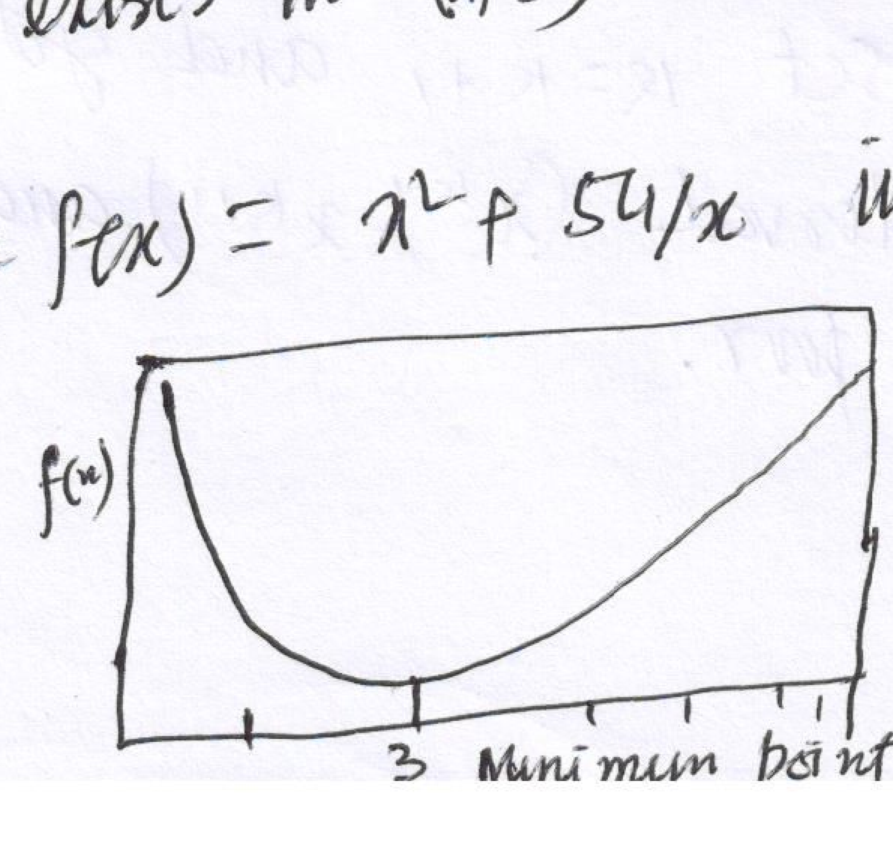
else $x_1 = x_2$, $x_2 = x_3$, $x_3 = x_2 + \Delta x$

Step 3

is $x_3 \leq b$ if yes goto step 2.

Else no minimum exists in (a, b) or a boundary point (a or b) is the minimum point.

Example: Minimize $f(x) = x^2 + 54/x$ in the interval $(0, 5)$



The unimodal single variable function used in the exercise problems

The plot show that the minimum lies at $x = 3$

$$f(3) = 27, \quad f'(3) = 0, \quad f''(3) = 6$$

According to sufficiency condition $x = 3$ is a local minimum

Now consider $n = 10$ for exhaustive search

Step 1 According to parameter chosen

$$x_1 = a = 0 \quad \text{and} \quad b = 5$$

$$\Delta x = (5-0)/10 = 0.5$$

we set $x_2 = 0.5 + 0 = 0.5, \quad x_3 = x_2 + 0.5 = 1.5$

Step 2 Computing function values at various point we have

$f(0) = \alpha \quad f(0.5) = 108.75, \quad f(1.0) = 55.00$

$f(x_1) > f(x_2) > f(x_3)$

So minimum does not lie between (x_1, x_3) ie $(0, 1)$

Set $x_1 = 0.5, \quad x_2 = x_3 = 1, \quad x_3 = x_2 + 0.5 = 1.5$

$$f(1.5) = 38.25$$

$f(x_1) > f(x_2) > f(x_3)$ and minimum does not lie between $(0.5, 1.5)$

Repeat the process till $f(2.5) = 27.85, \quad f(3.0) = 27.00 \quad f(3.5) = 27.68$

Solution lies between $(2.5, 3.5)$

The accuracy solution $2(a-b)/n = 2(5-0)/10 = 1.0$

If more accurate solution is required, divide into more number of parts by increasing n.

Bounding phase method

Step 1
Choose an initial guess $x^{(0)}$ and an increment Δx, set $k = 0$

Step 2
if $f(x^0 - |\Delta x|) \geq f(x^0) \geq f(x^0 + |\Delta x|)$ then Δx is +ve

$f(x^0 - |\Delta x|) \leq f(x^0) \leq f(x^0 + |\Delta x|)$ then Δx is −ve

else goto step 1

Step 3 set $x^{k+1} = x^k + 2^k \Delta$

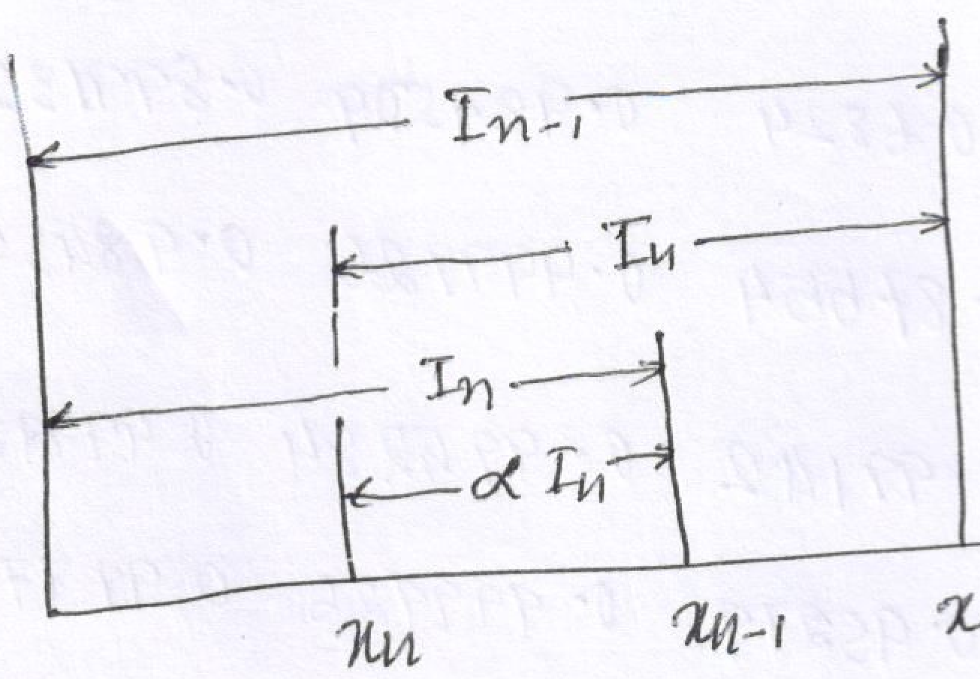
Step 4 if $f(x^{k+1}) < f(x^k)$. set $k = k+1$, and goto step 3

Else, the minimum lies in the interval (x^{k-1}, x^{k+1}) and terminate

if Δ is large accuracy is poor.

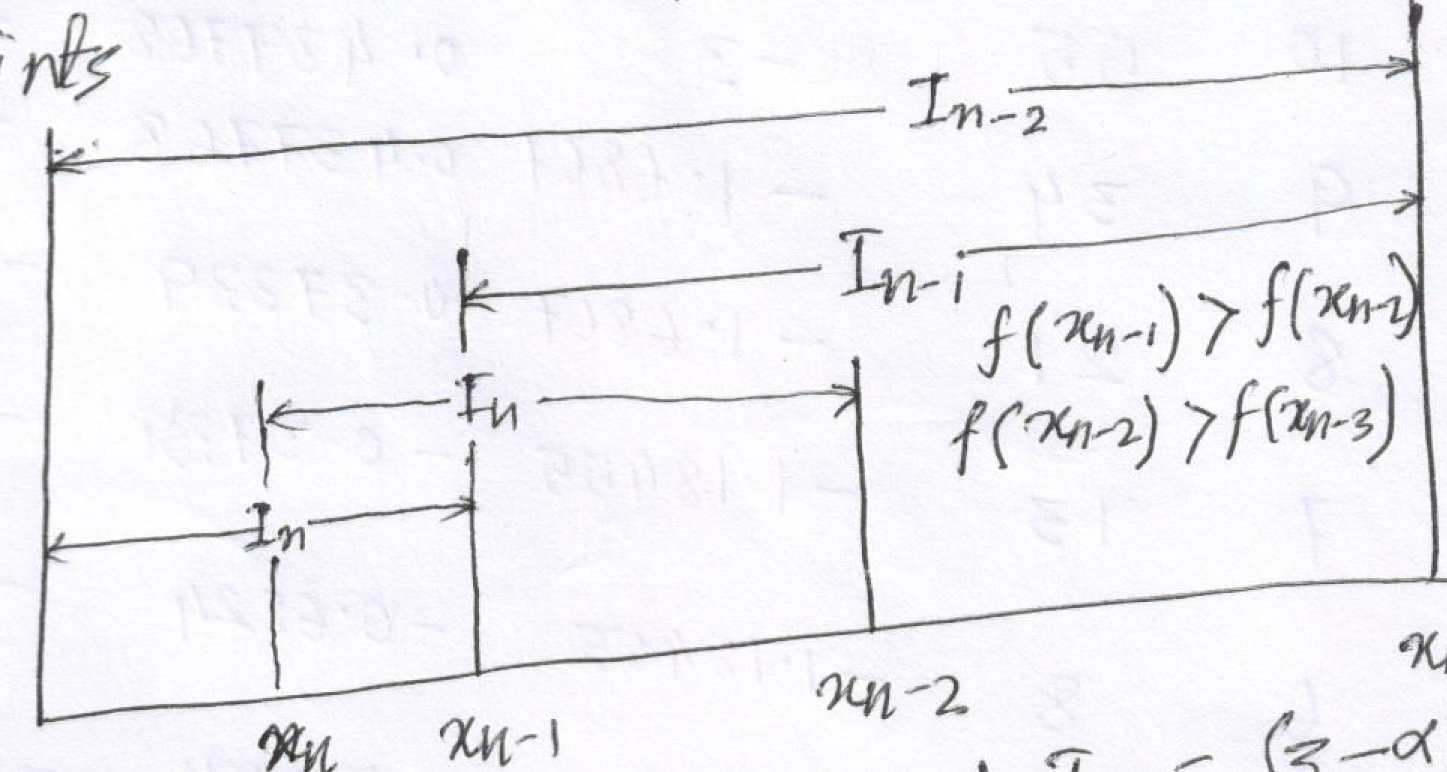
Fibonaci Search Method

- If a number of test points is specified in advanced, then we can do slightly better than the Golden Section Search Method
- This method has the largest interval reduction compared to other methods using the same number of test points



$f(x_{n-1}) > f(x_{n-2})$

$\alpha I_n = \varepsilon I_1$



$f(x_{n-1}) > f(x_{n-2})$
$f(x_{n-2}) > f(x_{n-3})$

$$I_n = \frac{1}{2} I_{n-1} + \frac{1}{2} \alpha I_n \Rightarrow I_{n-1} = (2-\alpha) I_n$$

$$I_{n-2} = I_{n-1} + I_n = (2-\alpha) I_n + I_n = (3-\alpha) I_n$$

$$\frac{\text{Length of final interval of uncertainty}}{2 \times \text{Length initial interval of uncertainty}} \leq \frac{5}{100}$$

$$I_{n-1} = (2-\alpha) I_n$$

$$I_{n-2} = I_{n-1} + I_n = (2-\alpha) I_n + I_n = (3-\alpha) I_n$$

$$I_{n-3} = I_{n-2} + I_{n-1} = (3-\alpha) I_n + (2-\alpha) I_n = (5-2\alpha) I_n$$

$$I_{n-4} = I_{n-3} + I_{n-2} = (5-2\alpha) I_n + (3-\alpha) I_n = (8-3\alpha) I_n$$

$$\vdots$$

$$I_{n-k} = (F_{k+2} - F_k \alpha) I_n$$

where $F_{k+2} = F_{k+1} + F_k$, $F_1 = F_2 = 1$

$$\frac{L_n}{2} \leq \frac{1}{20} L_0$$

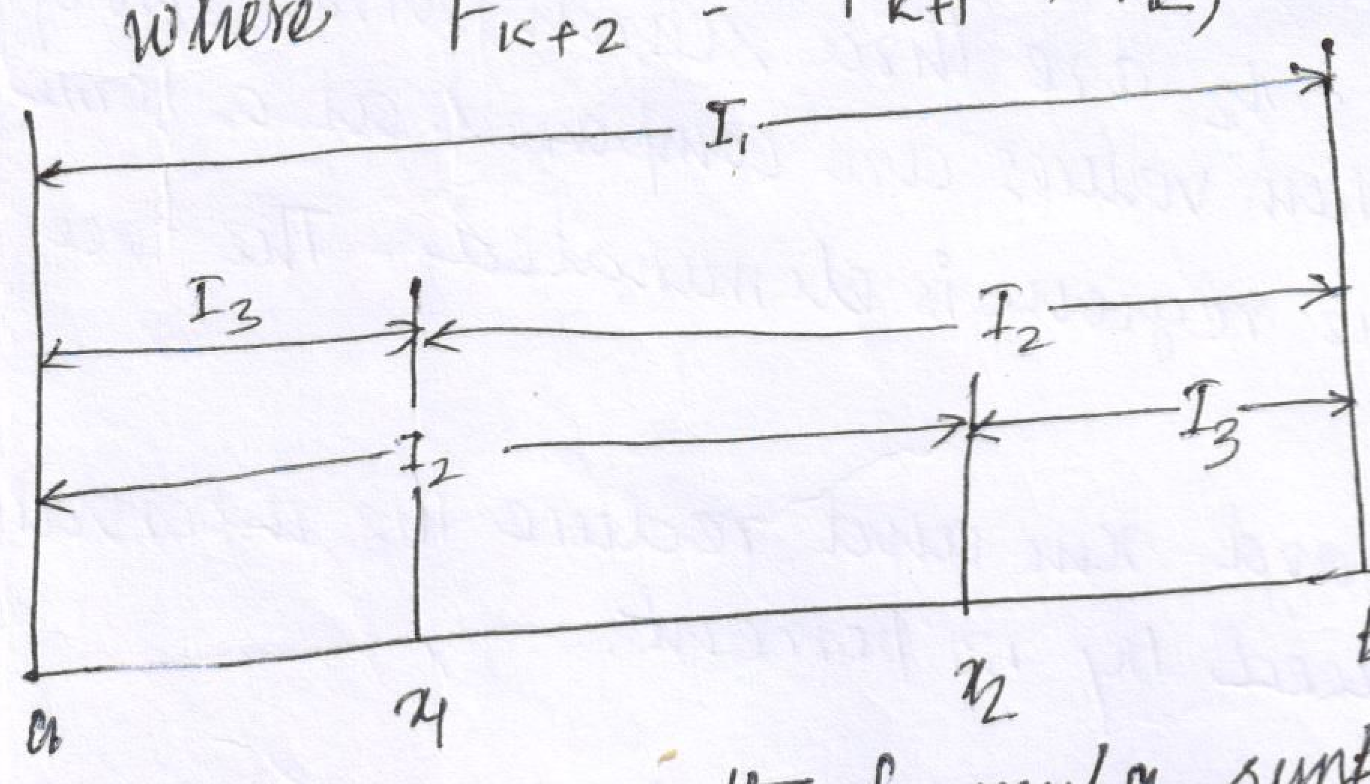
$$\frac{L_n}{L_0} \leq \frac{1}{10}$$

$$L_n = \frac{L_0}{F_n} \leq \frac{1}{10}$$

$$F_n \geq 10$$

$$I_1 = (F_{n+1} - F_{n-1}\alpha) I_n = F_{n+1} I_n - F_{n-1}(\varepsilon I_1)$$

$$I_n = \left(\frac{1 + F_{n-1}\varepsilon}{F_{n+1}}\right) I_1$$



$$x_1 = a + I_3 = a + (F_{n-1} + F_{n-3}\alpha) I_n = a + F_{n-1} I_n + F_{n-3}(\varepsilon I_1)$$

$$x_2 = a + I_2 = a + (F_n - F_{n-2}\alpha) I_n = a + F_n I_n + F_{n-2}(\varepsilon I_1)$$

If $\varepsilon = 0$, then the formula simplify to

$$I_n = \frac{I_1}{F_{n+1}}, \quad x_1 = a + \frac{F_{n-1}}{F_{n+1}}(b-a)$$

$$x_2 = a + \frac{F_n}{F_{n+1}}(b-a)$$

Fibonacci Search Method to maximize $f(x)$ over the interval $a \leq x \leq b$

Step 1 Initialize: choose the number of test points n

Step 2 Define the test points

$$x_1 = a + \frac{F_{n-1}}{F_{n+1}}(b-a), \quad x_2 = a + \frac{F_n}{F_{n+1}}(b-a)$$

Step 3: calculate $f(x_1)$ and $f(x_2)$

Step 4: For a maximization problem
if $f(x_1) \leq f(x_2)$ then $a = x_1$, $x_1 = x_2$
else $b = x_2$, $x_2 = x_1$
$n = n-1$. Find the new x_1 or x_2 using the formula in step 2

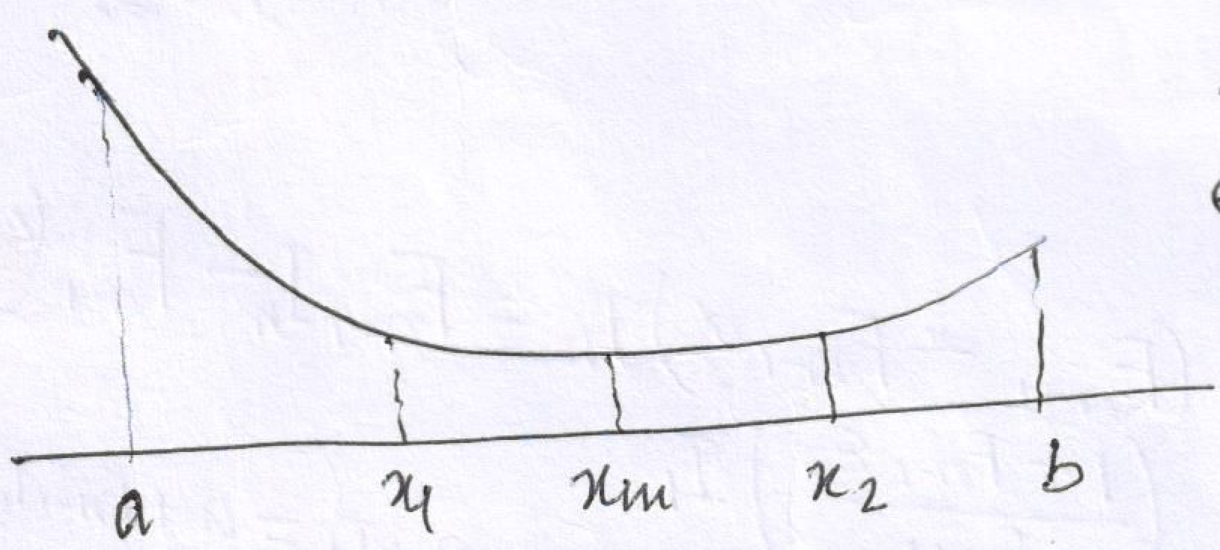
Step 5 If $n > 1$ return to step 3

Step 6 Estimate x^* as the midpoint of the final interval $x^* = \frac{a+b}{2}$ and compute $Max = f(x^*)$

n	Fn	a	b	x_1	x_2	$f(x_1)$	$f(x_2)$
13	233						
12	144	-3	6	0.437768	2.562232	-1.06718	-11.6895
11	89	-3	2.562232	-0.87554	0.437768	0.984509	-1.06718
10	55	-3	0.437768	-1.6867	-0.87554	0.52845	0.984509
9	34	-1.6867	0.437768	-0.87554	-0.37339	0.984509	0.607361
8	21	-1.6867	-0.37339	-1.18455	-0.87554	0.965942	0.984509
7	13	-1.18455	-0.37339	-0.87554	-0.6824	0.984509	0.899132
6	8	-1.18455	-0.6824	-0.99142	-0.87554	0.999926	0.984509
5	5	-1.18455	-0.87554	-1.06867	-0.99142	0.995284	0.999926
4	3	-1.06867	-0.87554	-0.99192	-0.95279	0.999926	0.997771
3	2	-1.06867	-0.95279	-1.03004	-0.99142	0.999097	0.999926
2	1	-1.03004	-0.95279	-0.99142	-0.99142	0.999926	0.999928
1	1	-1.03004	-0.99142				

$$x^* = -1.01073 \quad \text{and} \quad Max = 0.99985$$

Interval halving method



Three points x_1, x_m and x_2 used in the interval halving

Figure shows a region in the interval (a,b). Three points divide the search space into four regions. The fundamental rule for region elimination is used to eliminate a portion of search space based on function values at three choosen points. x_1, x_m, x_2 are three search points. Two of the function values are compared at a time and some region is eliminated. The three probabilities are

(i) If $f(x_1) < f(x_m)$ minimum cannot lie beyond x_m and reduce the interval from (a,b) to (a, x_m) search space is reduced by 50 percent.

(ii) If $f(x_1) > f(x_m)$ minimum cannot lie in the interval (a, x_1). The point x_1 is $1/4$ in search space, hence reduction is 25 percent

Then compare and to eliminate further 25% of search space. continue the process till small enough interval is found

-Algorithm-

Step 1 choose lower bound a and upper bound b and a small value ε for desired accuracy $x_m = (a+b)/2$, $L_0 = L = b-a$ compute $f(x_m)$

Step 2 set $x_1 = a + L/4$, $x_2 = b - L/4$. compute $f(x_1)$ and $f(x_2)$

Step 3 If $f(x_1) < f(x_m)$ set $b = x_m$, $x_m = x_1$, go to step 5, else goto step 4

Step 4 If $f(x_2) < f(x_m)$ set $a = x_m$, $x_m = x_2$ goto step 5, else $a = x_1$, $b = x_2$ go to step 5

Step 5 calculate $L = b-a$. If $|L| < \varepsilon$, terminate. Else goto step 2.