Module 5
COMPRESSIBLE FLUID FLOW

Adiabatic flow with friction

- Governing equations for steady state, adiabatic flow with friction of ideal gas
- Equation of state

$$p = \rho RT \ldots.(1)$$

- Mach no eqn

$$\frac{dM^2}{M^2} = \frac{du^2}{u^2} - \frac{dT}{T} \ldots.(2)$$

- Energy equation

$$\frac{dT}{T} + \frac{u^2}{2C_P T}\frac{du^2}{u^2} = 0$$

$$\Rightarrow \frac{dT}{T} + \frac{\gamma - 1}{2} M^2 \frac{du^2}{u^2} = 0 \ldots.(3)$$

- For constant mass flow rate per unit area

$$\rho u = \frac{\dot{m}}{A}$$

$$\Rightarrow \frac{d\rho}{\rho} + \frac{1}{2}\frac{du^2}{u^2} = 0 \ldots.(4)$$

- Momentun eqn

$$\rho A u du = -A dp - \tau_w dA_w$$

$$\Rightarrow \frac{dp}{p} + \frac{\gamma M^2}{2} 4f \frac{dx}{D} + \frac{\gamma M^2}{2}\frac{du^2}{u^2} = 0 \ldots.(5)$$

- From stagnation properties:

$$\frac{dp_0}{p_0} = \frac{dp}{p} + \frac{\frac{\gamma M^2}{2}}{1+\frac{\gamma-1}{2}M^2}\frac{dM^2}{M^2} \quad ....(6)$$

- For impulse input

$$F = pA + \rho A u^2$$

$$\Rightarrow \frac{dF}{F} = \frac{dp}{p} + \frac{\gamma M^2}{1+\gamma M^2}\frac{dM^2}{M^2} \quad ....(7)$$

- 7 equations with 8 variables:

$$\frac{dp_0}{p_0}, \frac{dp}{p}, \frac{dM^2}{M^2}, \frac{d\rho}{\rho}, \frac{dT}{T}, \frac{du^2}{u^2}, \frac{dF}{F} \;\&\; \frac{dx}{D}$$

- Solutions are

$$\frac{dp}{p} = \frac{\gamma M^2}{1-M^2}\left[\frac{1+(\gamma-1)M^2}{2}\right]4f\,\frac{dx}{D}$$

$$\frac{dM^2}{M^2} = -\frac{\gamma M^2}{1-M^2}\left[1+\frac{(\gamma-1)M^2}{2}\right]4f\,\frac{dx}{D}$$

$$\frac{du}{u} = \frac{\gamma M^2}{2(1-M^2)}4f\,\frac{dx}{D}$$

$$\frac{dT}{T} = -\frac{\gamma(\gamma-1)M^4}{2(1-M^2)}4f\,\frac{dx}{D}$$

$$\frac{d\rho}{\rho} = -\frac{\gamma M^2}{2(1-M^2)}4f\,\frac{dx}{D}$$

$$\frac{dp_0}{p_0} = -\frac{\gamma M^2}{2}4f\,\frac{dx}{D}$$

$$\frac{dF}{F} = -\frac{\gamma M^2}{2(1+\gamma M^2)}4f\,\frac{dx}{D}$$

$$\frac{\Delta s}{C_p} = \frac{(\gamma-1)}{2}M^2 4f\,\frac{dx}{D}$$

Effect of Mach no on flow properties

Change in properties in flow direction	Subsonic flow	Supersonic flow
P	decrease	increase
u	increase	decrease
T		
ρ		
po		

Isothermal friction flow

- Governing equations

$$\frac{dT_0}{T_0} = \frac{\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}\frac{dM^2}{M^2}$$

$$\frac{dp}{p} = \frac{d\rho}{\rho}$$

$$\frac{dM^2}{M^2} = 2\frac{du}{u}$$

$$\frac{d\rho}{\rho} + \frac{1}{2}\frac{du^2}{u^2} = 0$$

$$\frac{dp}{p} + \frac{\gamma M^2}{2}4f\frac{dx}{D} + \frac{\gamma M^2}{2}\frac{du^2}{u^2} = 0$$

$$\frac{dp_0}{p_0} = \frac{dp}{p} + \frac{\frac{\gamma M^2}{2}}{1+\frac{\gamma-1}{2}M^2}\frac{dM^2}{M^2}$$

- Solution

$$\frac{dp}{p}=\frac{d\rho}{\rho}=-\frac{du}{u}=-\frac{1}{2}\frac{dM^2}{M^2}=-\frac{\gamma M^2}{2(1-\gamma M^2)}4f\frac{dx}{D}$$

$$\frac{dp_0}{p_0}=\frac{\gamma M^2\left(1-\frac{\gamma+1}{2}M^2\right)}{2(\gamma M^2-1)\left(1+\frac{\gamma-1}{2}M^2\right)}4f\frac{dx}{D}$$

$$\frac{dT_0}{T_0}=\frac{\gamma(\gamma-1)M^4}{2(1-\gamma M^2)\left(1+\frac{\gamma-1}{2}M^2\right)}4f\frac{dx}{D}$$