Probability distributions of continuous variables. Probability density function

$$Pr[a < x < b] = \int_a^b f(x)\,dx.$$

$f(x)$ = Probability density function

$$A = Pr[a < x < b] = \int_a^b f(x)\,dx.$$

$$Pr[x < x_1] = \int_{-\infty}^{x_1} f(x)\,dx$$

$$\sum p(x_i) \sim \int f(x)\,dx.$$

Discrete continuous.

$$\therefore \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Binomial distribution

For only two possible outcomes: head or tail, success or failure, defective item or good item.

Let the probability that an item is defective be p. So the probability that an item is good be q. So $p+q=1$.

Let the fixed no. of trial be n.

Then the general expression for the probability of exactly r defective items in any order in n trial

$$Pr[R=r] = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$= \frac{n!}{r!\,(n-r)!}\, p^{r} q^{n-r}$$

Problem: A company is considering drilling four oil ~~oil~~ wells. Probability of success for each well is 0.4, indedendent of the results for any other well. The cost of each well is \$200,000. Each well that is successful will be worth \$600,000.

a) Determine probability distribution of success.?
b) What is expected number of success.?
c) What is expected gain.?
d) What is gain if only one well is successful!
e) What is probability of a loss.?
f) What is standard deviation?

Varience

$$\sum_{i=1}^{N} (x_i - \mu)^2 / N$$

$$\sigma_x^2 = E(x-\mu_x)^2 = \text{mean } (x-\mu_x)^2$$

$$= \sum (x-\mu_x)^2 Pr(x_i).$$

$$\sigma_x^2 = E[x^2] - \mu_x^2$$

$$= E[x^2 - 2\mu_x X + \mu_x^2]$$

$$= E[x^2] - 2\mu_x E[X] + \mu_x^2$$

$$= E[x^2] - 2\mu_x^2 + \mu_x^2$$

$$= E[x^2] - \mu_x^2$$

$$E[x^2] = \sum x_i^2 Pr(x_i).$$

standard deviation $\sqrt{\sigma_x^2} = \sigma_x.$

$$E(R^2] = 0^2 * \frac{1}{32} + (1)^2 \frac{5}{32} + 2^2 \frac{10}{32} + 3^2 \frac{10}{32} + 4^2 \frac{5}{32} + 5^2 \frac{1}{32}$$

$$= 7.5.$$

$$\therefore \sigma_x^2 = 7.5 - (\mu_R)^2 = 7.5 - (2.5)^2 = 1.25$$

Std. $\sigma_x = \sqrt{1.25} = 1.118$

Poisson Distribution (S.D Poisson, French Mathemati

↳ counts from Geiger counter.
collisions of cars at specific intersection under specific conditions,
flaws in casting

Probability of r occurrences in a fixed interval of time or space under particular condition is given by.

$$Pr[R=r] = \frac{(\lambda t)^r e^{-\lambda t}}{r!}$$

t = interval of time or space in which events occur.
λ = mean rate of occurrence per unit time or space.
λt = dimensionless

$$Pr[R=r+1] = \left(\frac{\lambda t}{r+1}\right) Pr[R=r]$$

Prob
The number of meteors found by a radar system in any 30-sec interval under specified conditions averages 1.81. Assume it appear randomly and independently.

a) What is the probability that no meteors are found in a one-minute interval?

b) Find. $Pr[8 \geq r \geq 5]$.

a) $\lambda = 1.81/0.5 \text{ min}^{-1} = 3.62 \text{ min}^{-1}$.

$\therefore t = 1;\ \mu = \lambda t = 3.62 \times 1 = 3.62$.

$Pr[r=0] = e^{-3.62} = 0.0268$

b) $\mu = \lambda t = 3.62 \times 2 = 7.24$ occurance in 2 minute.

$$Pr[R=5] = \frac{(7.24)^5 e^{-7.24}}{5!} = 0.1189$$

$$Pr[R=r+1] = \frac{\lambda t}{r+1} Pr[R=r]$$

$$Pr[R=6] = \frac{7.24}{6} \, 0.1189 = 0.1435$$

$$Pr[R=7] = \frac{7.24}{7} \times 0.1435 = 0.1484$$

$$Pr[R=8] = \frac{7.24}{8} \times 0.1484 = 0.1343$$

$$Pr[5 \le r \le 8] = 0.1189 + 0.1435 + 0.1484 + 0.1343$$

$$= \underline{\underline{0.545}}$$

Answer: First, let us calculate the sample mean as an estimate of the population mean, μ.

$$F_X = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

x_i	f_i		$x_i f_i$	
0	2	0.056505	0	0.056505
1	7	0.127825	7	0.127825
2	10	0.2050	20	0.2050
3	8	0.233262	24	0.233262
4	6	0.188169	24	
5	3	0.10764	15	0.1076
6	3	0.043665	18	0.0436
7	1	0.012561	7	0.012561
>8	0	0.002562 8	0	$1 - \int_0^7 F(x)\,dx$
Total	40		115	

Then $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{115}{40} = 2.875$. Then take $\mu = \lambda t = 2.875$ in 6 minutes.

Then $\lambda = \frac{\lambda t}{t} = \frac{2.875}{6} = 0.479$ cars / minute.

According to the Poisson Distribution, then, Pr $[R=r] = (2.875)^r e^{-2.875} / r!$. It was mentioned previously that once one of the Poisson probabilities is calculated, others can be calculated conveniently using the recurrence relation of equation 5.14,

$$\Pr[R=r+1] = \left(\frac{\lambda t}{r+1}\right)\Pr[R=r].$$

Calculation of Poisson probabilities and relative frequencies gives the following results:

r	f_i	**Pr $[R=r]$**	**Relative Frequency**	$x^2 f_i$
0	2	0.0564	0.0500	0
1	7	0.1622	0.1750	7
2	10	0.2332	0.2500	40
3	8	0.2234	0.2000	72
4	6	0.1606	0.1500	96
5	3	0.0923	0.0750	75
6	3	0.0442	0.0750	108
7	1	0.0182	0.0250	49
>8	0	0.0095	0	0
Total	40			Sum = 447

$$\sigma^2 \quad E(x^2) = \frac{447}{40} = 11.175$$

$$\sigma_x^2 = 11.175 - \mu_x^2 = 2.9093$$

$$\sigma_x = 1.7056$$

The frequencies from the problem statement are compared with the calculated expected frequencies in Figure 5.18. It can be seen that the agreement between

f(x) gaussian.

Probability → It is a measure of the likelihood that a particular event occur.

Probability distribution of discrete variable

Probability functions

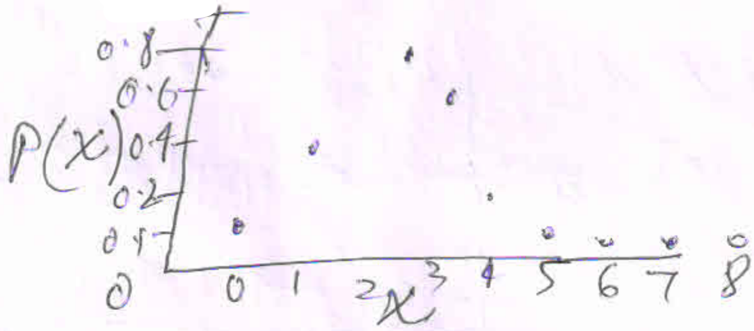
discrete random variable X are

$x_0 \ x_1 \ x_2 \ldots\ldots \ x_K$.

Probabilities $p(x_0)\ p(x_1)\ p(x_2) \ldots p(x_K)$.

$p(x_i) \geq 0$ and $\sum_{i=0}^{K} p(x_i) = 1$.

$p(x_i)$ = probability function



cumulative distribution function

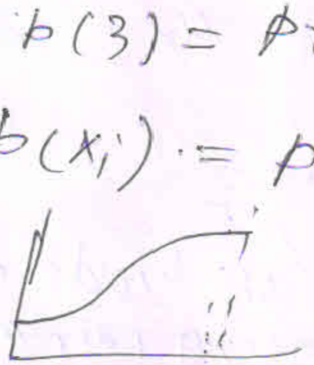
$$Pr[X \leq x] = \sum p(x_i).$$

$$Pr[X \leq 3] = p(x_0) + p(x_1) + p(x_2) + p(x_3)$$
$$= p(0) + p(1) + p(2) + p(3)$$

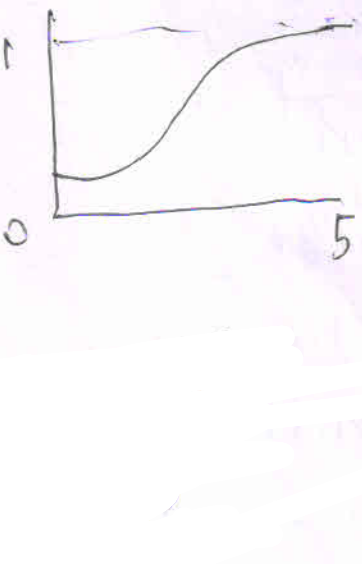
$$Pr[X \leq 2] = p(0) + p(1) + p(2).$$

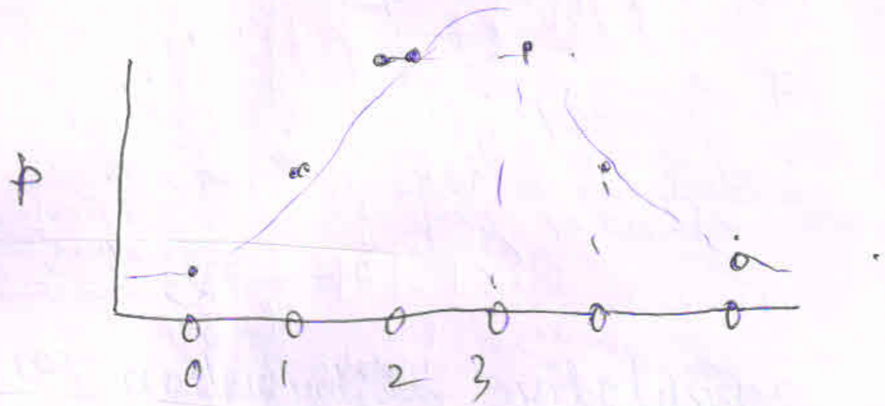
$$p(3) = Pr[X \leq 3] - Pr[X \leq 2].$$

$$p(x_i) = Pr[X < x_i] - Pr[X \leq x_{i-1}]$$



Five fair coin	no. of heads (r)	p(r)	
	0	$\frac{1}{32}$	${}^{5}C_{0}\,(0.5)^{0}.(0.5)^{5}$
	1	$5/32$	$\frac{5!}{5!\,0!}\,(0.5)^{5}$
	2	$\frac{10}{32}$	
	3	$\frac{10}{32}$	
	4	$5/32$	
	5	$1/32$	





$$p(3) = Pr[R \leq 3] - Pr[R \leq 2]$$

$$= \frac{26}{32} - \frac{16}{32} = 0.3125$$

$$P_r[x_i] = \frac{f(x_i)}{\Sigma f(x_i)}$$

Mean

$$E(x) = \mu_x = \Sigma x_i Pr[x_i]$$

$$E(R) = \mu_R = 0 \times \frac{1}{32} + 1 \times \frac{5}{32} + 2 \times \frac{10}{32} + 3 \times \frac{10}{32} + 4 \times \frac{5}{32} + 5 \times \frac{1}{32}$$

$= 2.5$ no. of heads obtained on tossing fair coin.

f). $\sigma_x^2 = E(x^2) - \mu_x^2$

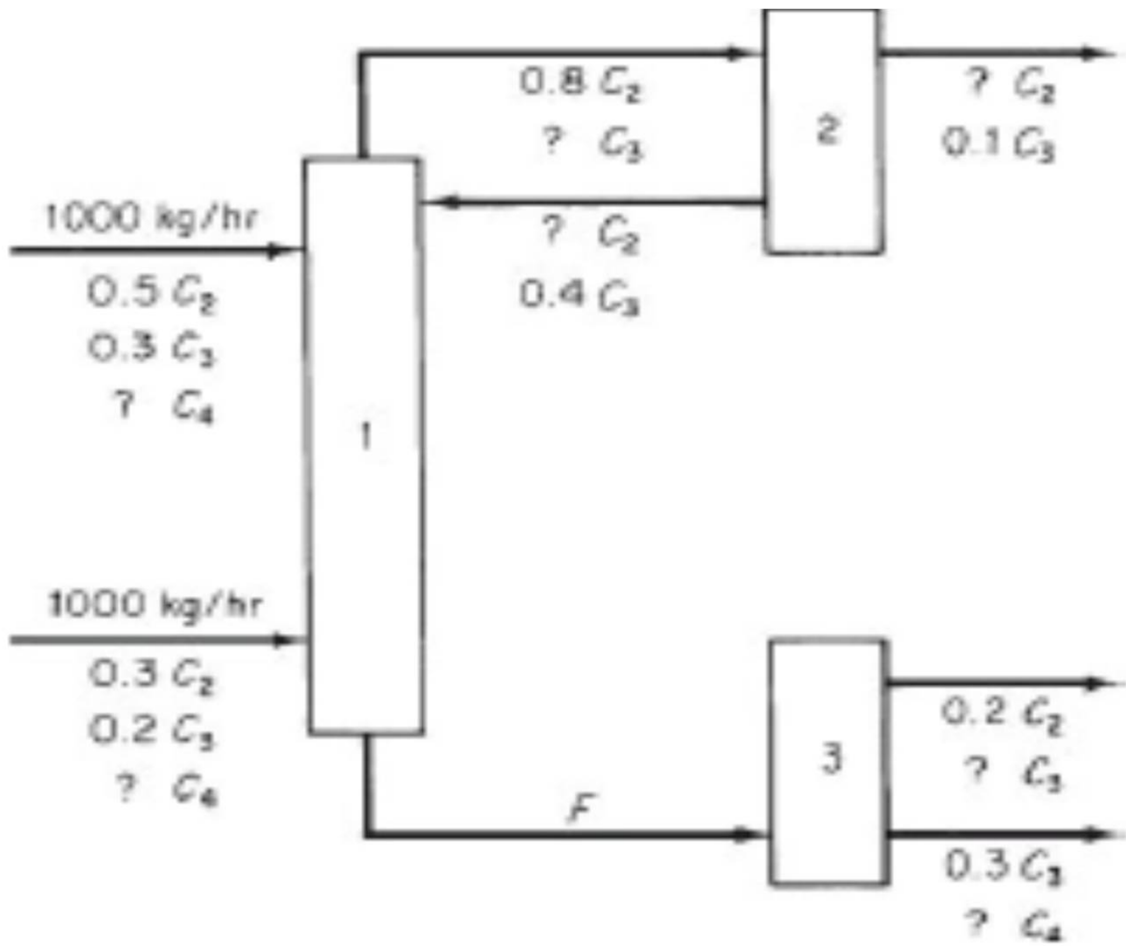
$E(x)^2 = 3.5200$. $\mu_x = 1.6$.

$\therefore \sigma_x^2 = 3.5 - (1.6)^2 = 0.96$

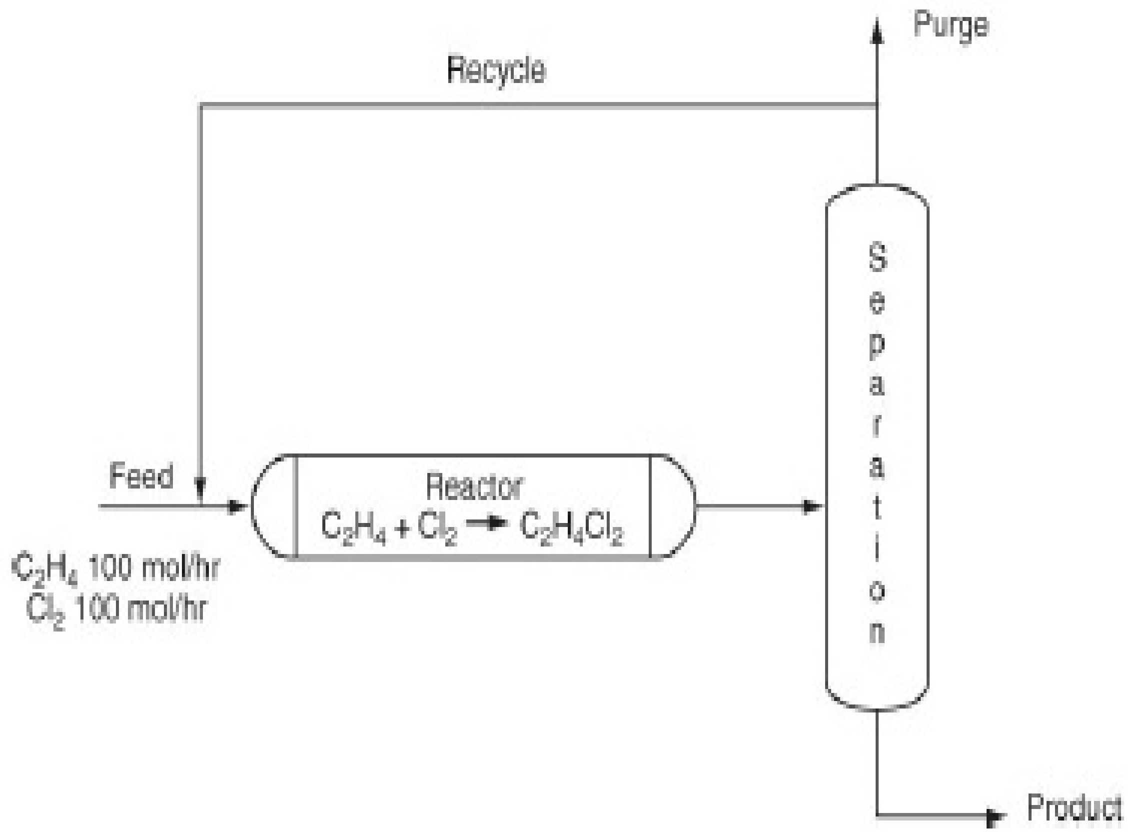
$\sigma_x = \sqrt{0.96} = \underline{\underline{0.98}}$

Reference: W. J. DeCoursey, Statistics and Probability for Engineering Applications.

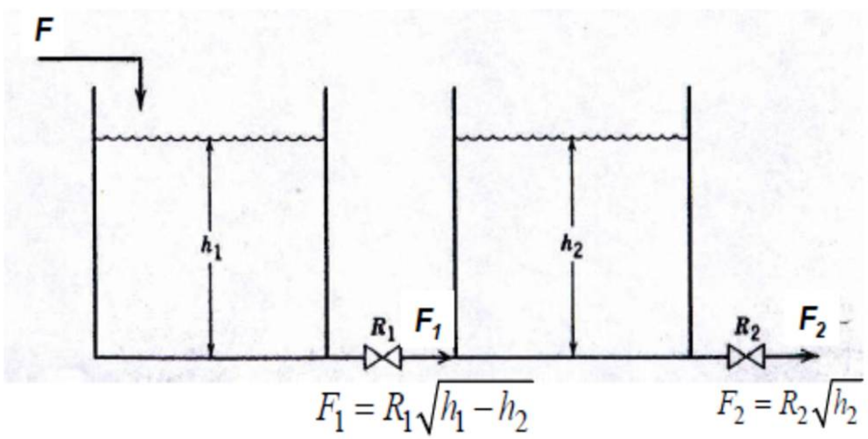
- A distillation process is shown in Figure. You are asked to solve for all of the values of the stream flows and compositions. Explain each answer and show all details of how you reached your decision. For each stream (except *F*), the only components that occur are labeled below the stream. Solve the system by ***Matlab*** function.



- Figure shows a simplified process to make ethylene dichloride ($C_2H_4Cl_2$). The feed data have been placed on the figure. Ninety percent conversion of the C_2H_4 occurs on each pass through the reactor. The overhead stream from the separator contains 98% of the Cl_2 entering the separator, 92% of the entering C_2H_4, and 0.1% of the entering $C_2H_4Cl_2$. Five percent of the overhead from the separator is purged. Calculate (a) the flow rate and (b) the composition of the purge stream using ***Matlab*** function.



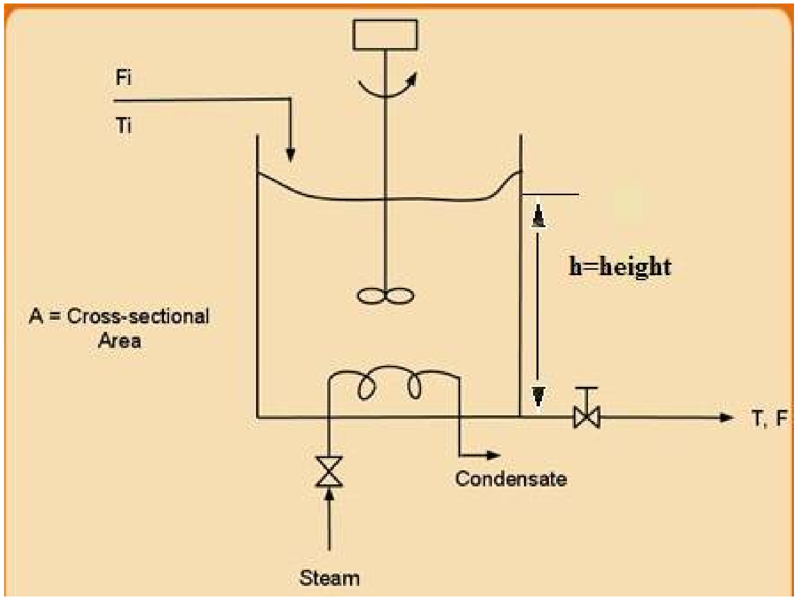
- **Two interacting tank in series with outlet flow rate being function of the square root of tank height. Write mass balance (ordinary differential equation) equations of two tanks assuming no change of density in the system.**



F=5 ft^3/min, A_1=5 ft^2, A_2=10 ft^2, R_1=2.5 ft$^{2.5}$/min,

Solve two equations with ode45 function in Matlab and find h_2 in the time range from t=0 min to t=150 min. At t=0, $h_1(0)$=12 and $h_2(0)$=7.

- Consider the stirred Tank Heater System (Figure): Total momentum of the system remains constant and will not be considered. Write total mass balance: Total mass in the tank at any time t =*r*V = *r*Ah where A represents cross sectional area, h represents height of liquid and *r* represents density of the liquid. Assuming that the density is independent of the temperature and remains constant. Take $F = 0.02236\sqrt{h}$. Write energy balance equation considering no change in kinetic energy and potential energy. For liquid system assume change of internal energy same as enthalpy change. Heat given through steam is *Q*=10 kW and it is remains unchanged. Solve total mass balance and energy balance equations with state variables h (in material balance) and T (in energy balance) by ***Matlab*** ode solver with initial gauss of T=30^{o}C and h=0.1 m. Find the steady state h and steady state T of the tank. Take inlet temperature of the tank T_i=30^{0}C, inlet flow rate of the tank F_i=0.01 m^3/min. A=1 m^2, *r*=1000 kg/m^3, C_p=2000 J/kg.



- **Vapor-Liquid equilibrium for ideal mixture**

1. A liquid mixture contains 50% pentane (1), 30% hexane (2) and 20% cyclohexane (3) (all in mol-%), i.e., $x_1 = 0.5$; $x_2 = 0.3$; $x_3 = 0.2$. At T = 400 K, the pressure is gradually decreased. Estimate bubble pressure and composition of the first vapor that is formed using ***Matlab-generated code.*** Assume ideal liquid mixture and Ideal gas (Raoult's law).

Components	A	B	C
pentane	3.97786	1064.840	-41.136
hexane	4.00139	1170.875	-48.833
cyclohexane	3.93002	1182.774	-52.532

Where, $log_{10}P_{sat}(bar) = A - \frac{B}{T(K)+C}$

2. Consider the liquid mixture with 50% pentane (1), 30% hexane (2) and 20% cyclohexane (3) (all in mol-%). At p = 5 bar, the temperature is gradually increased. Estimate bubble temperature and composition of the first vapor that is formed using ***Matlab-generated code.*** Assume ideal liquid mixture and Ideal gas (Raoult's law).

3. A vapor mixture contains 50% pentane (1), 30% hexane (2) and 20% cyclohexane (3) (all in mol-%), i.e., $y_1 = 0.5$; $y_2 = 0.3$; $y_3 = 0.2$. At T = 400 K, the pressure is gradually increased. Estimate dew-point pressure and composition of the first liquid that is formed using ***Matlab-generated code.*** Assume ideal liquid mixture and Ideal gas (Raoult's law).
4. A vapor mixture contains 50% pentane (1), 30% hexane (2) and 20% cyclohexane (3) (all in mol-%), i.e., $y_1 = 0.5$; $y_2 = 0.3$; $y_3 = 0.2$. At P = 5 bar, the temperature is gradually decreased. Estimate dew-point T and composition of the first liquid that is formed using ***Matlab-generated code.*** Assume ideal liquid mixture and Ideal gas (Raoult's law).

- **Vapor-Liquid equilibrium for non ideal mixture**

1. Estimate the bubble point temperature and vapor composition for a acetone (1) and water (2) liquid mixture with x_1 = 0.01 at a total pressure of 101.3 kPa6 with a ***Matlab*** code. Use the Wilson model with the parameters:

Components	Λ_{12}	Λ_{21}
Acetone	0.1173	0.4227
Water		

$$lnP^1{}_{sat}(kPa) = 14.71712 - \frac{2975.95}{T(K)-34.5228}$$

$$lnP^2{}_{sat}(kPa) = 16.5362 - \frac{3985.44}{T(K)-38.9974}$$

2. Construct a Txy diagram for a mixture of ethanol (1) with hexane (2) at a total pressure of 101.3 kPa with a ***Matlab*** code. Use the Wilson model with the parameters $\Lambda_{12} = 0.0952$, $\Lambda_{21} = 0.2713$. Vapor pressure: (P^i_{sat} in kPa and T in ^oK)

$$lnP^1{}_{sat}(kPa) = 16.1952 - \frac{3423.53}{T(K) - 55.7172}$$

$$lnP^2{}_{sat}(kPa) = 14.0568 - \frac{2825.42}{T(K) - 42.7089}$$

Wilson	$\ln \gamma_1 = -\ln[x_1 + x_2\Lambda_{12}] + x_2\left[\frac{\Lambda_{12}}{x_1 + x_2\Lambda_{12}} - \frac{\Lambda_{21}}{x_2 + x_1\Lambda_{21}}\right]$ $\ln \gamma_2 = -\ln[x_2 + x_1\Lambda_{21}] + x_1\left[\frac{\Lambda_{21}}{x_2 + x_1\Lambda_{21}} - \frac{\Lambda_{12}}{x_1 + x_2\Lambda_{12}}\right]$	Aromatics, alcohol, ketones, ethers, C_4-C_{18} hydrocarbons