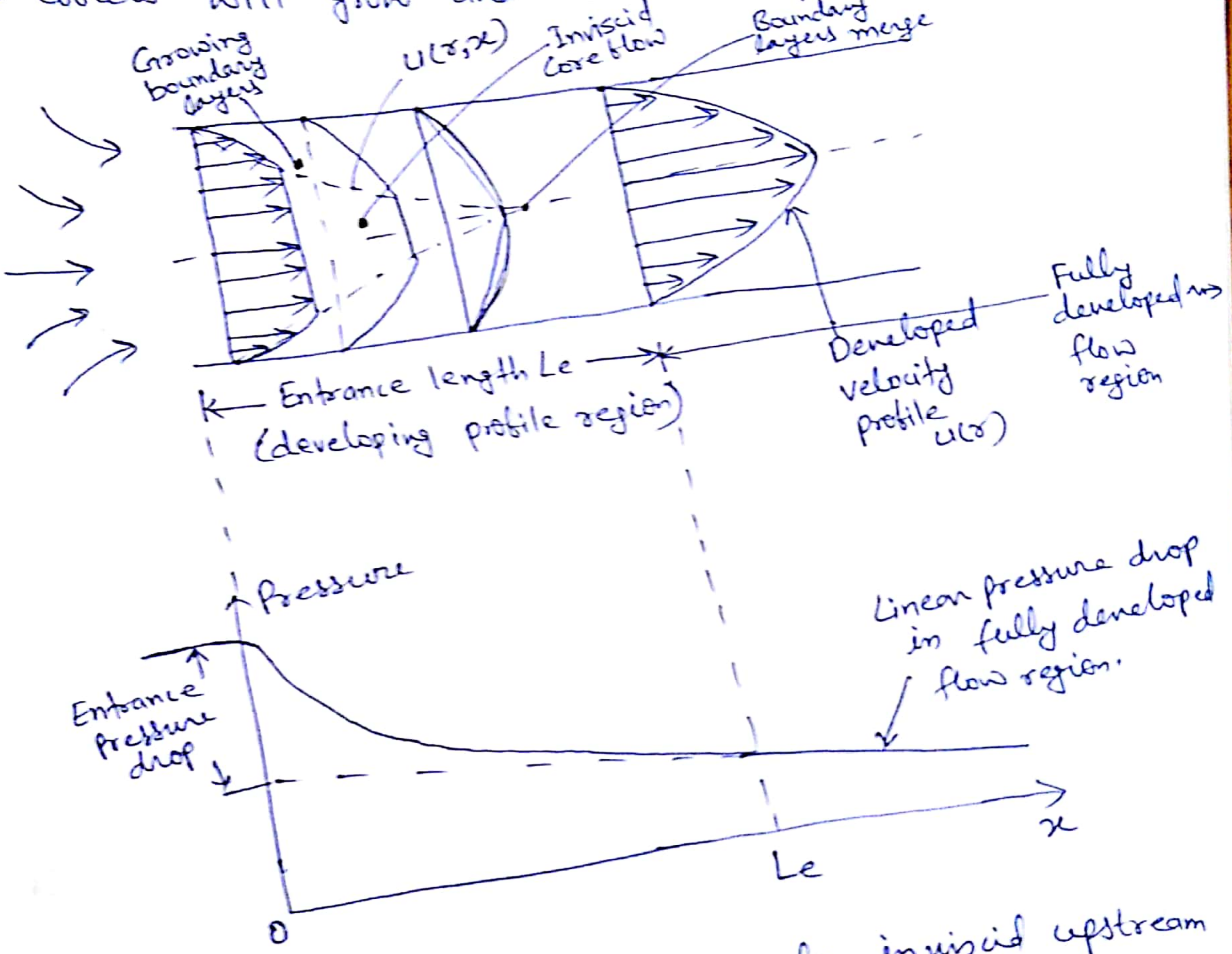
Internal, External Flows

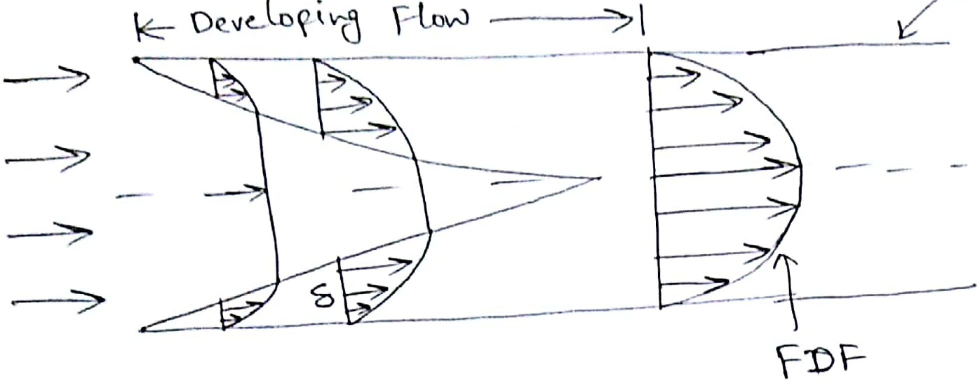
Both laminar and turbulent flow may be either internal, i.e., "bounded" by walls, or external and unbounded.

An internal flow is constrained by the bounding walls, and the viscous effects will grow and meet and permeate the entire flow.



There is an <u>entrance region</u> where a nearly inviscid upstream flow converges and enters the tube. viscous boundary layers grow downstream, retarding the axial flow $u(r,x)$ at the wall and there by ~~resulting~~ accelerating the center-core flow to maintain the incompressible continuity eqn.

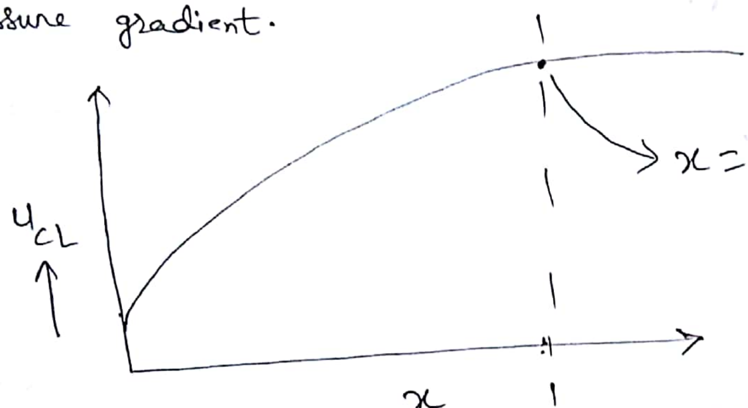
Pipe Flow



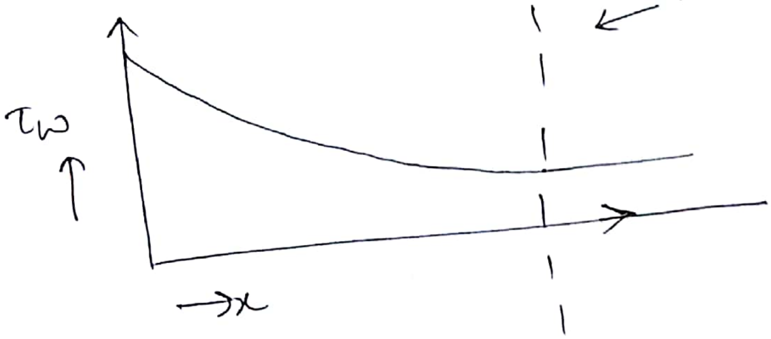
Pipe flow (Internal flow)

In the core region, the velocity of fluid is going to increasing due to pressure gradient.

U_{CL} = velocity of fluid at centre line



$x = l_{entrance}$ where FDF flow and U_{CL} is constant.



wall shear stress

$\frac{\mu U_c \uparrow}{\delta \uparrow}$ (Both increasing as we go inside pipe)

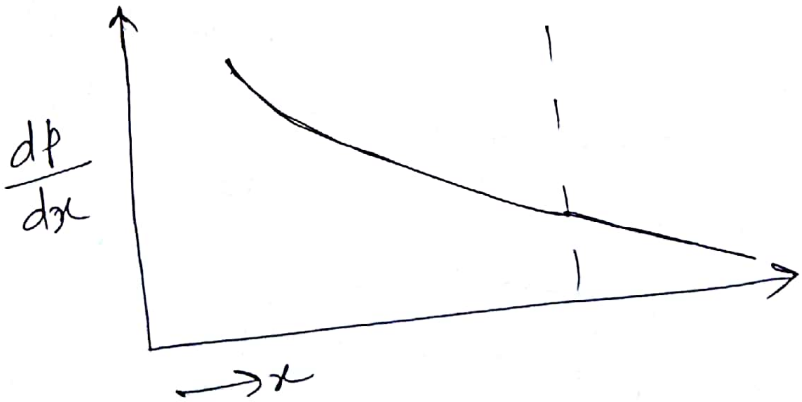
But $\delta \uparrow$ with faster rate

(as $\delta \to 0$ at entering

δ = finite, large at some δ

thus $\mu \left(\frac{U_c}{\delta}\right)$ is decrease

$\therefore$ wall shear stress decreases



After FDF, τ_w constant.

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Flow of viscous fluid Through circular pipe

$$Re_e = \frac{\rho V D}{\mu}$$

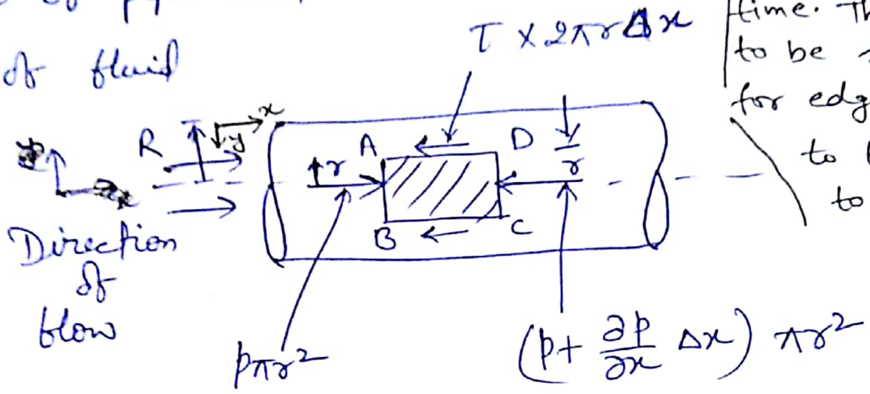
Where, ρ = Density of fluid flowing through pipe

V = Average velocity of fluid

D = Diameter of pipe and

μ = viscosity of fluid

A long tube carries a fluid which is initially at rest. A sudden pr. gradient is applied across the tube. Its magnitude is $\frac{dp}{dx}$ and it remains constant for all time. The tube is assumed to be sufficiently long for edge effects related to flow development to be insignificant.



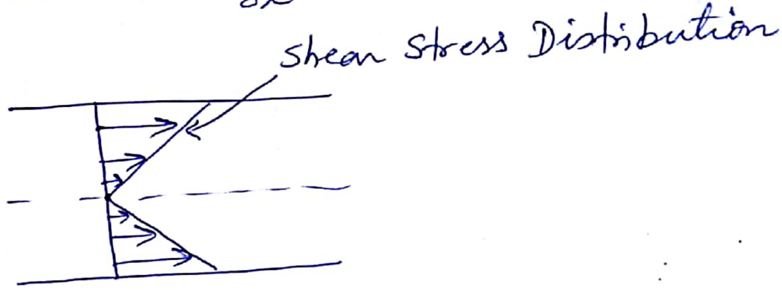
Newton's 2nd law of motion

$$\Sigma F = m a_x$$

$$p\pi r^2 - p\pi r^2 - \frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau 2\pi r \Delta x = 0$$

$$\Rightarrow -\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau 2\pi r \Delta x = 0 \Rightarrow -\frac{\partial p}{\partial x} r - 2\tau = 0$$

$$\therefore \tau = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{---(i)}$$



⇒ Velocity Distribution

$$\tau = \mu \frac{du}{dy}$$

$$y = R - r \text{ and } dy = -dr \Rightarrow \tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

Now, from (i), $-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2}$ or, $\frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x} r$

Integrating w.r.to 'r'

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{r^2}{2} + C \quad \text{---(ii)}$$

BC at $r = R$, $u = 0$

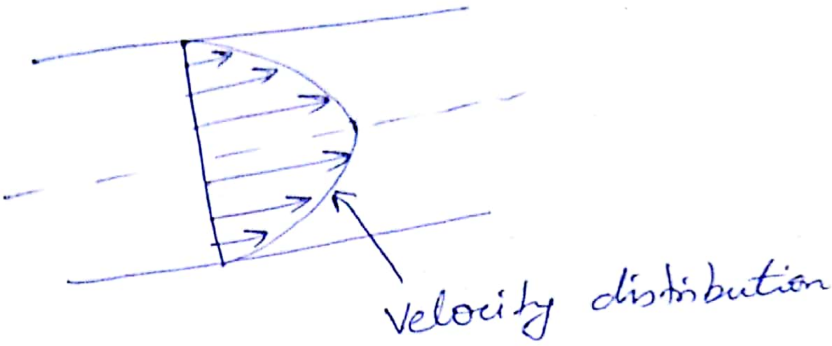
$$0 = \frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{R^2}{2} + C$$

$$\Rightarrow C = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{R^2}{2}$$

Now from (ii)

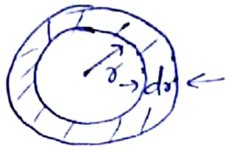
$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right]$$



⇒ Ratio of Max^m vel to Avg vel.

$r=0$, $\boxed{u_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2}$



dQ = velocity at a radius r × area of ring element

$$= u \times 2\pi r\,dr$$
$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] 2\pi r\,dr$$

$$Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2)\,2\pi r\,dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)2\pi\int_0^R (R^2 - r^2)\,r\,dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)2\pi\int_0^R (R^2 r - r^3)\,dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)2\pi\left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)2\pi\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)2\pi\frac{R^4}{4} = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4$$

∴ Average velocity, $\bar{u} = \frac{Q}{Area} = \frac{\frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4}{\pi R^2}$

or, $\boxed{\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2}$

$$Q = \frac{\pi \Delta P R^4}{8\mu L}$$

$$\frac{\Delta P}{L} = \frac{8Q\mu}{\pi R^4}$$

$Q \propto R^4$

$Q \propto \frac{1}{\mu}$

$$\bar{V} = \frac{Q}{\pi R^2}$$

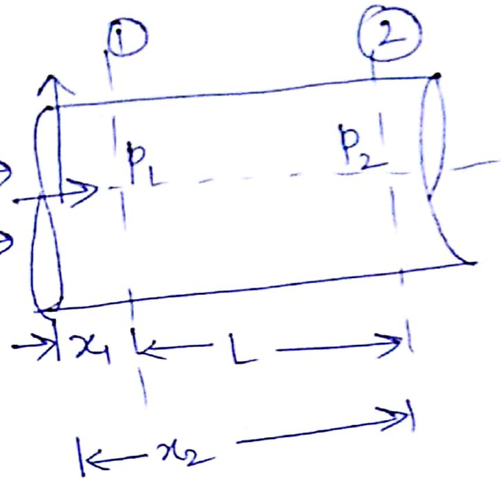
$$\frac{\Delta P}{L} = \frac{8\bar{V}\pi R^2\mu}{\pi R^4} = \frac{8\bar{V}\mu}{R^2}$$

$$\boxed{\frac{\Delta P}{L} = \frac{32\mu\bar{V}}{D^2}}$$

$$\frac{u_{max}}{\bar{u}} = \frac{\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2} = 2.0 \Rightarrow \boxed{\frac{u_{max}}{\bar{u}} = 2}$$

⇒ Drop of Pressure for a given length (L) of pipe

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$



or, $$\left(-\frac{\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

Integrating, $$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\,dx$$

$$\Rightarrow -[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2] \text{ or}$$

$$(p_1 - p_2) = \frac{8\mu\bar{u}}{R^2}(x_2 - x_1)$$

$$= \frac{8\mu\bar{u}}{R^2}L = \frac{8\mu\bar{u}L}{(D/2)^2}$$

$$\Rightarrow (p_1 - p_2) = \frac{32\mu\bar{u}L}{D^2}$$, where $p_1 - p_2$ is the drop of Pr.

loss of pressure head $= \frac{p_1 - p_2}{\rho g}$

$$\therefore \frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$$

Hagen – Poiseuille Formula ⇒

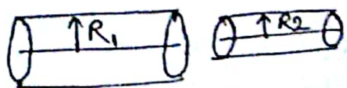
$Q = AV_{av}$

$$p_1 - p_2 = \frac{32\mu QL}{A\,D^2} = \frac{32\mu QL}{\pi R^2\,R^2 4} = \frac{8\mu LQ}{\pi R^4}$$

valid for Re < 2000 $$\boxed{\Delta P = \frac{8\mu LQ}{\pi R^4}}$$ $$\boxed{Q = \frac{\pi\Delta P R^4}{8\mu L}}$$ Volumetric flow rate in a pipe when there is laminar flow

So, volumetric flow rate directly proportional to radius of pipe as 4th power $Q \propto R^4$; $Q \propto \frac{1}{\mu}$ when $\Delta P \to$ const.

$Q \to$ fix



$R_2 = \frac{R_1}{2}$; $\frac{\Delta P}{L} \to \frac{1}{2^4}$

So, when half pipe dia ΔP increases 16 times. 16 times increase

3

If there is a θ vel. that is going to break the symmetry in θ-dir so, not all pts. in θ-dir equivalent.

Ass^m

1) Axis-symmetric flow

$\Rightarrow \frac{\partial}{\partial\theta}(v_z) = 0;\ v_\theta = 0$

(no variation in θ-direction)

2) Steady flow, $\frac{\partial}{\partial t}(\,) = 0$

3) Fully developed flow

$\Rightarrow \frac{\partial}{\partial z}(v_z) = 0$

4) Laminar flow

~~fact~~ Hagen-Poiseuille Flow

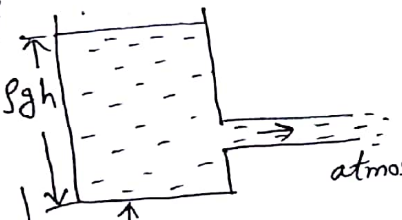
Pipe Flow

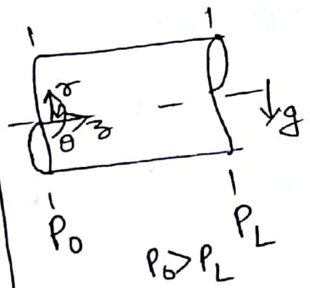
→ what is vel. distribution of fluid in pipe.

→ what is relation b/w pr. drop & flow rate (volumetric)?

→ why do this?

This is one of the most practically encountered flow geometry in chemical processing industries that you have plants pipe flow in pipes of various dimensions diameters in any chemical plant.





$P_0 > P_L$

$\underset{\downarrow}{\Delta P} \equiv P_0 - P_L$

due to this pr. diff.; across ends of pipe in pipe, there is flow in pipe.

Pipe flow:

Reynolds number: $Re = \frac{\overset{\text{avg. vel}}{\bar{V}} D \rho}{\mu}$

$\bar{V} = \frac{Q}{\pi R^2}$

Experiments $Re > 2000 \Rightarrow$ NOT laminar

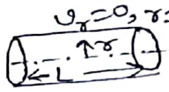
$Re < 2000 \Rightarrow$ Laminar

Mass (continuity eq^n): $\nabla \cdot v = 0$

$$\frac{1}{r}\frac{\partial}{\partial r}(r \cdot v_r) + \frac{1}{r}\underbrace{\cancel{\frac{\partial v_\theta}{\partial \theta}}}_{\text{symm.}\ 0} + \underbrace{\cancel{\frac{\partial v_z}{\partial z}}}_{\text{FDF}\ 0} = 0$$

$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(r v_r) = 0 \Rightarrow \frac{\partial}{\partial r}(r v_r) = 0$

$\Rightarrow r v_r = \text{const.} \Rightarrow v_r = \frac{c_1}{r}$



$v_r = 0 = \text{const.}$ at $r = R$ for $0 \le x \le L$

$\Rightarrow 0 = \frac{c_1}{R} \Rightarrow \boxed{c_1 = 0}$; $v_r = 0$ everywhere in flow (there is no normal vel.)

$v_r = 0 = v_\theta$; $v_z(r, \cancel{\theta}, \cancel{z}, \cancel{t})$ — θ-symm, FDF, 0 steady

$\boxed{v_z(r)}$

N-S eq^n

cylindrical coordinates (r, θ, z)

r-mom

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

θ-mom

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

Z-mom

$$\rho\left(\underbrace{\cancel{\frac{\partial v_z}{\partial t}}}_{\text{S.S.}\ 0} + \underbrace{\cancel{v_r}}_{0}\frac{\partial v_z}{\partial r} + \underbrace{\cancel{\frac{v_\theta}{r}}}_{\text{axisymm}\ 0}\frac{\partial v_z}{\partial \theta} + v_z\underbrace{\cancel{\frac{\partial v_z}{\partial z}}}_{\text{FDF}\ 0}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\underbrace{\cancel{\frac{\partial^2 v_z}{\partial \theta^2}}}_{\text{axisymm}\ 0} + \underbrace{\cancel{\frac{\partial^2 v_z}{\partial z^2}}}_{\text{FDF}\ 0}\right] + \cancel{\rho g_z}^{\,0}$$

The energy eqⁿ

RTT

$$\frac{dN_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, dV + \int_{CS} \eta \rho \, \underline{v} \cdot d\underline{A} \quad \text{——— (1)}$$

This is beautiful but only occasionally useful, when the coordinate system is ideally suited to the control volume selected.

- If heat dQ is added to the system or work dW is done by the system, the system energy dE must change according to the energy relation, or first law of thermodynamics,

$$dQ - dW = dE$$

We are going to generalize this to flowing system where things change with time continuously.

- $$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} \quad \text{——— (II)}$$

rate at which heat is transferred to system

rate at which work is done by system

Rate of energy change of system.

Like mass conservation, this is a scalar relation having only a single component.

⇒ We apply the Reynolds Transport Theorem (RTT), eqⁿ (1) to the first law of thermodynamics, eqⁿ (II). The dummy variable N becomes energy E, and the energy per unit mass is $\eta = \frac{dE}{dm} = e$. Eqⁿ (1) can then be written for a fixed control volume as follows:*

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\left(\int_{CV} e\rho \, d\mathcal{V}\right) + \int_{CS} e\rho(V\cdot n)\, dA \quad \text{(B)}$$

Positive Q denotes heat added to the system and positive W denotes work done by the system.

The system energy per unit mass e may be of several types:

$$e = e_{internal} + e_{kinetic} + e_{potential} + e_{other}$$

Potential and Kinetic ~~Ergte~~ Energies

In thermostatics the only energy in a substance is that stored in a system by molecular activity and molecular bonding forces. This is commonly denoted as internal energy $\hat{u}$. A commonly accepted adjustment to this static ~~eq~~ situation for fluid flow is to add two more energy terms which arise from newtonian mechanics: the potential energy and kinetic energy.

The potential energy equals the work required to move the system of mass m from the origin to a position vector $\underline{r} = \underline{i}x + \underline{j}y + \underline{k}z$ against a gravity field $\underline{g}$.

* The energy eq^n for a deformable control volume is rather complicated and is not discussed here.

Its values is $-m\underline{g}\cdot\underline{r}$ or $-\underline{g}\cdot\underline{r}$ per unit mass.

The kinetic energy equals the work required to change the speed of the mass from zero to velocity V. Its values is $\frac{1}{2}mV^2$ or $\frac{1}{2}V^2$ per unit mass.

The molecular internal energy $\hat{u}$ is a function of T and p for the single-phase pure substance. whereas the potential and kinetic ~~erg~~ energies are kinematic properties X —— X —— X —— X —

$$e = e_{internal} + e_{kinetic} + e_{potential} + e_{other}$$

where e_{other} could encompass chemical reactions, nuclear reactions, and electrostatic or magnetic field effects. We neglect e_{other} here and consider only the first three terms, with Z defined as "up":

$$e = \hat{u} + \frac{1}{2}V^2 + gZ \qquad \text{—— (A)}$$

The heat and work terms could be examined in detail. If this were a heat transfer, $\frac{dQ}{dt}$ would be broken down into conduction, convection and radiation effects. Here we leave the term untouched and consider it only occasionally.

using for convenience the overdot to denote the ti[me]
derivative, we divide the work term into three part[s]

$$\dot{W} = \dot{W}_{shaft} + \dot{W}_{press} + \dot{W}_{viscous\ stresses}$$

$$= \dot{W}_s + \dot{W}_p + \dot{W}_v$$

The work do gravitational forces has already been included as potential energy in Eq^n Ⓐ. Other types of work, e.g.; those due to electromagnetic forces, are excluded here.

The shaft work isolates that portion of the work which is deliberately done by a machine (pump impeller, fan blade, piston, etc.) protruding through the control surface into the control volume. No further specification other than $\dot{W}_s$ is desired at this point, but calculations of the work done by turbomachines (pumps) will be performed.

The rate of work $\dot{W}_p$ done on pressure forces occurs at the surface only; all work on internal portions of the material in the control volume is by equal and opposite forces and is self-cancelling. The pressure work equals the pressure force on a small surface element dA times the normal velocity component into the control volume:

Part

$$d\dot{W}_p = -(p\,dA)\,V_{n,in} = -p(-\underline{V}\cdot\underline{n})\,dA$$

The total pressure work is the integral over the control ~~volume~~ surface

$$\dot{W}_p = \int_{CS} p(\underline{V}\cdot\underline{n})\,dA.$$

A cautionary remark: If part of the control surface is the surface of a machine part, we prefer to delegate that portion of the pressure to the shaft work term $\dot{W}_s$, not to $\dot{W}_p$, which is primarily meant to isolate the fluid-flow pressure-work terms.

Finally, the shear work due to viscous stresses occurs at the control surface, the internal work terms again being self-canceling, consists of the product of each viscous stress (one normal and two tangential) and the respective velocity component:

$$d\dot{W}_v = -\underline{\tau}\cdot\underline{V}\,dA \quad \text{or,} \quad \dot{W}_v = -\int_{CS} \underline{\tau}\cdot\underline{V}\,dA$$

where τ is the stress vector on the elemental surface dA.

This term may vanish or be negligible according to the particular type of surface at that part of the control volume.

Solid surface: For all parts of the control surface which are solid confining walls, $\underline{V} = 0$ from the viscous no-slip condition

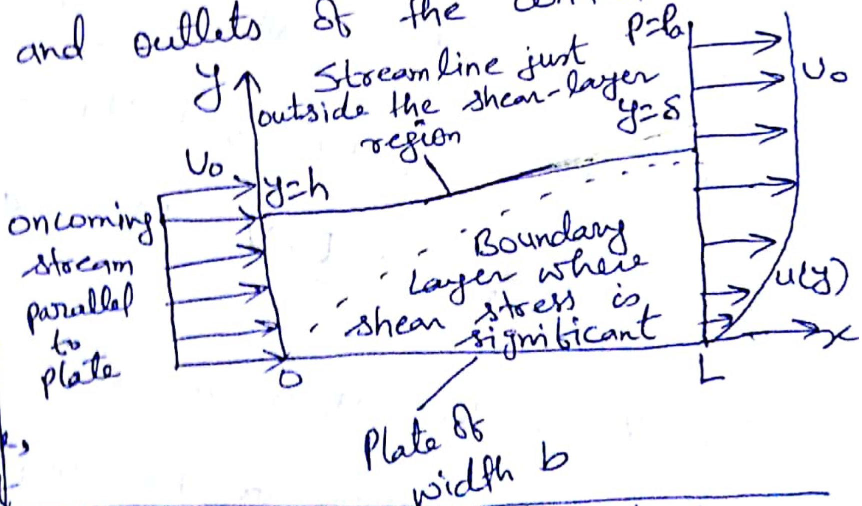
hence $\dot{W}_v = 0$ identically.

Surface of a machine: Here the viscous work is contributed by the machine, and so we absorb this work in the term $\dot{W}_s$.

An inlet or outlet. At inlet or outlet, the flow is approximately normal to the element dA; hence the only viscous-work term comes from the normal stress $\tau_{nn} V_n \, dA$. Since viscous normal stresses are extremely small in all but rare cases, e.g., the interior of a shock wave, it is customary to neglect viscous work at inlets and outlets of the control volume.

Streamline surface.

If the control surface is a streamline such as the upper curve in the boundary-layer analysis of Figure, the viscous-work term must be evaluated and retained if shear stresses are significant along this line. In this particular case, the streamline is outside the boundary layer, and viscous work is negligible.



The net result of the above discussion is that the rate-of-work term in eqⁿ (B) consists essentially of

$$\dot{W} = \dot{W}_s + \int_{CS} p(\underline{V}\cdot\underline{n})\,dA - \int_{CS} (\underline{\underline{\tau}}\cdot\underline{V})_{SS}\,dA \qquad \text{(D)}$$

where the subscript SS stands for stream surface. When we introduce eqⁿ (D) and (A) into (B), we find that the pressure-work term can be combined with the energy-flux term since both involve surface integrals of $\underline{V}\cdot\underline{n}$.

The control-volume energy eqⁿ thus becomes

$$\dot{Q} - \dot{W}_s - \dot{W}_p - \dot{W}_v = \dot{Q} - \dot{W}_s - \int_{CS} p(\underline{V}\cdot\underline{n})\,dA - \dot{W}_v = \frac{\partial}{\partial t}\left(\int_{CV} e\rho\,d\forall\right) + \int_{CS} (e)\,\rho(\underline{V}\cdot\underline{n})\,dA$$

$$\dot{Q} - \dot{W}_s - (\dot{W}_v)_{SS} = \frac{\partial}{\partial t}\left(\int_{CV} e\rho\,d\forall\right) + \int_{CS}\left(e + \frac{p}{\rho}\right)\rho(\underline{V}\cdot\underline{n})\,dA$$

using e from (A), we see that the enthalpy $\hat{h} = \hat{u} + p/\rho$ occurs in the control-surface integral. The final general form of for the energy eqⁿ for a fixed control volume becomes

$$\boxed{\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\left[\int_{CV}\left(\hat{u} + \tfrac{1}{2}V^2 + gz\right)\rho\,d\forall\right] + \int_{CS}\left(\hat{h} + \tfrac{1}{2}V^2 + gz\right)\rho(\underline{V}\cdot\underline{n})\,dA} \qquad \text{(C)}$$

As mentioned above, the shear work term $\dot{W}_v$ is rarely important.

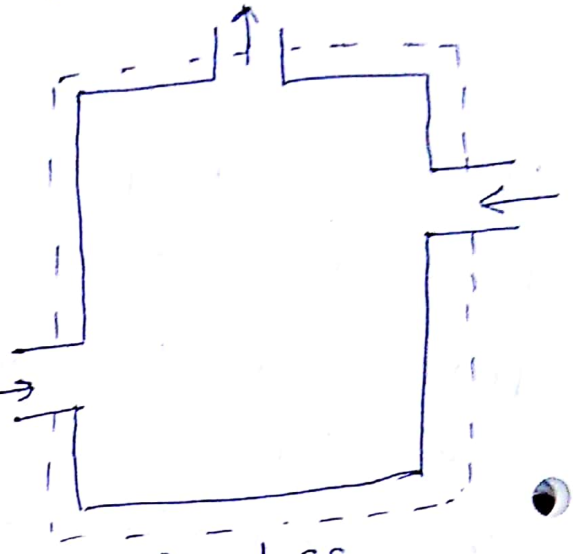
One-Dimensional Energy-Flux Terms

If the c.v. has a series of one-dimensional inlets and outlets, as in Figure, the surface integral in eqn © reduces to a summation of outlet fluxes minus inlet fluxes

$$\int_{CS} (\hat{h} + \tfrac{1}{2}V^2 + gz)\,\rho(\underline{V}\cdot\underline{n})\,dA$$

$$= \sum (\hat{h} + \tfrac{1}{2}V^2 + gz)_{out}\,\dot{m}_{out} -$$

$$- \sum (\hat{h} + \tfrac{1}{2}V^2 + gz)_{in}\,\dot{m}_{in} \quad \text{—} (X)$$



Fixed CS

where the values of $\hat{h}$, $\frac{1}{2}V^2$, and gz are taken to be averages over each cross section.

The Steady-Flow Energy Equation

For steady flow with one inlet and one outlet, both assumed one-dimensional, Eqn © reduces to a celebrated relation used in many engineering analysis. Let section 1 be the inlet and section 2 the outlet. Then

$$\dot{Q} - \dot{W}_s - \dot{W}_v = -\dot{m}_1(\hat{h}_1 + \tfrac{1}{2}V_1^2 + gz_1) + \dot{m}_2(\hat{h}_2 + \tfrac{1}{2}V_2^2 + gz_2) \quad \text{—} (E)$$

But, from continuity, $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and we can rearrange eqn (E) as follows:

$$\hat{h}_1 + \tfrac{1}{2}V_1^2 + gz_1 = (\hat{h}_2 + \tfrac{1}{2}V_2^2 + gz_2) - q + w_s + w_v \quad \text{—} (F)$$

where $q = \frac{\dot{Q}}{\dot{m}} = \frac{dQ}{dm}$, the heat transferred to the fluid per unit mass. Similarly, $w_s = \dot{W}_s/\dot{m} = \frac{dW_s}{dm}$ and $w_v = \dot{W}_v/\dot{m} = \frac{dW_v}{dm}$.

eq^n (F) is a general form of the steady-flow energy eq^n, which states that the upstream stagnation enthalpy $H_1 = (\hat{h} + \frac{1}{2}V^2 + gz)_1$ differs from the downstream value H_2 only if there is heat transfer, shaft work, or viscous work as the fluid passes between sections 1 and 2. q is positive if heat is added to the C.V. and that w_s and w_v are positive if work is done by the fluid on the surroundings.

FMW PP166

Each term in eq^n (F) has the dimensions of energy per unit mass, or velocity squared, which is a form commonly used by mechanical engineers. If we divide through by g, each term becomes a length, or head, which is a form preferred by civil engineers. The traditional symbol for head is h, which we do not wish to confuse with enthalpy. Therefore we use internal energy in rewriting the head form of the energy relation:

$$\frac{P_1}{\gamma} + \frac{\hat{u}_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{\hat{u}_2}{g} + \frac{V_2^2}{2g} + z_2 - h_q + h_s + h_v \qquad (m)$$

where $h_q = q/g$, $h_s = w_s/g$ and $h_v = w_v/g$ are the head forms of the heat added, shaft work done, and viscous work done, respectively. The term P/γ is called pressure head and the term $V^2/2g$ is denoted as velocity head.

Friction losses in low-speed Flow

A very common application of the steady-flow energy e... for low-speed flow with no shaft work and negligible viscous work, such as liquid flow in pipes. For this case, Eq^n (m) may be written in the form.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \left(\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right) + \frac{\hat{u}_2 - \hat{u}_1 - q}{g} \qquad (n)$$

The term in parentheses is called the useful head or available head or total head of the flow, denoted as h_0. The last term on the right is the difference between the available head upstream and downstream and is normally positive, representing the loss in head due to friction, denoted as h_f. Thus, in low-speed (nearly incompressible) flow with one inlet and one exit, we may write

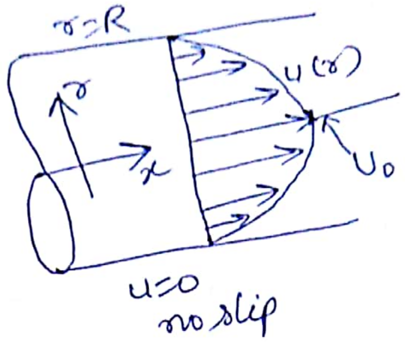
$$\left(\frac{P}{\gamma} + \frac{V^2}{2g} + z\right)_{in} = \left(\frac{P}{\gamma} + \frac{V^2}{2g} + z\right)_{out} + h_{friction} - h_{pump} + h_{turbine} \qquad (o)$$

Most of our internal-flow problems will be solved with the aid of eq^n (o). The h terms are all positive; that is, friction loss is always positive in real (viscous) flows, a pump adds energy (increases the left hand side), and a turbine extracts energy from the flow. If h_p and/or h_t are included, the pump and/or turbine must lie between points 1 and 2.

Kine... &

Kinetic - Energy Correction Factor

Often the flow entering or leaving a port is not strictly one-dimensional. In particular, the velocity may vary over the cross section as in figure.



In this case the kinetic energy term in eqⁿ ⓧ for a given port should be modified by a dimensionless correction factor α so that the integral can be proportional to the square of the average velocity through the port

$$\int_{port} \left(\tfrac{1}{2}V^2\right) \rho\,(V\cdot n)\,dA = \alpha\left(\tfrac{1}{2}V_{av}^2\right)\dot{m}$$

where $V_{av} = \frac{1}{A}\int u\,dA$ for incompressible flow

If the density is also variable, the integration is very cumbersome; we shall not treat this complication. By letting u be the velocity normal to the port, the first eqn above becomes, for incompressible flow,

$$\frac{1}{2}\rho\int u^3\,dA = \frac{1}{2}\rho\,\alpha\,V_{av}^3 A$$

$$\text{or, } \alpha = \frac{1}{A}\int\left(\frac{u}{V_{av}}\right)^3 dA$$

The term α is the kinetic-energy correction factor, having a value of about 2.0 for fully developed laminar pipe flow and from 1.04 to 1.11 for tubulent pipe flow. The complete incompressible steady-flow steady-flow energy eqn ⓔ, including pumps, turbines, and losses would generalize to

$$\left(\frac{p}{\rho g}+\frac{\alpha}{2g}V^2+z\right)_{in}=\left(\frac{p}{\rho g}+\frac{\alpha}{2g}V^2+z\right)_{out}+h_{turbine}-h_{pump}+h_{friction} \quad \text{—— (Y)}$$

where the head terms on the right (h_t, h_p, h_f) are all numerically positive. All additive terms in eqn (Y) have dimensions of length [L]. In problems involving turbulent pipe flow, it is common to assume that $\alpha \approx 1.0$.

Laminar flow: $u = U_0\left[1-\left(\frac{r}{R}\right)^2\right]$

$$V_{av} = 0.5\, U_0$$

$$\boxed{\alpha = 2.}$$

$$\alpha = \frac{1}{A}\int_0^R\int_0^{2\pi}\frac{\cancel{U_0^3}\left[1-\frac{r^2}{R^2}\right]^3}{0.5^3\,\cancel{U_0^3}}\, r\,dr\,d\theta \qquad (a-b)^3 = a^3-b^3-3a^2b+3ab^2$$

$$= \frac{2\pi}{\pi R^2}\frac{1}{0.5^3}\int_0^R\left[1-\frac{r^6}{R^6}-3\frac{r^2}{R^2}+3\frac{r^4}{R^4}\right]r\,dr$$

$$= \frac{2\times 8}{R^2}\left[\frac{r^2}{2}-\frac{r^8}{8R^6}-3\frac{r^4}{4R^2}+3\frac{r^6}{6R^4}\right]_0^R$$

$$= \frac{16}{R^2}\left[\frac{1}{2}R^2-\frac{1}{8}R^2-\frac{3}{4}R^2+\frac{1}{2}R^2\right]$$

$$= \frac{16}{R^2}\left[R^2-\frac{3}{4}R^2-\frac{1}{8}R^2\right]$$

$$= \frac{16}{\cancel{R^2}}\,\frac{8-6-1}{8}\,\cancel{R^2}$$

$$\boxed{\alpha = 2}$$

Turbulent Flow: $u \approx U_0\left(1-\frac{r}{R}\right)^m$

$$m \approx \frac{1}{7}$$

$$V_{av} = \frac{2U_0}{(1+m)(2+m)}$$

Frictionless Flow: The Bernoulli Equation

Closely related to the steady-flow energy eqn is a relation between pressure, velocity and elevation in a frictionless flow, now called the Bernoulli equation. It was stated (vaguely) in words in 1738 in a textbook by Daniel Bernoulli. A complete derivation of the eqn was given in 1755 by Leonhard Euler. The Bernoulli eqn is very famous and very widely used, but one should be wary of its restrictions — all fluids are viscous and thus all flows have friction to some extent. To use the Bernoulli equation correctly, one must confine it to regions of the flow which are nearly frictionless.

$$\Rightarrow \boxed{\frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + v_s\frac{dv_s}{ds} = 0}$$

$$\boxed{\frac{dp}{\rho} + g\,dz + v_s\,dv_s = 0}$$ Euler's eqn

Bernoulli's Equation from Euler's Eqn

Bernoulli's eqn is obtained by integrating the Euler's eqn. of motion

$$\int\frac{dp}{\rho} + \int g\,dz + \int v_s\,dv_s = \text{const}$$

If flow is incompressible, $\rho \to$ const

$$\therefore \frac{p}{\rho} + gz + \frac{v_s^2}{2} = \text{const}$$

$$\Rightarrow \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{const}$$

or, $$\boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{const}}$$

potential energy per unit wt or potential head.

Kinetic energy per unit wt. or kinetic head

pr. energy per unit wt. of fluid or pr. head

Assumptions

The following are the assumptions made in the derivation of Bernoulli's eqn.

i) The fluid is ideal, i.e., viscosity is zero

ii) The flow is steady; $\frac{\partial}{\partial t}() = 0$

iii) The flow is incompressible

iv) The flow is irrotational, $\nabla \times \underline{V} = 0$

curl of velo vector

vector product

friction factor is actually non-dimensionalization of pr. drop.

$$f = \frac{\Delta P}{\frac{1}{2}\rho \bar{V}^2 \frac{L}{D}} = \text{Darcy friction factor}$$

Dimensional analysis

$$\boxed{f = fn\left(Re, \frac{\epsilon}{D}\right)}$$ → wall roughness

this is for fully developed flow

$$Re = \frac{\rho \bar{V} D}{\mu}$$

Laminar Flow:

$$\boxed{\frac{\Delta P}{L} = \frac{8 Q \mu}{\pi R^4}}$$ H-P Formula

$$\Rightarrow \bar{V} = \frac{Q}{\pi R^2}$$

$$\frac{\Delta P}{L} = \frac{\bar{V} 8 \mu}{R^2} \Rightarrow R^2 = \left(\frac{D}{2}\right)^2 = \frac{D^2}{4}$$

$$\frac{\Delta P}{L} = \frac{32 \mu \bar{V}}{D^2}$$ Laminar flow

Divide by $\frac{1}{2} \frac{\rho \bar{V}^2}{D}$

$$\frac{\Delta P}{\frac{1}{2}\rho \bar{V}^2 \frac{L}{D}} = \frac{32 \mu \bar{V}}{\frac{1}{2}\frac{\rho \bar{V}^2}{D} D^2} = \frac{64 \mu}{\rho \bar{V} D}$$

Darcy friction factor,

$$\boxed{f = \frac{64}{Re}}$$ for laminar flow.

Fanning friction factor,

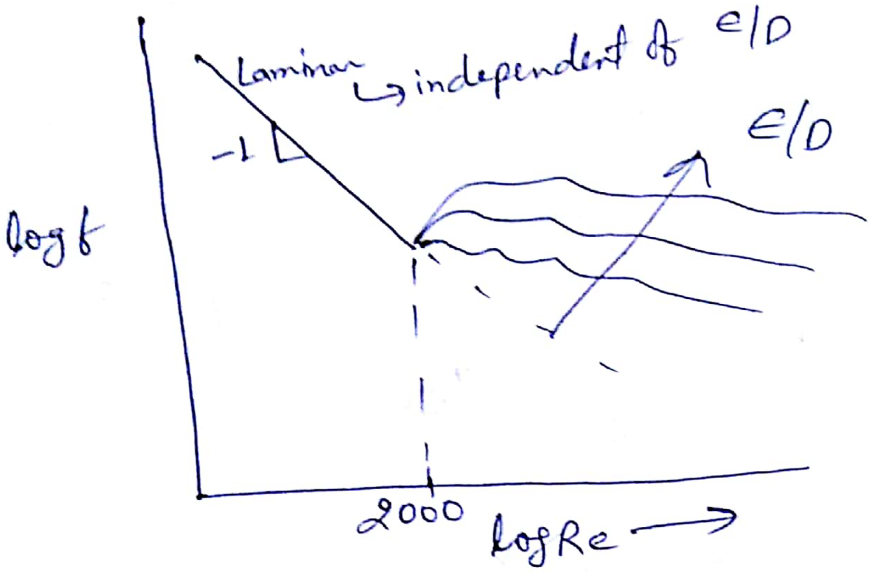
$$f_{fanning} = \frac{\Delta P}{2 \rho \bar{V}^2 \frac{L}{D}}$$

Similarly, $\boxed{f_{fanning} = \frac{16}{Re}}$, this is also for laminar flow.

For turbulence flow; friction factor chart

Moody's Friction factor chart

⇒



Variation of Friction Factor

- In case of a laminar fully developed flow through pipes, the friction factor, f is found from the exact solution of the Navier-Stokes equation as discussed in lecture 26. It is given by

$$f = \frac{64}{\text{Re}} \tag{35.7}$$

- In the case of a turbulent flow, friction factor depends on both the Reynolds number and the roughness of pipe surface.
- Sir Thomas E. Stanton (1865-1931) first started conducting experiments on a number of pipes of various diameters and materials and with various fluids. Afterwards, a German engineer Nikuradse carried out experiments on flows through pipes in a very wide range of Reynolds number.
- A comprehensive documentation of the experimental and theoretical investigations on the laws of friction in pipe flows has been presented in the form of a diagram, as shown in Fig. 35.2, by **L.F. Moody** to show the variation of friction factor, f with the pertinent governing parameters, namely, the Reynolds number of flow and the relative roughness ε / D of the pipe. This diagram is known as **Moody's Chart** which is employed till today as the best means for predicting the values of f.

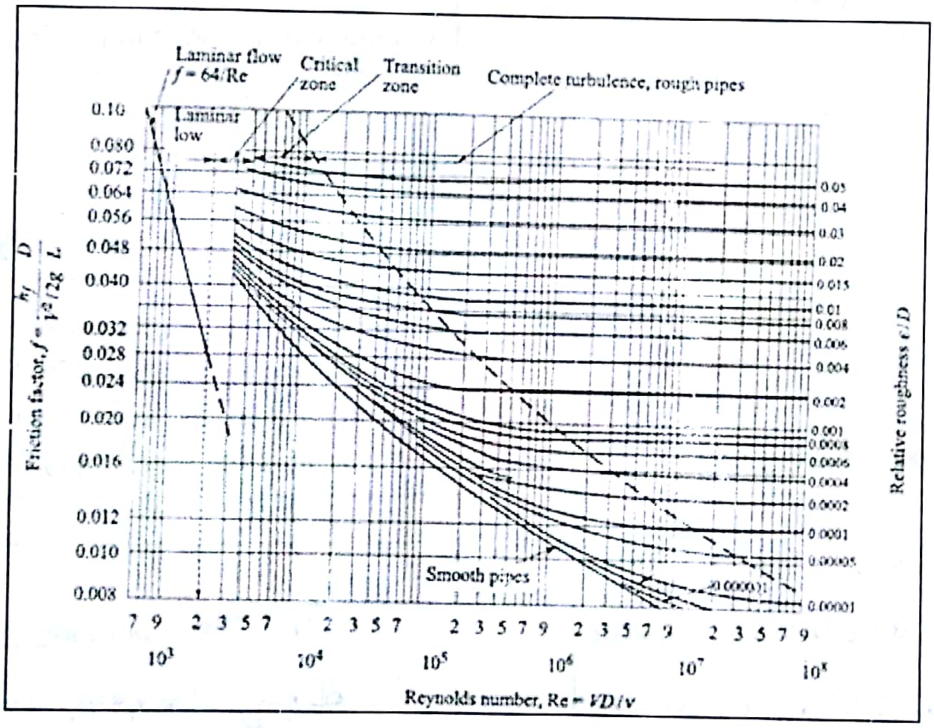


Fig. 35.2 Friction Factors for pipes (adapted from Trans. ASME, 66,672, 1944)

Figure 35.2 depicts that

- The friction factor f at a given Reynolds number, in the turbulent region, depends on the relative roughness, defined as the ratio of average roughness to the diameter of the pipe, rather than the absolute roughness.
- For moderate degree of roughness, a pipe acts as a smooth pipe up to a value of Re where the curve of f vs *Re* for the pipe coincides with that of a smooth pipe. This zone is known as the **smooth zone of flow** .
- The region where f vs Re curves (Fig. 35.2) become horizontal showing that f is independent of *Re*, is known as the **rough zone** and the intermediate region between the smooth and rough zone is known as the **transition zone.**

loss of energy in pipes

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as:

Energy losses

- Major Energy losses
  - This is due to friction and is calculated by the formula Darcy - Weisbach Formulae
- Minor Energy losses
  - This is due to
    - a) Sudden expansion of pipe
    - b) Sudden contraction of pipe
    - c) Bend in pipe
    - d) Pipe fittings etc.

loss of Energy (or Head) due to Friction

Darcy Weisbach Formula

$$f\frac{L}{D}\frac{V^2}{2g} = h_f = \frac{\cancel{4}fLV^2}{d2g} = \frac{8fLQ^2}{gD^5\pi^2}$$

(f = Darcy friction factor)

Where, h_f = loss of head due to friction

f = coefficient of friction which is function of Reynolds n.

L = length of pipe

V = mean velocity of flow

d = diameter of pipe

$$\boxed{\frac{fLV^2}{2gD} = h_f}$$

$$\frac{P_1 - P_2}{\rho g} = h_f$$

$$f = \frac{\Delta P}{\frac{1}{2}\rho V^2 \frac{L}{D}} \Rightarrow \frac{1}{2}f\rho V^2 \frac{L}{D} = \frac{\Delta P}{\rho g}$$

$$f = \frac{\Delta P}{\cancel{2}\rho V^2 \frac{L}{D}}$$

$$f\,\cancel{2}\,\rho V^2 \frac{L}{D}\frac{1}{\rho g} = h_f$$

$$h_f = \frac{2fLV^2}{Dg}$$

$$\Rightarrow \frac{\Delta P}{\rho g} = h_f = \frac{2fLV^2}{Dg}$$

$$\Rightarrow \boxed{\Delta P = \frac{2fL\rho V^2}{D}}$$

The Darcy-Weisbach eqn is valid for FD, SS and Incompressible flow.

Minor Energy (Head) losses

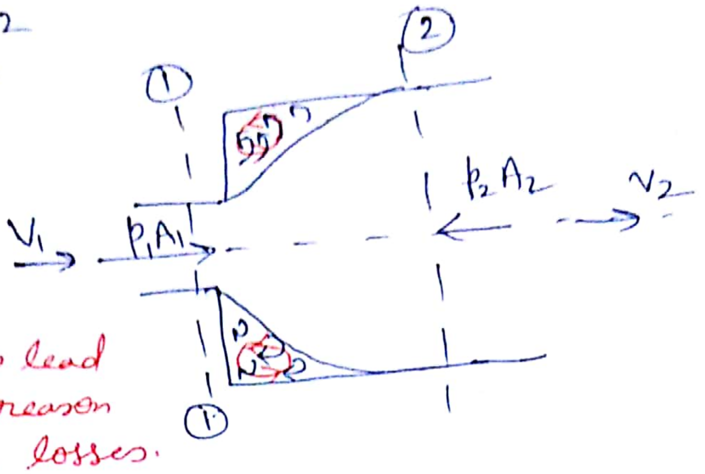
The loss of head or Energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy.

$$\boxed{\frac{\Delta P}{L} = \frac{32\mu\bar{V}}{D^2}}$$

loss of head due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

Energy are trying to supply to make the fluid flow in this direction, is lost in making the fluid go round and round in this sort of dead pocket. So this lead to loss. So this is the reason why expansion always has losses.

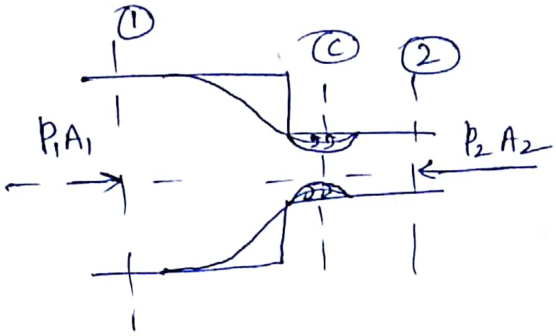


loss of head due to sudden contraction

$$h_c = \frac{kV_2^2}{2g}$$

$$k = \left[\frac{1}{C_c} - 1\right]^2$$

$$C_c = A_c/A_2$$



loss of head due to Bend in pipe

when there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary and also formation of eddies takes place. Thus the energy is lost. Loss of head in pipe due to bend is expressed as

$$h_b = \frac{kV^2}{2g}$$ where, h_b = loss of head due to bend

V = velocity of flow

k = coefficient of bend

loss of head in various pipe Fittings

Then loss of head in the various pipe fittings such as valves, couplings etc. is expressed as $\frac{kV^2}{2g}$

V = velocity of flow

k = co-efficient of pipe fittings