Basic concept of Fluid Mechanics

The fluid is a term given to matter which can flow, i.e., liquids and gases. Mechanics is the branch of science under which we study about the bodies (stationary and moving) under the impact of force. Fluid mechanics is the study of static and dynamic conditions of the fluids

There are three broad states of the matter viz, solid, liquid and gas. The shape and size of the solid do not change easily due to strong intermolecular forces. This force is weak in liquids, they acquire the shape of the vessel in which they are kept but their volume remain constant. The intermolecular forces are very small in gases and they acquire the shape and entire volume of the vessel in which they are kept. The density of the gases can be easily changed but it is difficult to change the density of the liquids.

Fluid mechanics is divided into two parts

Fluid Mechanics

Fluid Statics
Study of static (at rest) conditions of fluids

Fluid Dynamics
Study of dynamic (moving) conditions of fluids

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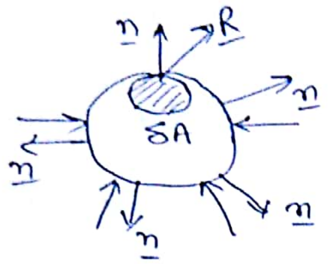
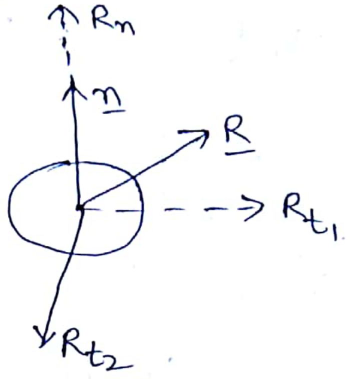
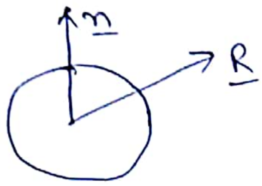
Fluid Statics (No motion / no flow) shear stresses = 0



Arbitrary volume element in a fluid flow is surrounded by fluid





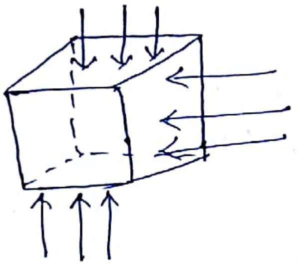
⇒ Surface force per unit area

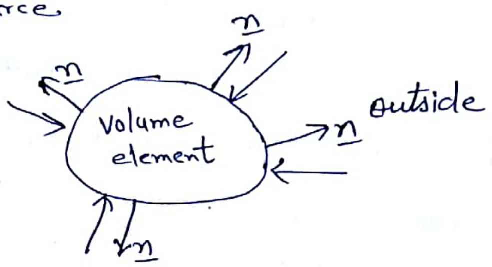
R_n : normal stress, R_{t_1}, R_{t_2} : tangential/Shear stress

Static $\boxed{R_{t_1}=0\ ,\ R_{t_2}=0}$ zero tangential stress

only surface force normal force.

Static





normal force exerted per unit area] "compressive" in nature in a fluid

pressure (N/m²)

↑ compressive normal force

Static $R = p(-n)$ in a static fluid

R is the force per unit area exerted by the fluid outside on the surface.

Static fluid can not support any shear stress.

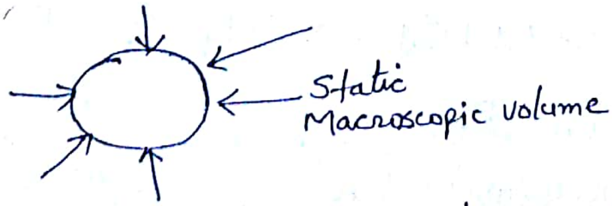
unit of pressure $1\ N/m^2 = 1\ Pa$.

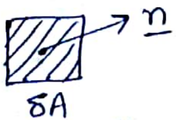
1 atmospheric pr. $\simeq 10^5$ Pa $\simeq$ 760 mm Hg ; $1\ kPa = 10^3\ Pa$

$1\ MPa = 10^6\ Pa$

↑ Mega

⇒ What is the pressure at a point?





What is the pressure at this point?

Does value of p at a point in a static fluid → depend on n?

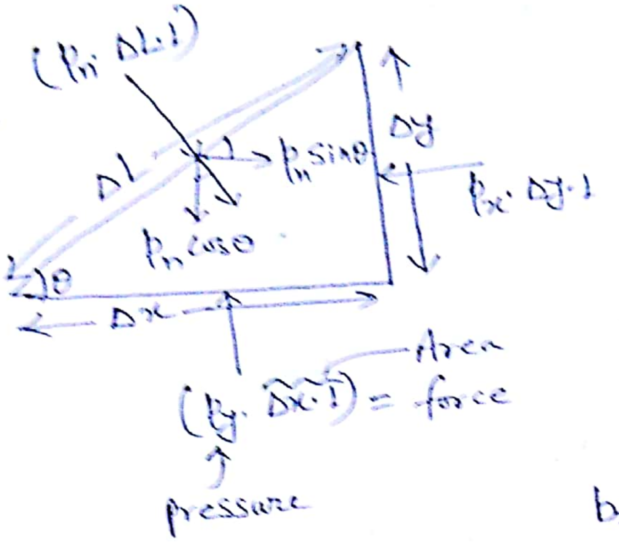
Ans. Not, because pressure is a scalar.

Pressure is acting on fluid element as static as well as in motion.

Consider fluid element at rest - No shear component, only normal component.

⇒ Does value of p at a point in a static fluid ⇒ depend on $\underline{n}$?
⇓
orientation of surface.

⇒



$$b_x = \frac{\text{body force}}{\text{mass along } x}$$

Total body force $= b_x \cdot \frac{1}{2} \cdot \Delta x \cdot \Delta y \cdot 1 \cdot \rho$

[Even if fluid element is not at rest but internal deformation is negligeble but the fluid element is moving like rigid body then also similar consideration may be valid. (No shear component only normal).] For x- component:

$$\sum F_x = m a_x \leftarrow \text{zero (as fluid is at rest)}$$

$$-P_x \cdot \cancel{\Delta y} \cdot 1 + P_n (\Delta L) \cdot 1 \cdot \sin\theta + \frac{1}{2} \Delta x \cdot \Delta y \cdot \rho \cdot b_x = \frac{1}{2} \Delta x \Delta y \cdot \rho \cdot a_x$$

Volume shrinks to a point, $\Delta x, \Delta y \to 0$; $\sin\theta = \frac{\Delta y}{\Delta L}$

IP.

$$-P_x \cancel{\Delta y} + P_n \cancel{\Delta y} + \frac{1}{2} \Delta x \cancel{\Delta y} \rho b_x = \frac{1}{2} \Delta x \cancel{\Delta y} \rho a_x$$

$$\Rightarrow -P_x + P_n + \frac{1}{2} \underset{0}{\cancel{\Delta x}} \rho b_x = \frac{1}{2} \underset{0}{\cancel{\Delta x}} \rho a_x$$

In a stationary fluid the pr. is exerted equally in all directions and is referred to as the static pressure.

$$\Rightarrow \boxed{P_n = P_x}$$

For y- component, $P_y \cdot \Delta x - P_n \cos\theta \, \Delta L - mg = 0$; $\cos\theta = \frac{\Delta x}{\Delta L}$

$$\Rightarrow P_y \cancel{\Delta x} - P_n \cancel{\Delta x} - \frac{1}{2} \cancel{\Delta x} \Delta y \cdot 1 \rho g = 0 \Rightarrow P_y - P_n - \frac{1}{2} \Delta y \rho g = 0$$

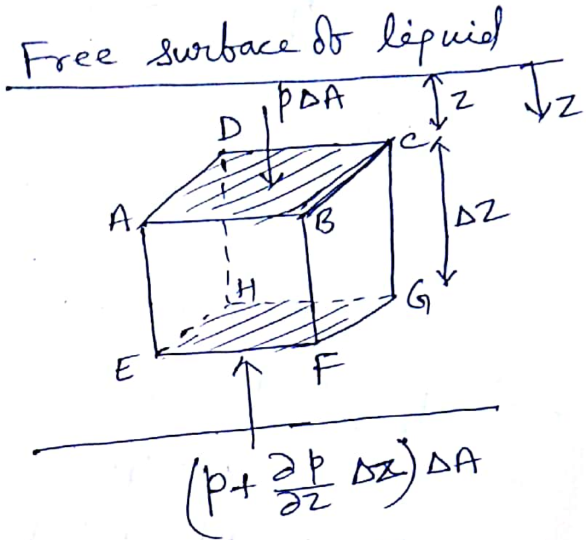
$\Delta y \to 0$; $P_n = P_y$.

$\boxed{P_x = P_y = P_n}$ PASCAL'S Law

Thus, unlike the other space tensor, pressure is tensor of zero order. Thus required no index for specification. Although stress and pressure both are expressed as force per unit area.

PASCAL's Law It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.

Pressure variation in a Fluid at rest



Forces on a fluid element

consider a small fluid element as shown in figure.

Let,

ΔA = Cross-sectional area of element

Δz = Height of fluid element

p = Pressure on face ABCD

z = Distance of fluid element from free surface.

The forces acting on fluid element ~~from free surface~~ are:

→ Pressure force on ABCD $= p \times \Delta A$ and acting perpendicular to face ABCD in the downward direction.

→ Pressure force on EFGH $= (p + \frac{\partial p}{\partial z}\Delta z)\Delta A$, acting perpendicular to face EFGH, vertically upward direction

→ Weight of fluid element $= mg$ = volume density g $= (\Delta A \Delta z)\rho g$

→ Pressure forces on surfaces AEHD and BCGF are equal and opposite. For equilibrium of fluid element, we have

$$p\Delta A - (p + \frac{\partial p}{\partial z}\Delta z)\Delta A + \rho g(\Delta A \Delta z) = 0$$

$$\Rightarrow \cancel{p\Delta A} - \cancel{p\Delta A} - \frac{\partial p}{\partial z}\Delta z \Delta A + \rho g \Delta A \Delta z = 0$$

Ⓐ

$$\Rightarrow \frac{\partial p}{\partial z}\cancel{\Delta z}\cancel{\Delta A} = \rho g \cancel{\Delta A}\cancel{\Delta z} \quad \Rightarrow \boxed{\frac{\partial p}{\partial z} = \rho g} = w \leftarrow \text{weight density of fluid}$$

This eqn states that the rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point.

This is Hydrostatic law. $\frac{\partial p}{\partial x} = 0$, $\frac{\partial p}{\partial y} = 0$ Thus, $\frac{dp}{dz} = \rho g$

The pressure at any point in a fluid at rest is obtained by the Hydrostatic law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point.

By integrating the eqn Ⓐ for liquids, we get

$$\int dp = \int \rho g \, dz \Rightarrow p = \rho g z$$

where, p → pressure above atmospheric pressure and

z → is the height of the point from free surfaces

$z = \frac{p}{\rho g}$, z → pressure head.

⇒ P_{atm}: Pressure due to the air that is present in the atmosphere.

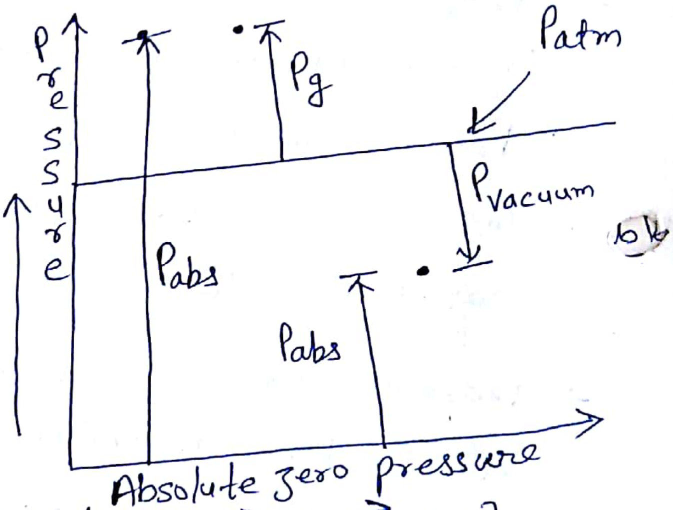
the

P_{abs}: Absolute pressure ; P_g: Gauge pressure

$P_{abs} - P_{atm} = P_g$ (if $P_{abs} > P_{atm}$)

$P_{atm} - P_{abs} = P_{vacuum}$ (if $P_{abs} < P_{atm}$).

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero, it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus,



[The measured pr. may be higher or lower than the local atmosphere]

Absolute Pressure: is defined as the pressure which is measured with reference to absolute vacuum pressure.

Gauge Pressure: is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

Nature of fluids

A fluid is a substance that does not permanently resist distortion. An attempt to change the ~~change~~ shape of a mass of fluid results in a layers of fluid sliding over one another until a new shape is attained. During the change in shape, shear stresses (shear is the lateral displacement of one layer of material relative to another layer by an external force, shear stress is defined as the ratio of this force to the area of the layer) exist, the magnitudes of which depend upon the viscosity of the fluid and the rate of sliding; but when a final shape has been reached, all shear stresses will have disappeared. A fluid in equilibrium is free from shear stresses.

At a given temperature and pressure, a fluid possesses a definite density, which in engineering practice is usually measured in kilograms per cubic meter. Although the density of all fluids depends on the temp and pr, the variation in density with changes in these variables may be small or large. If the density changes only slightly with moderate changes in temp. and pr, the fluid is said to be incompressible; if the changes in density are significant, the fluid is said to be compressible. Liquids are generally considered to be incompressible and gases compressible. The terms are relative, however, and the density of a liquid can change appreciably if pr and temp are changed over wide limits. Also, gases subjected to small percentage changes in pr and temp act as incompressible fluids, and density changes under such conditions may be neglected without serious error.

The pressure in a static fluid is familiar as a surface force exerted by the fluid against a unit area of the walls of its container. Pressure also exists at every point within a volume of fluid. It is a scalar quantity; at any given point its magnitude is the same in all directions.

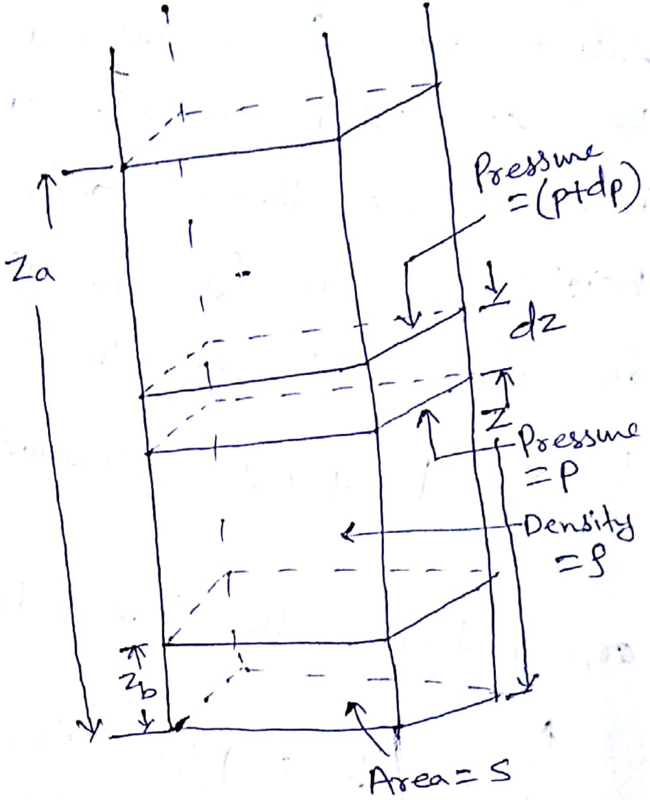
Sol^n 1 ⓐ

Sol^n 1.(a)

Hydrostatic Equilibrium

In a stationary mass of a single static fluid, the pressure is constant in any cross section parallel to the earth's surface but varies from height to height. Consider the vertical column of fluid shown in Figure:

Assume the cross-sectional area of the column is S. At a height Z above the base of the column let the pressure be p and the density be ρ. The resultant of all forces on the small volume of fluid of height dz and cross-sectional area S must be zero. Three vertical forces are acting on this volume: 1) the force from pressure p acting in an upward direction, which is pS; 2) the force from pressure p+dp acting in a downward direction, which is (p+dp) S; 3) the force of gravity acting downward, which is $g\rho S$~~ρ~~dz. Then



Hydrostatic equilibrium
[Pr. is the same at all pts. shown on the cross-sectional area S]

$$pS - (p+dp)S - g\rho S dz = 0$$

In this eqn, forces acting upward are taken as positive and those acting downward as negative. After simplification and division by S, Now,

$$dp + g\rho\, dz = 0$$

Above eqn cannot be integrated for compressible fluids unless the variation of density with pressure is known throughout the column of fluid. However, it is often satisfactory for engineering calculations to consider ρ to be essentially constant. The density is constant for incompressible fluids and, except for large changes in height, is nearly so for compressible fluids. Integration of above eqn on the assumption that ρ is constant gives

$$\frac{p}{\rho} + gZ = \text{constant} \quad \{\text{condition of hydrostatic eqlbm}\}.$$

or, between the two definite heights Z_a and Z_b

$$\frac{p_b}{\rho} - \frac{p_a}{\rho} = g(Z_a - Z_b)$$

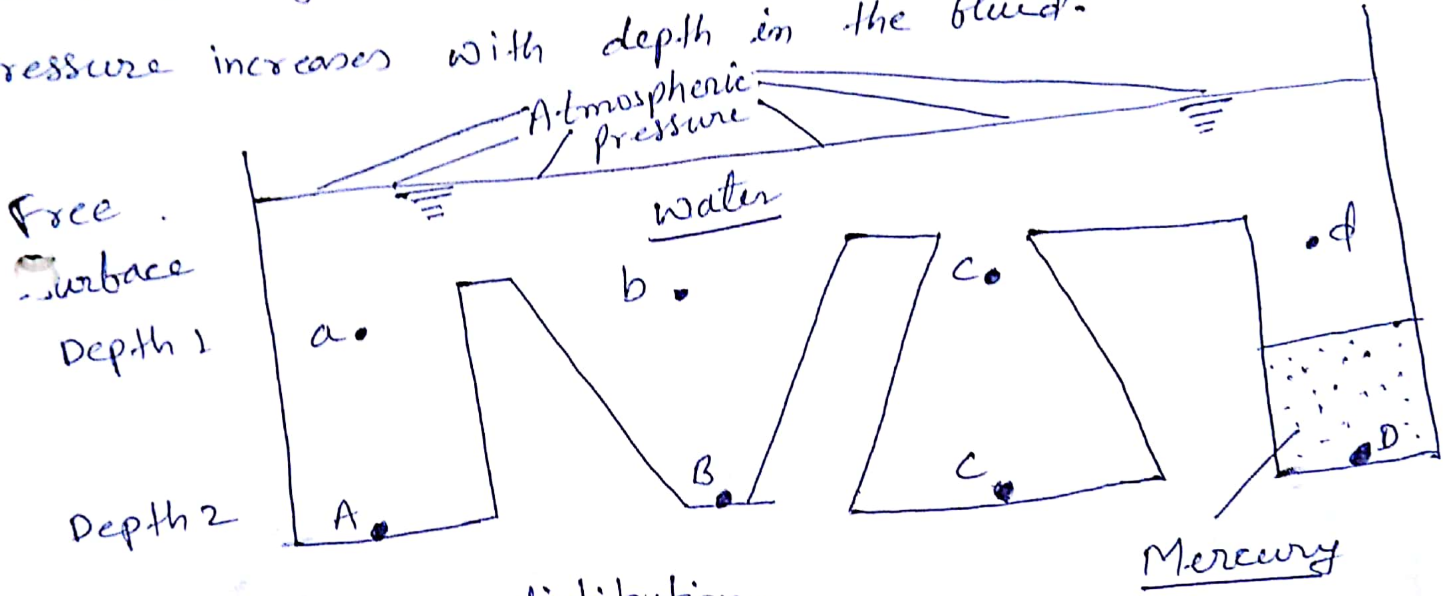
Head of a Fluid

Pressures are given in many different sets of units, such as psia, dyn/cm², and N/m², However, a common method of expressing pressures in terms of head in m or feet of a particular fluid.

$$h(\text{head}) = \frac{p}{\rho g}$$

We state ~~or~~the following conclusions about a hydrostatic Condition

Pressure in a continuously distributed uniform static fluid varies only with vertical distance and is independent of the shape of the container. The pressure is the same at all points on a given horizontal plane in the fluid. The pressure increases with depth in the fluid.



Hydrostatic pressure distribution.

Points a, b, c and d are at equal depths in water and therefore have identical pressures.

Points A, B and C are also at equal depths in water and have identical pressures higher than a, b, c and d.

Point D has a different pressure from A, B and C because it is not connected to them by a water depth.

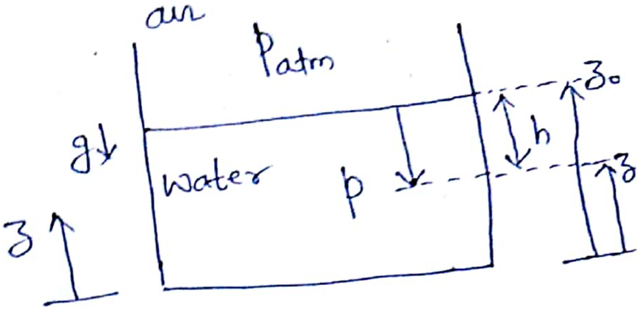
Vacuum pressure: is defined as the pressure below the atmospheric pressure.

Pressure Depth relationship for incompressible fluids

Atmospheric pressure $= p_{atm} = p_0$

Pressure at depth h from free surface, $p = p_0 + \rho g h$

$\rho \rightarrow$ constant, independent of pressure



$$\frac{dp}{dz} = -\rho g \Rightarrow \int_{p_0}^{p} dp = -\rho g \int_{z_0}^{z} dz$$

$$\Rightarrow p - p_0 = -\rho g (z - z_0) \Rightarrow p = p_0 + \rho g (z_0 - z) \Rightarrow \boxed{p = p_0 + g\rho h}$$

Pressure Depth Relationship for compressible fluid

compressible ideal gas $p\mathcal{V} = nRT$

$$\Rightarrow p = \frac{W}{M}\frac{1}{\mathcal{V}}RT = \frac{W}{\mathcal{V}}\frac{R}{M}T \Rightarrow p = \rho \underline{R_g T} \text{ ⓧ} ; R_g = \frac{R}{M}$$

where, R_g = specific gas constant

$$\Rightarrow \frac{dp}{dz} = -\rho g \Rightarrow \frac{dp}{dz} = -\frac{p}{R_g T} g$$

Isothermal process, $\frac{dp}{p} = -\frac{g}{R_g T_0} dz$, $T = T_0$ (constant)

Integrating on both sides,

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{g}{R_g T_0} \int_{z_0}^{z} dz \Rightarrow \ln \frac{p}{p_0} = -\frac{g}{R_g T_0} [z - z_0]$$

where p_0 is the pressure and z_0 is height.

$$Ⓐ \leftarrow p = p_0 \exp\left(-\frac{g}{R_g T_0}(z - z_0)\right) = p_0 \exp\left(\frac{g}{R_g T_0}(z_0 - z)\right)$$

$$\boxed{p = p_0 \exp\left(\frac{gh}{R_g T_0}\right)}$$

If the datum line is taken at z_0, then $z_0 = 0$ and p_0 becomes the pressure at datum line.

from Ⓐ $p = p_0 \exp\left(-\frac{gz}{R_g T_0}\right)$

$\left[e^{-x} = 1 - x + \frac{x^2}{\lfloor 2} - \cdots \right.$

If $x << 1$,

$e^{-x} = 1 - x$

from Taylor's series

$$\Rightarrow p = p_0 \left[1 - \frac{gz}{R_g T_0}\right] = p_0 - \frac{p_0}{R_g T_0} gz$$

from (X) $p_0 = \rho_0 R_g T_0$

$$\Rightarrow \boxed{p = p_0 - \rho_0 g z}$$

[At STP, atmospheric pressure $= 101.325 \ kN/m^2$

Temperature $= 15°C$

$$\frac{gz}{R_g T_0} << 1 \Rightarrow \frac{gz}{R_g T_0} \le 0.1$$

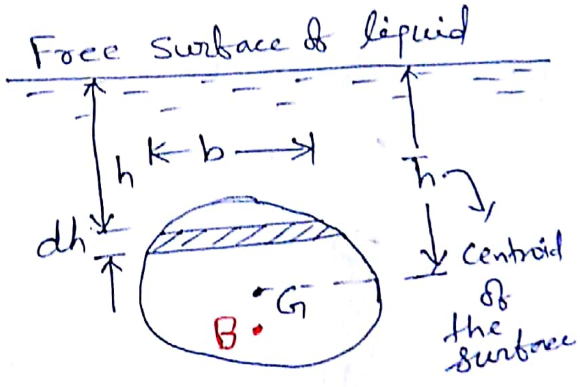
For air, $z < 800m$.

(68)

Forces on Submerged Bodies

Vertical plane Surface submerged in liquid

consider a plane vertical surface of arbitrary shape immersed in a liquid



Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of ~~free~~ plane surface

Pressure ~~intensity~~ on the strip, $p = \rho g h$

[Taking gage pr. for simplicity, with $p=0$ at the free surface]

Area of strip, $dA = b \times dh$

~~Total~~ pressure force on strip, $dF = p \times Area = \rho g h \times b \times dh$

Total pressure force on the whole surface,

$\therefore$ Total pr. force on the whole surface

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

$$\Rightarrow \int b \times h \times dh = \int h \times dA \Rightarrow \boxed{y \int dA = y_c A}$$

$$\boxed{\int h \, dA = \bar{h} A}$$

moment of surface area about the free surface of liq.

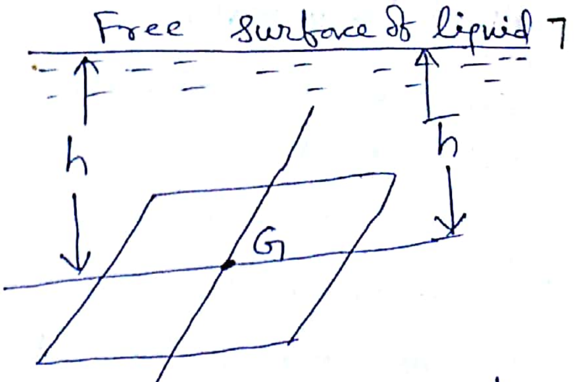
$$= A\bar{h} \Rightarrow \boxed{F = \rho g A \bar{h}}$$

Horizontal Plane Surface Submerged in liquid

Consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, Pressure intensity will be equal on the entire surface and equal to,

$p = \rho g h$, h = depth of surface

A = Total area of surface.

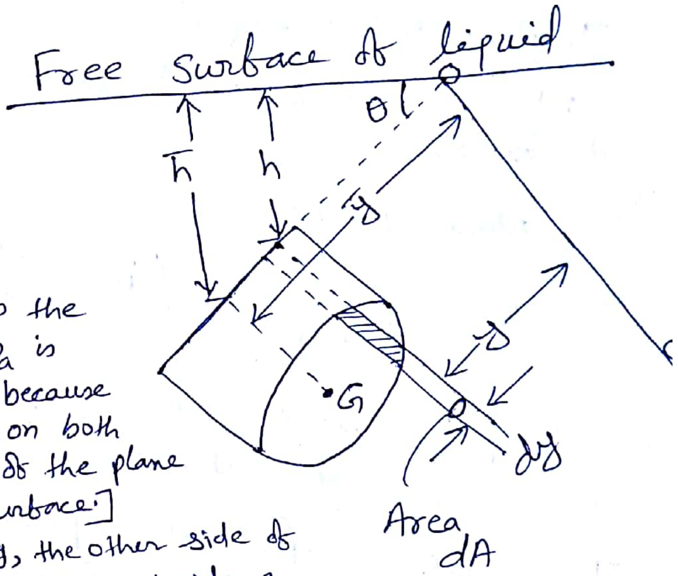


Then, total force F, on the surface $= p \times Area = \rho g \bar{h} A$ [This force will be centred on the centre of mass or centre of gravity of the plane surface]

where, $\bar{h}$ = Depth of centre of gravity from free surface of liquid = h. Also h^* = Depth of centre of pr. from free surface = h.

Inclined Plane Surface Submerged in Liquid

consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of surface makes an angle θ with the free surface of the liquid.



[In most cases the ambient pr. P_a is neglected because it acts on both sides of the plane surface.] e.g., the other side of the plate is inside a ship or on the dry side of a gate or dam]

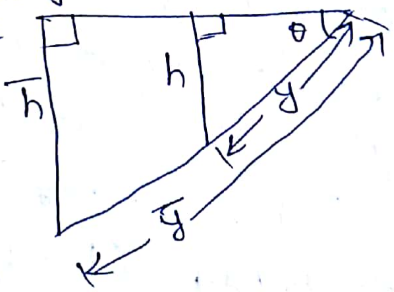
Pressure intensity on strip

$$p = \rho g h$$

pressure force, dF, on the strip,

$$dF = p \times \text{Area of strip}$$

$$= \rho g h \times dA$$



Total pressure force on the whole areas

$$F = \int dF = \int \rho g h \, dA$$

$$\Rightarrow F = \int \rho g y \sin\theta \, dA = \rho g \sin\theta \int y \, dA$$

$$\Rightarrow \sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}}$$

$$\Rightarrow \int y \, dA = A\bar{y} \leftarrow$$ Distance of C.G. from O–O axis

$$F = \rho g \sin\theta \, \bar{y} A$$

$$\boxed{F = \rho g A \bar{h}}$$

[The force on one side of any plane submerged surface in a uniform fluid equals the pr. at the plate centroid times the plate area, independent of the shape of the plate or the angle θ at which it is slanted.]

Hydrostatic Forces on Surfaces

Total Pressure and centre of pressure

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of Pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the ~~total~~ pressure force and centre of pressure is to be determined. The submerged surfaces may be :

1) Vertical plane surface
2) Horizontal plane surface
3) Inclined plane surface
4) Curved surface.

If a body is immersed in a liquid, the total pr. force acting on the body is the force exerted by the liquid (or fluid) on entire surface area in contact with the liquid. This total pr. is centred on a point called centre of pressure

Centre of pressure (h^*) for vertical plane surface

Centre of pressure is calculated by using the "Principle of Moments', which states that the moment of the hydrostatic ~~resultant~~ force about an axis is equal to the sum of moments of the components about the same axis.

The ~~resultant~~ force F is acting at P, at a distance h^* from free surface of the liquid. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ——— ①

Moment of force dF, acting on a strip about free surface of liquid $= dF \times h$ $\quad [\because dF = \rho g h \times b \times dh]$

$$= \rho g h \times b \times dh \times h$$

Sum of moments of all such forces about free surface of Liquid $= \int \rho g h \times b \times dh \times h = \rho g \int b \times h h\, dh$

$$= \rho g \int b h^2 dh = \rho g \int h^2 dA \qquad (\because b\,dh = dA)$$

$\int h^2 dA = \int b h^2 dA$

= Moment of Inertia of the surface about free surface of Liquid

$= I_o$

∴ Sum of moments about free surface

$= \rho g I_o$ ——— (II)

from ① & (II) we have

$F \times h^* = \rho g I_o$ $\qquad$ But $F = \rho g A \bar{h}$

$\rho g A \bar{h} \times h^* = \rho g I_o$

$$h^* = \frac{\rho g I_o}{\rho g A \bar{h}} = \frac{I_o}{A \bar{h}} \quad \text{——— (III)}$$

By the theorem of parallel axis, we have

$$I_o = I_G + A\bar{h}^2$$

where, I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_G in eqn (III), we get

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid.

Hence, i) Centre of pressure (h^*) lies below the C.G. of the vertical surface

ii) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

Inclined plane surface submerged in a liquid

Centre of Pressure (h^*)

Pressure force on the strip, $dF = \rho g h \, dA$
$= \rho g y \sin\theta \, dA$

Moment of the force, dF, about axis 0–0

$= dF \times y = \rho g y \sin\theta \, dA \, y = \rho g \sin\theta \, y^2 dA$

Sum of moments of all such forces about 0–0

$= \int \rho g \sin\theta \, y^2 dA = \rho g \sin\theta \int y^2 dA$

[However, to balance the bending-moment portion of the stress, the resultant force Facts not through the centroid but below it toward the high-pr. side.]

But $\int y^2 dA$ = M.O.I. of the surface about 0–0 $= I_0$

∴ Sum of moments of all forces about 0–0 $= \rho g \sin\theta \, I_0$ —— ①

Moment of the force F, about 0–0 is also given by —— (II)

$= F \times y^*$

$= F y^*$ where y^* = Distance of centre of pressure from 0–0.

from (I) & (II)

$\rho g \sin\theta \, I_0 = F y^*$

$\Rightarrow y^* = \dfrac{\rho g \sin\theta \, I_0}{F}$ —— (III)

$y^* = \dfrac{h^*}{\sin\theta}$, $F = \rho g \bar{h} A$

I_o by the theorem of parallel axis $= I_G + A\bar{y}^2$

from (iii)

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

$$\Rightarrow h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

$$\frac{\bar{h}}{\bar{y}} = \sin\theta$$

$$\Rightarrow h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A\frac{\bar{h}^2}{\sin^2\theta}\right]$$

$$\boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$$

If $\theta = 90^\circ$

for vertical plane.

Drive expressions for total pressure and ~~C.G.~~ centre of pressure for a vertically immersed surface

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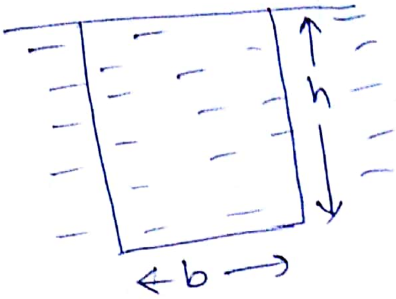
Ellipse

If rectangular surface



$\bar{h} = \frac{h}{2}$, $A = b \cdot h$

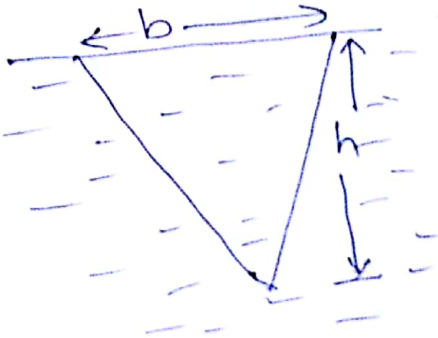
$I_G = \frac{bh^3}{12}$

$h^* = \dots$

$\bar{h} \rightarrow$ depth of centre of gravity

$h^* \rightarrow$ depth of centre of pressure.

If Triangular surface

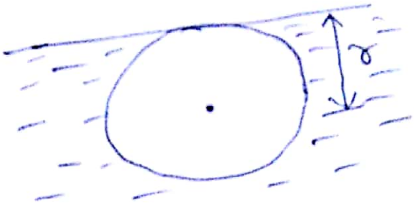


$\bar{h} = \frac{h}{3}$, $A = \frac{1}{2} b \cdot h$

$I_G = \frac{bh^3}{36}$

$h^* = \dots$

For circular shape



$\bar{h} = r$

$A = \pi r^2$, $I_G = \frac{\pi r^4}{4}$

$h^* = \dots$

Moment of the net force about free surface of the liquid will be equal to the sum of moments of forces on all strips of small thickness about the free surface of liquid

i.e., $F \cdot h^* = \int dF \cdot h$

A common problem in the design of structures which interact with fluids is the computation of the hydrostatic force on a plane surface. If we ~~neglet~~ neglect density changes in the fluid, $(P_2-P_1) = -\gamma(z_2-z_1)$ applies and, the pressure on any submerged surface varies linearly with depth. For a plane surface, the linear stress distribution is exactly analogous to combined bending and compression of a beam in strength-of-materials theory. The hydrostatic problem thus reduces to simple formulas involving the centroid and moments of inertia of the plate cross-sectional area.

Devices to Measure pressure and Pressure differences ||

→ In chemical and other industrial processing plants it is often important to measure and control the pressure in a vessel or process and/or the liquid level in a vessel. Also, since many fluids are flowing in a pipe or conduit, it is necessary to measure the rate at which the fluid is flowing. Many of these flow meters depend upon devices to measure a pressure or pressure difference.

→ Flowing fluid and manometric fluid is immiscible.

The Mercury Barometer

The simplest practical application of the hydrostatic formula is the barometer, which measures atmospheric pressure. A tube is filled with mercury and inverted while submerged in a reservoir. This causes a near vacuum in the closed upper end because mercury has an extremely small vapor pressure at room temperature (0.16 Pa at 20°C). Since atmospheric pressure forces a mercury column to rise a distance h into the tube, the upper mercury surface is at zero pressure.

→ Mercury is used because it is the heaviest common liquid. A water barometer would be 34ft high. In the United States the weather service reports this as an atmospheric "pressure" of 29.96 in Hg (inches of mercury).

Motivation

Many fluid problem do not involve motion. They concern the pressure distribution in a static fluid and its effect on solid surfaces and on floating and submerged bodies. When the fluid velocity is zero, denoted as the hydrostatic condition, the pressure variation is due only to the weight of the fluid. Assuming a known fluid in a given gravity field, the pressure may easily be calculated by integration.

If the fluid is moving in rigid-body motion, such as a tank of liquid which has been spinning for a long time, the pressure also can be easily calculated, because the fluid is free of shear stress.

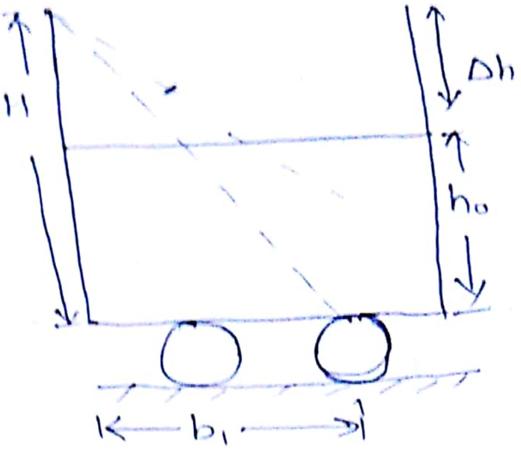
If the fluid is at rest or at constant velocity, $a=0$ and $\nabla^2 V=0$. The pressure distribution is

$$\nabla p = \rho g$$

This is a hydrostatic distribution and is correct for all fluids at rest, regardless of their viscosity.

If $\Delta h + h_0 > H$??

Spill out of water.



Applications of Fluid Statics

Measurement of pressure

Manometers

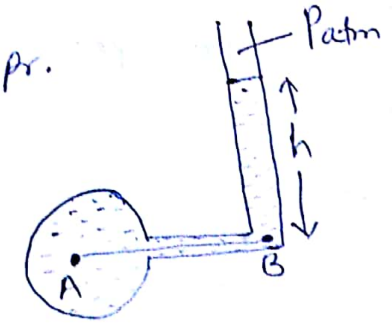
Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.

Simple Manometers

A simple manometer consists of a glass tube having one of its ends connected to a point, where pressure is to be measured and other end remains open atmosphere.

Type of Simple Manometers

1. Piezometer It is the simplest form of manometer used for measuring gauge pressure. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere. The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in Piezometer tube, then pressure at A,



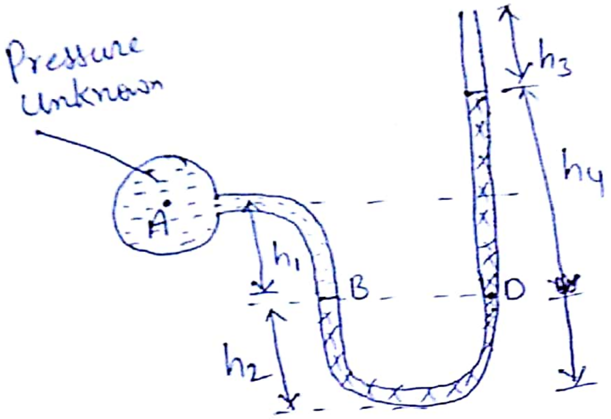
$$P_A = P_B$$ ← Because hydrostatic pr. is only function of height

$$= P_{atm} + \rho g h$$

$$\Rightarrow P_A - P_{atm} = \rho g h$$

$$\boxed{P_g = \rho g h} \quad N/m^2$$

2. U-Tube Manometer It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in figure. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



[Idea is to relate the unknown pressure to atmospheric pressure

$P_B = P_D$ ⇒ Because hydrostatic pressure is only function of height

$$p = P_{atm} + \rho g h$$

$$\Rightarrow P_A + \rho g h_1 = P_{atm} + \rho_m g (h_4 - h_2)$$

$$\Rightarrow (P_A - P_{atm}) = \rho_m g (h_4 - h_2) - \underset{\text{usually negligible}}{\rho_1 g h_1}$$

$$\boxed{\underbrace{(P_A - P_{atm})}_{p_g \rightarrow \text{gauge pr.}} = \rho_m g \Delta h}$$

[Because density of manometric liquid is very high compared to density of the working fluid;

$$\rho_m >> \rho_l$$

{usually the manometric liquid is mercury}

$$\rho_m = 13.6 \times 10^3 \text{ kg/m}^3$$
mercury

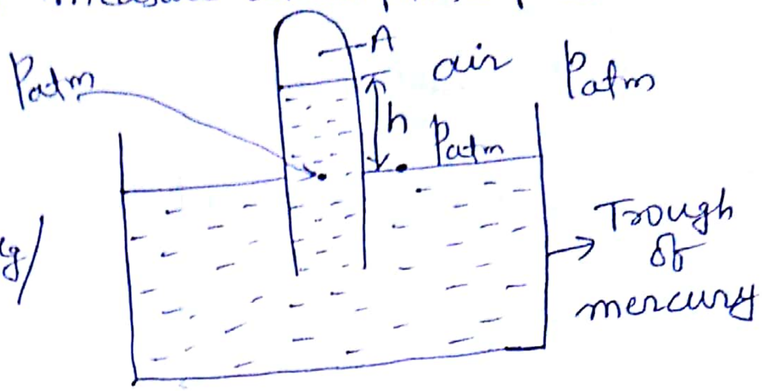
[Δh is the reading of the manometer]

Barometer ⇒ use to measure atmospheric pressure

$$\boxed{p_{atm}} = p_A + \rho_m g \boxed{h}$$

p_{atm} → Want this

h → 760mmHg / 76 cm



$$p_{atm} = \underbrace{13.6 \times 10^{3}}_{\text{mercury}} \times 9.8 \frac{m}{s^2} \times 0.76 m$$

$$p_{atm} = 1.03 \times 10^{5} \frac{N}{m^2} = 1.03 \times 10^{5} \text{ Pa}$$

$p_{atm} = 1.013 \times 10^{5}$ Pa (or) 760 mm Hg ≡ 1 atm.

Differential Manometers

Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are: 1. U-tube differential manometer

Two pipes at different levels.

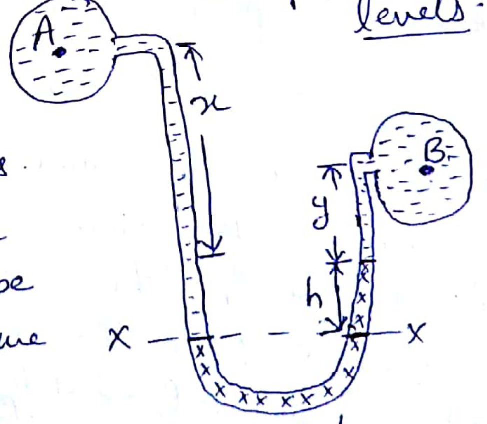


Fig. shows the differential manometers of U-tube type. Let the two points A and B are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb.

x = Distance of the centre of A, from the mercury level in the right ~~right~~ ~~left~~ limb.

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury

Taking datum line X–X

Pressure above X–X in the left limb $= \rho_1 g(h+x) + p_A$

Where p_A = pressure at A.

Pressure above X–X in the right limb $= \rho_g g h + \rho_2 g y + + p_B$

Where, p_B = pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h+x) + p_A = \rho_g g h + \rho_2 g y + p_B$$

$$\Rightarrow p_A - p_B = \rho_g g h + \rho_2 g y - \rho_1 g (h+x)$$

$$= hg(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

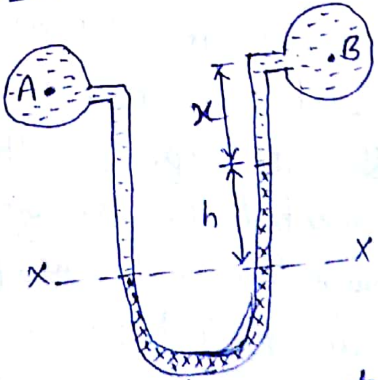
∴ Difference of pressure at A and B

$$= hg(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x.$$

⇒ A and B are at the same level and contains the same liquid of density ρ_1. Then pressure above X–X in the right limb $= \rho_g g h + \rho_1 g x + p_B$

Pressure above X–X in left limb $= \rho_1 g (h+x) + p_A$

Equating the two pressure

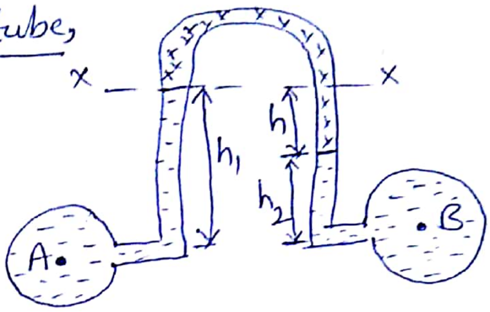


A and B are at the same level.

$$\rho_g gh + \rho_1 gx + p_B = \rho_1 g(h+x) + p_A$$

$$\therefore\ p_A - p_B = \rho_g gh - \rho_1 gh = gh(\rho_g - \rho_1).$$

2. Inverted U-tube Differential Manometer



It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig shows an inverted U-tube differential Manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.

Let h_1 = Height of liquid in left limb below the datum line X–X.

h_2 = Height of liquid in right limb.

h = Difference of light liquid

ρ_1 = Density of liquid at A, ρ_2 = Density of liquid at B.

ρ_s = Density of light liquid

p_A = pressure at A, p_B = pressure at B.

Taking X–X as datum line. Then pressure in the left limb below X–X $= p_A - \rho_1 g h_1$

pressure in the right limb below X–X $= p_B - \rho_2 g h_2 - \rho_s g h$

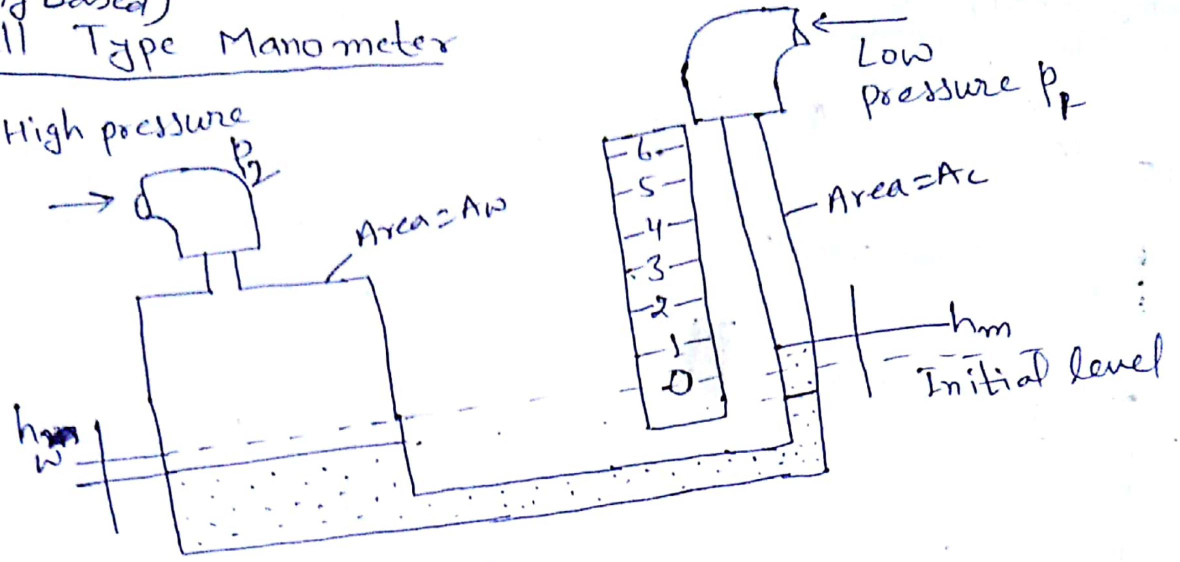
Equaling the two pressure

$$p_A - \rho_1 g h_1 = p_B - \rho_2 g h_2 - \rho_s g h$$

$$\Rightarrow \boxed{p_A - p_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_s g h}$$

(Gravity based)

Well Type Manometer



$$P_2 - P_1 = \rho g h_m$$

- The well type manometer is widely used because of inconvenience; the reading of only a single leg is required in it.

It consists of a very large-diameter vessel (well) connected on one side to a very small-sized tube. Thus the zero level moves very little when pressure is applied.

- → In order to avoid the inconvenience of having to read two limbs, the well-type manometer shown in Fig. can be used.

If A_w and A_c are the cross-sectional areas of the well and the column and h_m is the increase in the level of the column and h_w the decrease in the level of the well, then:

$$P_2 = P_1 + \rho g (h_m + h_w)$$

$$P_2 - P_1 = \rho g (h_m + h_w)$$

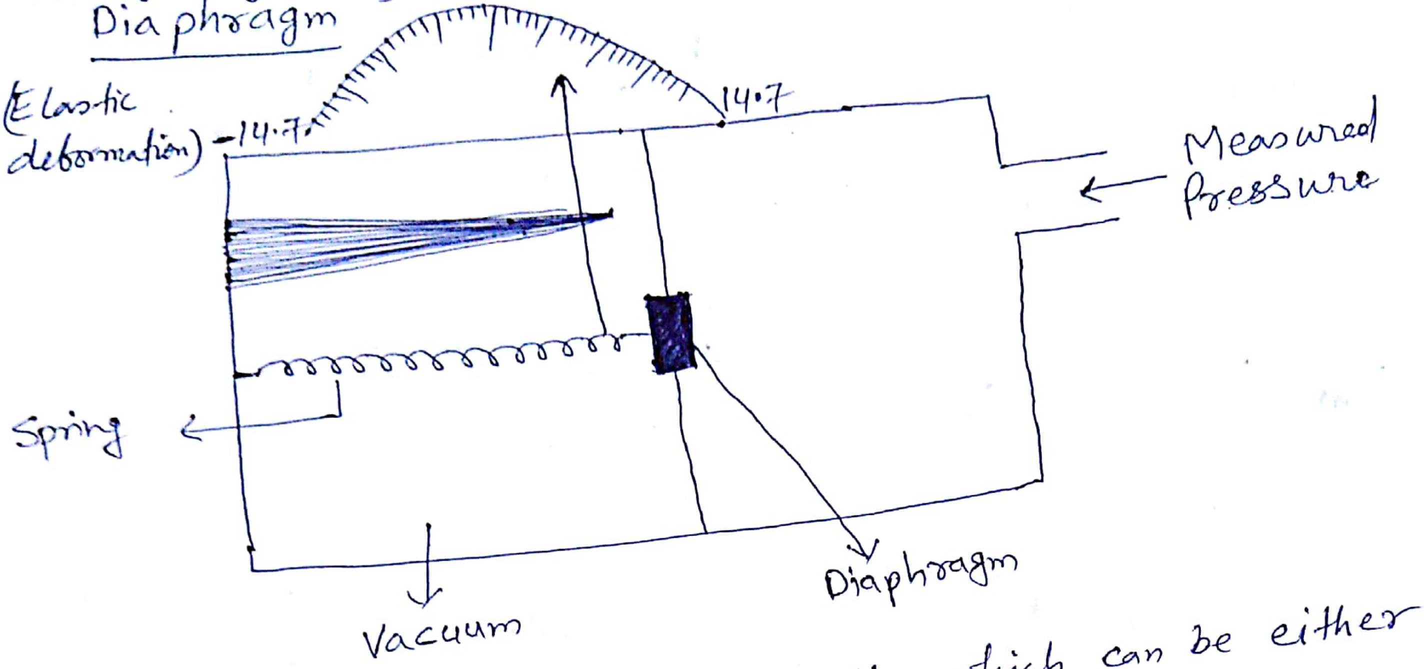
[Substituting $P_2 - P_1 = \rho g h_m \left(1 + \frac{A_c}{A_w}\right)$
If the well is large in comparison to the column then
$P_2 - P_1 = \rho g h_m$]

The quantity of liquid expelled from the well is equal to the quantity pushed into the column so that:

$$A_w h_w = A_c h_m \text{ and } h_w = \frac{A_c}{A_w} h_m$$

(Not gravity based)

Diaphragm



→ The diaphragm is a flexible disc, which can be either flat or with concentric corrugations and is made from sheet metal with high tolerance dimensions.

→ The diaphragm can be used as a means of isolating the process fluids, or for high pressure applications.

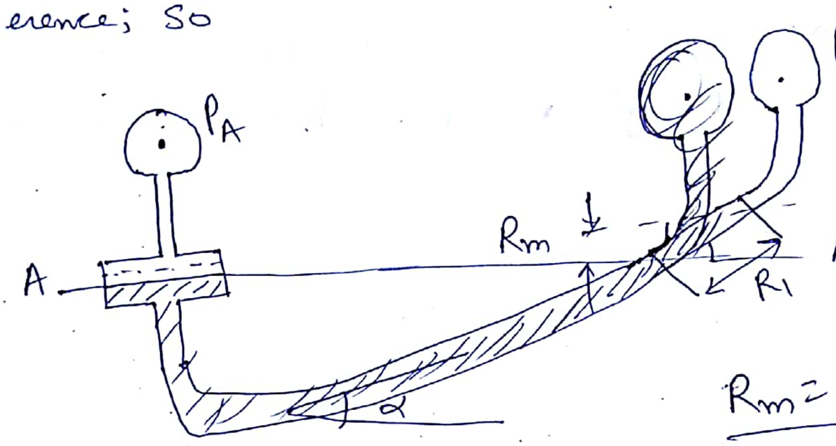
→ Advantages of Diaphragm

- Good for low pressure (Pressure differential even in the range of 0 to 4mm).
- Inexpensive
- wide range
- Reliable and proven
- used to measure gauge, atmospheric and differential pressure.

→ Diaphragm will deflect under pressure and can either be sensed directly or used to drive another sensor.

Inclined Manometer   Gravity based

For measuring small differences in pressure, the inclined manometer may be used. In this type, one leg of the manometer is inclined in such a manner that, for a small magnitude of Rm, the meniscus in the inclined tube must move a considerable distance along the tube. This distance is Rm divided by the sine of α, the angle of inclination. By making α small, the magnitude of Rm is multiplied into a long distance R1, and a large reading becomes equivalent to a small pressure difference; so



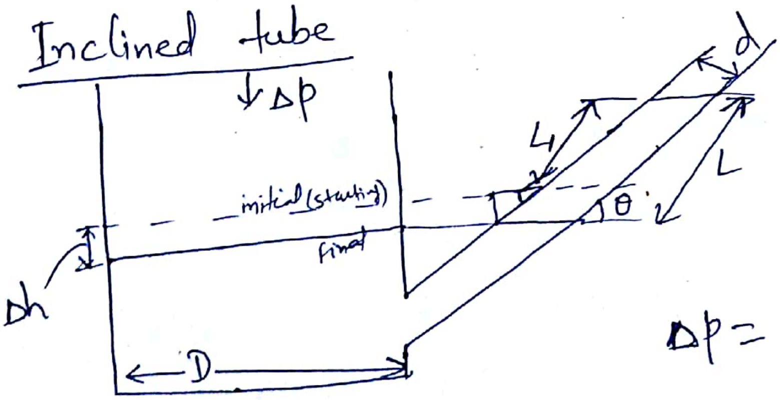
$$R_m = R_1 \sin\alpha$$

AA'

$$P_A' + R_1 \sin\alpha \, \rho_A g = P_B + R_1 \sin\alpha \, \rho_B g$$

$$\boxed{P_A - P_B = g R_1 (-\rho_A + \rho_B) \sin\alpha}$$

Inclined tube



$$V_1 = V_2$$

$$\Rightarrow \Delta h \frac{\pi D^2}{4} = L_1 \frac{\pi d^2}{4}$$

$$\Rightarrow L_1 = \Delta h \frac{D^2}{d^2}$$

$$\Delta p = (\underbrace{\boxed{L}}_{L_1 + \frac{\Delta h}{\sin\theta}} \sin\theta) \rho g$$

$$\Delta p = \left(\Delta h \frac{D^2}{d^2} \sin\theta + \Delta h\right) \rho g$$

$$\Delta p = \boxed{\Delta h}\,\rho g \left[\frac{D^2}{d^2}\sin\theta + 1\right]$$

$\downarrow$

Express in terms of L

~~$\sin\theta = \frac{\Delta h}{L_1}$~~

~~$\Delta h = (L_1)\sin\theta$~~

→ The manometer is a simple and inexpensive hydrostatic principle device with no moving parts except the liquid column itself. Manometer measurements must not disturb the flow. The best way to do this is to take the measurement through a static hole in the wall of the flow.