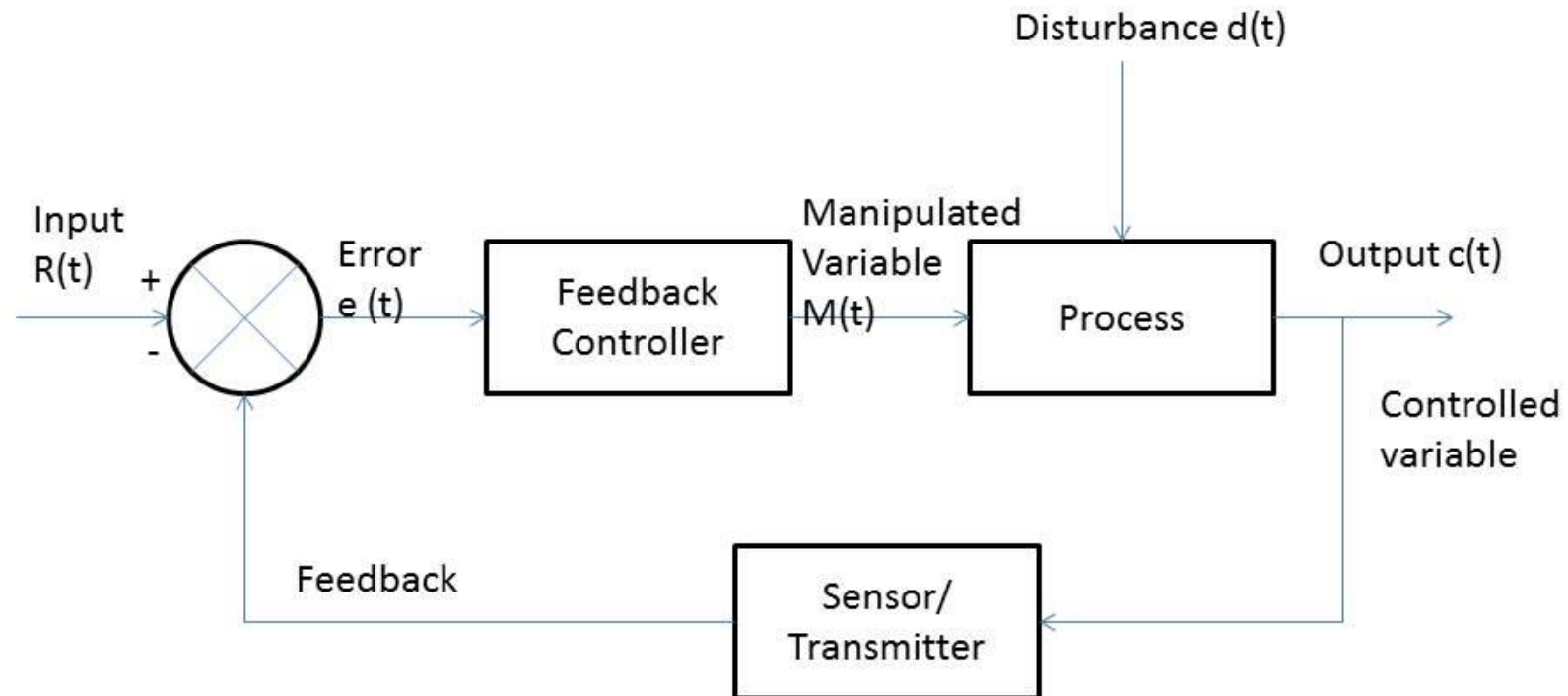


MODULE 3

Feed-back loop

Feedback Control System



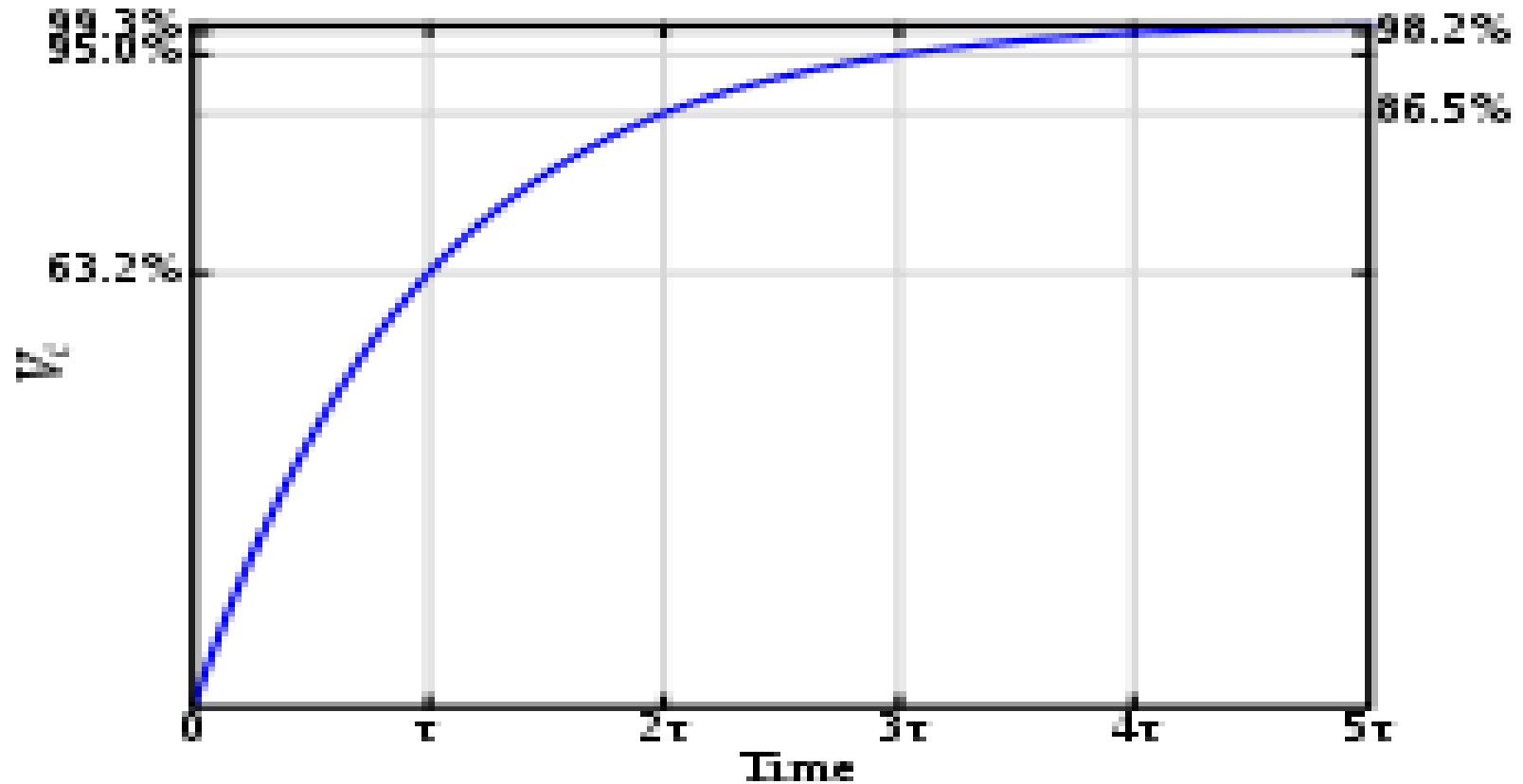
Transfer function

- Introduction and derivation of 1st and 2nd order transfer function
- Laplace transformation for different forcing function step, ramp etc
- Zero & poles of transfer function
- Stability analysis from pole values

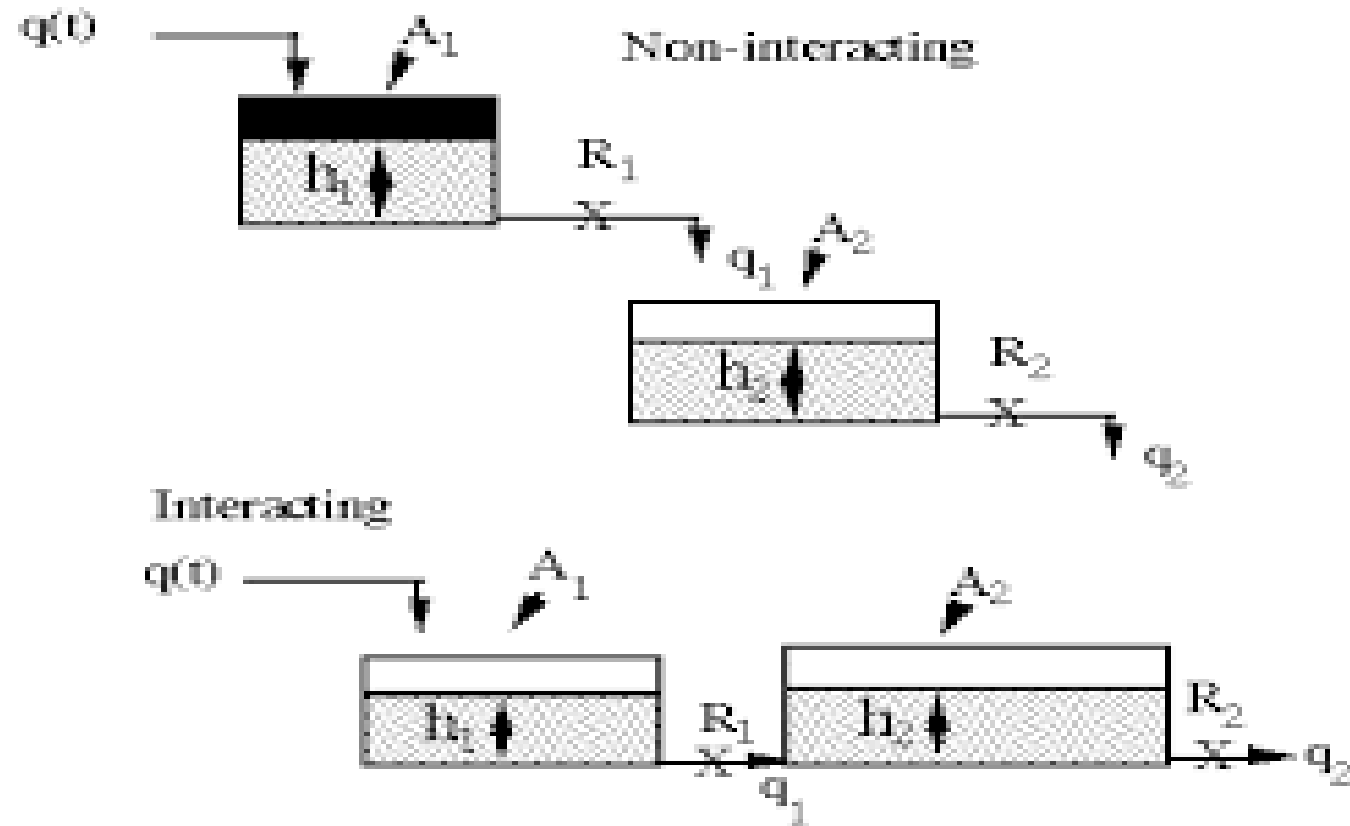
- Dynamics of 1st order lag system for unit step change

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{\tau_p s + 1}$$

$$y(t) = K_p (1 - e^{-t/\tau_p})$$



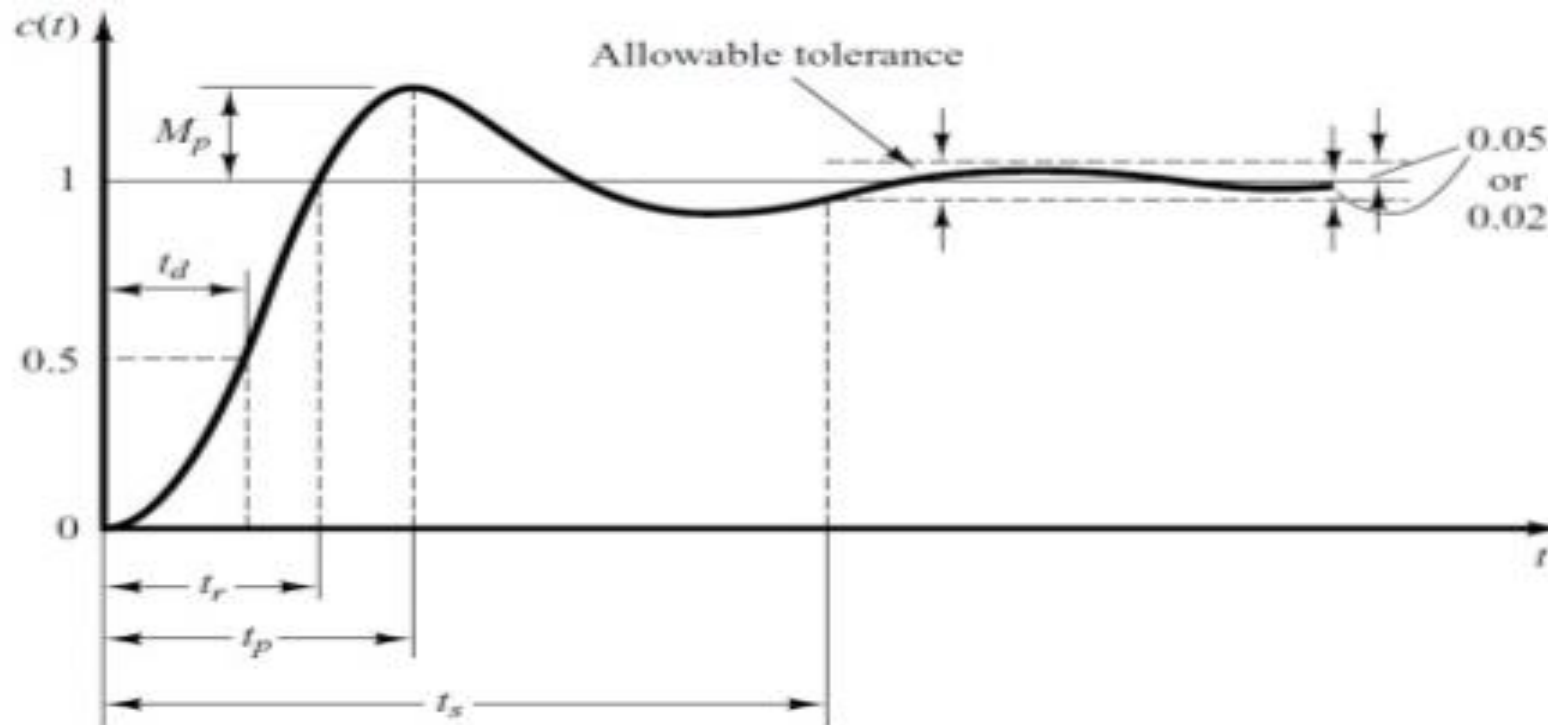
- Dynamics of 2nd order system
- Interacting and non interacting tanks in series



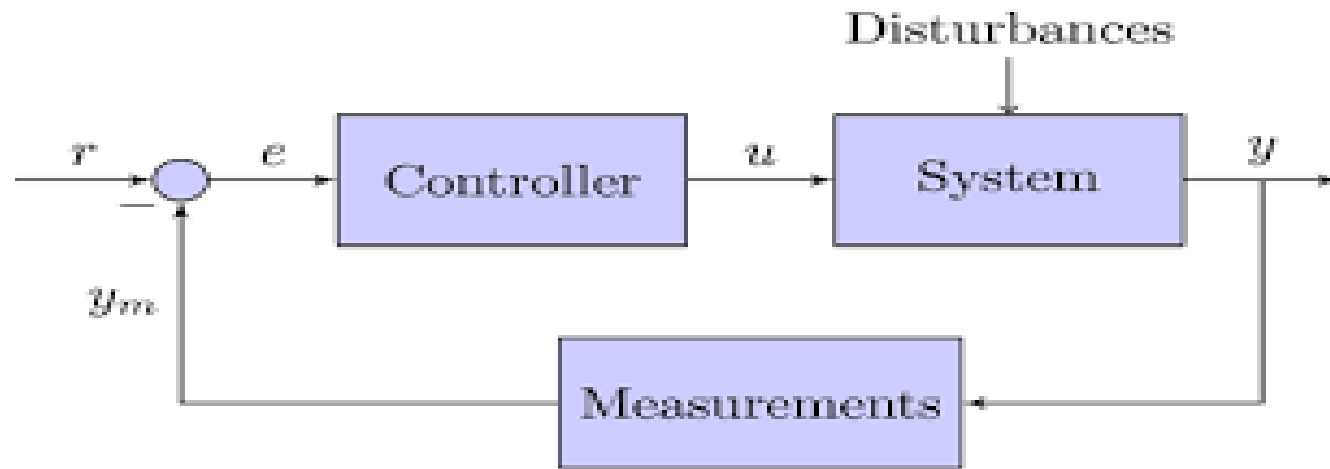
- Dynamics of 2nd order system
- $\zeta < 1$ under damp system
- $\zeta = 1$ critically damp system
- $\zeta > 1$ over damp system
- Characteristics of an under damped system

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{\tau^2 s^2 + 2\tau\zeta s + 1}$$

$\zeta = \text{damping_coefficient}$



Block diagram for feedback control system



- Introduction servo problem and regulator problem
- offset , overall gain
- Effect of feedback control action on 1st order lag and pure capacitive systems
- Effect of setpoint change and controller action on output.

$$\bar{y}(s) = \frac{G_p G_f G_c}{1 + G_p G_f G_c G_m} \bar{y}_{sp}(s) + \frac{G_d}{1 + G_p G_f G_c G_m} \bar{d}(s)$$

Stability analysis

- Introduction to Routh-Hurwitz stability
- Introduction to time integral performance criteria
- Simple performance criteria
- Cohen Coon model
- Frequency response analysis and Bode stability criterion.

Routh-Hurwitz Stability Criterion

- The characteristic equation of the n th order continuous system can be written as:

$$a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 = 0$$

- The stability criterion is applied using a Routh table which is defined as;

s^n	a_n	a_{n-2}	a_{n-4}	\dots
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	\dots
\cdot	b_1	b_2	b_3	\dots
\cdot	c_1	c_2	c_3	\dots
\cdot	\dots	\dots	\dots	\dots

- Where a_n, a_{n-1}, \dots, a_0 are coefficients of the characteristic equation.

Routh-Hurwitz Criterion

s^n	a_n	a_{n-2}	a_{n-4}	a_{n-6}	\dots
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	a_{n-7}	\dots
s^{n-2}	b_1	b_2	b_3	b_4	\dots
s^{n-3}	c_1	c_2	c_3	c_4	\dots
\vdots	\vdots	\vdots			
s^2	k_1	k_2			
s^1	l_1				
s^0	m_1				

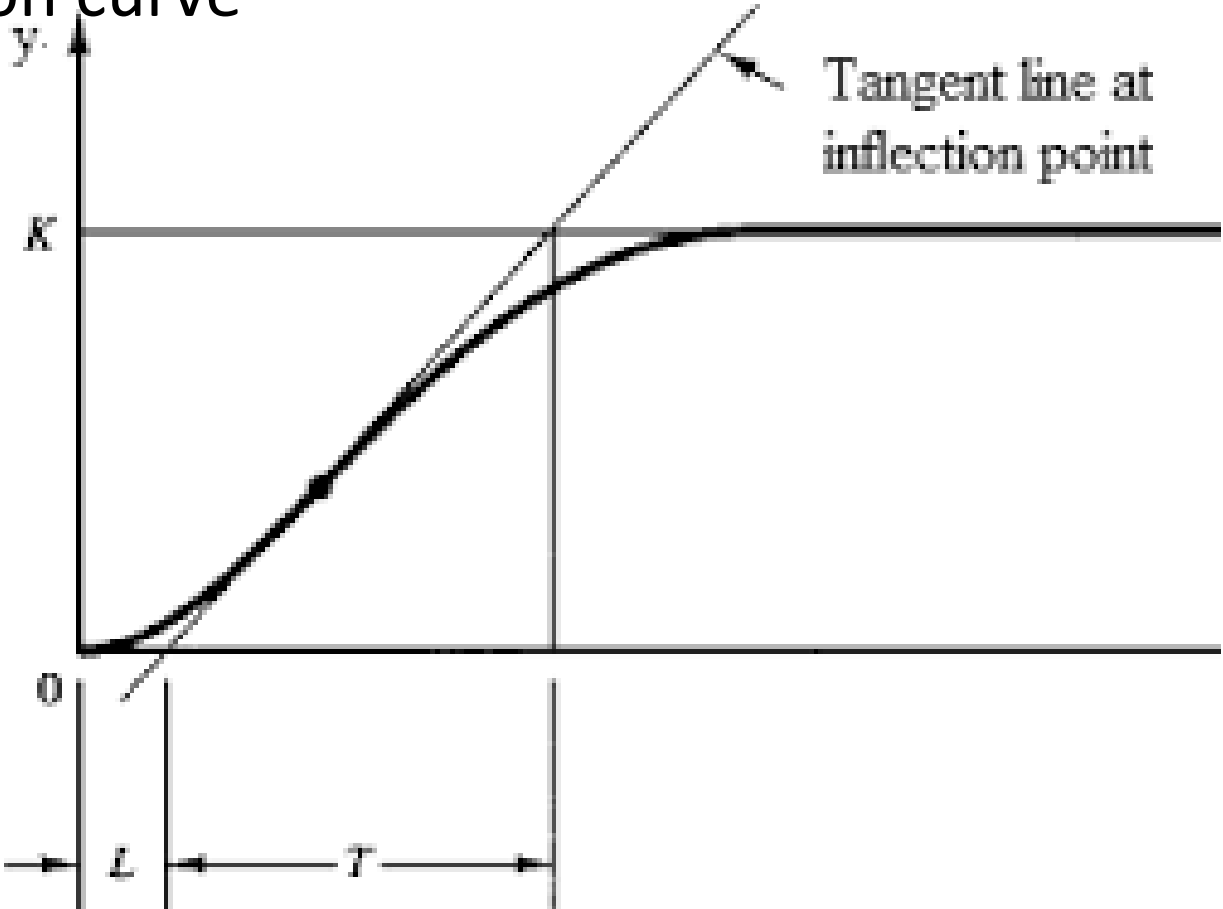
The number of roots in the open right half-plane is equal to the number of sign changes in the **first column** of Routh array.

Routh-Hurwitz stability criteria

- If any of the elements of 1st column is negative, at least one root to be right of imaginary axis and the system is unstable.
- The number of sign changes in the elements of the first column is equal to the number of roots to the right of the imaginary axis.

Cohen coon model

Process reaction curve



Cohen coon model

- Design of P, PI and PID controller

	K_c	τ_I	τ_D
P	$\frac{1}{K} \frac{\tau}{t_d} \left(1 + \frac{t_d}{3\tau} \right)$	—	—
PI	$\frac{1}{K} \frac{\tau}{t_d} \left(0.9 + \frac{t_d}{12\tau} \right)$	$t_d \frac{30 + 3 t_d / \tau}{9 + 20 t_d / \tau}$	—
PID	$\frac{1}{K} \frac{\tau}{t_d} \left(\frac{4}{3} + \frac{t_d}{4\tau} \right)$	$t_d \frac{32 + 6 t_d / \tau}{13 + 8 t_d / \tau}$	$t_d \frac{4}{11 + 2 t_d / \tau}$

Bode diagram

