Design of Experiments

Design Of Experiments

- Design of Experiments is a method of experimenting with complex processes with the objective of optimizing the process.

Design of Experiments

- Dr. Genichi Taguchi (1924-   )
  - Loss Function
    - Quality, or the lack of it, is a loss to society
  - Experiment Design
  - Four Basic Steps to Experiments
    - Select the process/product to be studied
    - Identify the important variables
    - Reduce variation on the important process improvement
    - Open up tolerances on unimportant variables

Design Of Experiments

- Design of experiments seeks to:
  - Determine which variables affect the system.
  - Determine how the magnitude of the variables affects the system.
  - Determine the optimum levels for the variables.
  - Determine how to manipulate the variables to control the response.

Design Of Experiments

- Methods of Experimentation
  - Trial and Error
  - Single Factor Experiment
    - one change at a time
  - Fractional Factorial Experiment
    - change two or more things at a time
  - Full Factorial Experiment
    - change many things at a time
  - Others (Box-Jenkins, Taguchi, etc.)

Design Of Experiments

- Trial and Error Experiments
  - Lack direction and focus
  - Guesswork

Design Of Experiments

- Trial and Error Experiment Example

Problem: Selecting copying settings to prepare a document

Contrast	Size
7	93
6	85
5	78

- How many different permutations exist?
- What would happen if we added three settings for location (center, left flush, right flush)?

Design Of Experiments

- Single Factor Experiment
  - A single factor experiment allows for the manipulation of only one factor during an experiment.
    - Select one factor and vary it, while holding all other factors constant.
  - The objective in a single factor experiment is to isolate the changes in the response variable as they relate to the single factor.

Design Of Experiments

- Single Factor Experiment
  - These types of experiments are:
    - Simple to Analyze
      - Only one thing changes at a time and you can see what affect that change has on the system.
    - Time Consuming
      - Changing only one thing at a time can result in dozens of repeated experiments.

Design Of Experiments

- Single Factor Experiment
  - In these types of experiments:
    - Interactions between factors are not detectable.
      - These experiments rarely arrive at an optimum setup because a change in one factor frequently requires adjustments to one or more of the other factors to achieve the best results.
  - Life isn't this simple
    - Single factor changes rarely occur that are not inter-related to other factors in real life..

Design Of Experiments

- Single Factor Experiment Example
- Problem: What combination of factors avoids tire failure?

Speed	Temperature	Tire Pressure	Chassis Design
65	75	32	A
70	75	32	A
65	75	32	B
70	75	32	B
65	85	32	A
70	85	32	A
65	85	32	B
70	85	32	B
65	75	27	A
70	75	27	A
65	75	27	B
70	75	27	B
65	85	27	A
70	85	27	A
65	85	27	B
70	85	27	B

Design Of Experiments

- Fractional Factorial Experiment
  - Studies only a fraction or subset of all the possible combinations.
    - A selected and controlled multiple number of factors are adjusted simultaneously.
      - This reduces the total number of experiments.
      - This reveals complex interactions between the factors.
      - This will reveal which factors are more important than others.

Design Of Experiments

- Fractional Factorial Experiment Example
- Problem: What combination of factors avoids tire failure?

Speed	Temperature	Tire Pressure	Chassis Design
70	75	32	A
65	75	32	B
65	85	32	A
70	85	32	B
70	75	27	A
65	75	27	B
65	85	27	A
70	85	27	B

Design Of Experiments

- Full Factorial Experiment
  - A full-factorial design consists of all possible combinations of all selected levels of the factors to be investigated.
    - Examines every possible combination of factors at all levels.

Design Of Experiments

- Full Factorial Experiment
  - A full-factorial design allows the most complete analysis
    - Can determine main effects of the factors manipulated on response variables
    - Can determine effects of factor interactions on response variables
    - Can estimate levels at which to set factors for best result
  - Time consuming

Design Of Experiments

- Full Factorial Experiment Example
- Problem: What combination of factors avoids tire failure?
- Speed Temperature Tire Pressure Chassis Design
- 65 75 32 A
- 70 85 32 A
- 70 85 27 A
- 65 75 32 B
- 70 85 32 B
- 70 85 27 B
-
- 65 85 32 A
- 65 85 27 A
-
- 65 85 32 B
- 65 85 27 B
-
- 70 75 27 A
- 70 85 27 A
- 70 85 32 A
- 70 85 27 A
- 70 85 27 B

Design Of Experiments

- Conducting an Experiment: The Process
  - Plan your experiment!
    - Successful experiments depend on how well they are planned.
      - What are you investigating?
      - What is the objective of your experiment?
      - What are you hoping to learn more about?
      - What are the critical factors?
      - Which of the factors can be controlled?
      - What resources will be used?

Design Of Experiments

- Conducting an Experiment: The Process
  - Setting up your experiment.
    - Determine the factors
      - How many factors will the design consider?
      - How many levels (options) are there for each factor?
      - What are the settings for each level?
      - What is the response factor?

Design Of Experiments

- Conducting an Experiment: The Process
  - Select a study for your experiment
    - Full Factorial
    - Fractional Factorial
    - Other

Design Of Experiments

- Conducting an Experiment: The Process
  - Run your experiment!
    - Complete the runs as specified by the experiment at the levels and settings selected.
    - Enter the results into analysis program.

Design Of Experiments

- Conducting an Experiment: The Process
  - Analyze your experiment!
    - Use statistical tools to analyze your data and determine the optimal levels for each factor.
      - Analysis of Variance
      - Analysis of Means
      - Regression Analysis
      - Pairwise comparison
      - Response Plot
      - Effects Plot
      - Etc.

Design Of Experiments

- Conducting an Experiment: The Process
  - Apply the knowledge you gained from your experiment to real life.

Design Of Experiments

- An ANOM is an analysis of means.
  - A one-way analysis of means is a control chart that identifies subgroup averages that are detectably different from the grand average.
    - The purpose of a one-way ANOM is to compare subgroup averages and separate those that represent signals from those that do not.
      - Format: a control chart for subgroup averages, each treatment (experiment) is compared with the grand average.

Design Of Experiments

- An ANOVA is an Analysis of Variance
  - Used to determine whether or not changes in factor levels have produced significant effects upon a response variable.
    - An ANOVA estimates the variance of the X using two-three different methods.
      - If the estimates are similar, then detectable differences between the subgroup averages are unlikely.
      - If the differences are large, then there is a difference between the subgroup averages that are not attributable to background noise alone.
      - ANOVA compares the between-subgroup estimate of variance of x with the within subgroup estimate.

Design Of Experiments

- Definitions:
  - Factor:
    - The variable that is changes and results observed.
      - A variable which the experimenter will vary in order to determine its effect on a response variable.
        - » (Time, temperature, operator…)
  - Level:
    - A value assigned to change the factor.
        - » Temperature; Level 1: 110, Level 2: 150

Design Of Experiments

- Definitions:
  - Effect:
    - The change in a response variable produced by a change in the factor level.
  - Degree of Freedom:
    - The number of levels of a factor minus 1.
  - Interaction:
    - Two or more factors that, together, produce a result different that what the result of their separate effects would be.

Design Of Experiments

- Definitions:
  - Noise factor:
    - An uncontrollable (but measurable) source of variation in the functional characteristics of a product or process.
  - Response variable:
    - The variable(s) used to describe the reaction of a process to variations in control variables (factors).
    - The Quality characteristic under study.

Design Of Experiments

- Definitions:
  - Treatment:
    - A set of conditions for an experiment
      - factor x level used for a particular run.
  - Run:
    - An experimental trial. The application of one treatment to one experimental unit.

Design Of Experiments

- Definitions:
  - Replicate:
    - Repeat the treatment condition.
  - Repetition:
    - Multiple results of a treatment condition.
  - Significance:
    - The importance of a factor effect in either a statistical sense or in a practical sense.

Design Of Experiments

- Types of Errors
  - Type I Error:
    - A conclusion that a factor produces a significant effect on a response variable when, in fact, its effect is negligible (a false alarm).
  - Type II Error:
    - A conclusion that a factor does not produce a significant effect on a response variable when, in fact, its effect is meaningful.

Design Of Experiments

- Experiment Errors
  - lack of uniformity of the material
  - inherent variability in the experimental technique

Design Of Experiments

- Characteristics of a Good Experiment Design
  - The experiment should provide unbiased estimates of process variable and treatment effects (factors at different levels).
  - The experiment should provide the precision necessary to enable the experimenter to detect important differences.
  - The experiment should plan for the analysis of the results.

Design Of Experiments

- Characteristics of a Good Experiment Design
  - The experiment should generate results that are free from ambiguity of interpretation.
  - The experiment should point the experimenter in the direction of improvement.
  - The experiment should be as simple as possible.
    - Easy to set up and carry out
    - Simple to analyze and interpret
    - Simple to communicate or explain to others

Design and Analysis of Multi-Factored Experiments

DOE - I

Introduction

Design of Engineering Experiments Introduction

- Goals of the course and assumptions
- An abbreviated **history** of DOE
- The **strategy** of experimentation
- Some basic **principles** and terminology
- **Guidelines** for planning, conducting and analyzing experiments

Assumptions

- You have
  - a first course in statistics
  - heard of the normal distribution
  - know about the mean and variance
  - have done some regression analysis or heard of it
  - know something about ANOVA or heard of it
- Have used Windows or Mac based computers
- Have done or will be conducting experiments
- Have not heard of factorial designs, fractional factorial designs, RSM, and DACE.

Some major players in DOE

- Sir Ronald A. Fisher - pioneer
  - invented ANOVA and used of statistics in experimental design while working at Rothamsted Agricultural Experiment Station, London, England.
- George E. P. Box - married Fisher's daughter
  - still active (86 years old)
  - developed response surface methodology (1951)
  - plus many other contributions to statistics
- Others
  - Raymond Myers, J. S. Hunter, W. G. Hunter, Yates, Montgomery, Finney, etc..

Four eras of DOE

- The **agricultural** origins, 1918 – 1940s
  - R. A. Fisher & his co-workers
  - Profound impact on agricultural science
  - Factorial designs, ANOVA
- The **first industrial** era, 1951 – late 1970s
  - Box & Wilson, response surfaces
  - Applications in the chemical & process industries
- The **second industrial** era, late 1970s – 1990
  - Quality improvement initiatives in many companies
  - Taguchi and robust parameter design, process robustness
- The **modern** era, beginning circa 1990
  - Wide use of computer technology in DOE
  - Expanded use of DOE in Six-Sigma and in business
  - Use of DOE in computer experiments

References

- D. G. Montgomery (2008): Design and Analysis of Experiments, 7th Edition, John Wiley and Sons
  - one of the best book in the market. Uses Design-Expert software for illustrations. Uses letters for Factors.
- G. E. P. Box, W. G. Hunter, and J. S. Hunter (2005): Statistics for Experimenters: An Introduction to Design, Data Analysis, and Model Building, John Wiley and Sons. 2nd Edition
  - Classic text with lots of examples. No computer aided solutions. Uses numbers for Factors.
- Journal of Quality Technology, Technometrics, American Statistician, discipline specific journals

Introduction: What is meant by DOE?

- Experiment -
  - a test or a series of tests in which purposeful changes are made to the *input variables or factors* of a system so that we may observe and identify the reasons for changes in the *output* response(s).
- Question: 5 factors, and 2 response variables
  - Want to know the effect of each factor on the response and how the factors may interact with each other
  - Want to predict the responses for given levels of the factors
  - Want to find the levels of the factors that optimizes the responses - e.g. maximize Y_1 but minimize Y_2
  - Time and budget allocated for 30 test runs only.

Strategy of Experimentation

- Strategy of experimentation
  - Best guess approach (trial and error)
    - can continue indefinitely
    - cannot guarantee best solution has been found
  - One-factor-at-a-time (OFAT) approach
    - inefficient (requires many test runs)
    - fails to consider any possible interaction between factors
  - Factorial approach (invented in the 1920’s)
    - Factors varied together
    - Correct, modern, and most efficient approach
    - Can determine how factors interact
    - Used extensively in industrial R and D, and for process improvement.

- This course will focus on three very useful and important classes of factorial designs:
  - 2-level full factorial (2^k)
  - fractional factorial (2^{k-p}), and
  - response surface methodology (RSM)
- I will also cover split plot designs, and design and analysis of computer experiments if time permits.
- Dimensional analysis and how it can be combined with DOE will also be briefly covered.
- All DOE are based on the same statistical principles and method of analysis - ANOVA and regression analysis.
- *Answer to question: use a 2^{5-1} fractional factorial in a central composite design = 27 runs (min)*

Statistical Design of Experiments

- All experiments should be designed experiments
- Unfortunately, some experiments are poorly designed - valuable resources are used ineffectively and results inconclusive
- Statistically designed experiments permit efficiency and economy, and the use of statistical methods in examining the data result in scientific objectivity when drawing conclusions.

- DOE is a methodology for systematically applying statistics to experimentation.
- DOE lets experimenters develop a mathematical model that predicts how input variables *interact* to create output variables or responses in a process or system.
- DOE can be used for a wide range of experiments for various purposes including nearly all fields of engineering and even in business marketing.
- Use of statistics is very important in DOE and the basics are covered in a first course in an engineering program.

- In general, by using DOE, we can:
  - Learn about the process we are investigating
  - Screen important variables
  - Build a mathematical model
  - Obtain prediction equations
  - Optimize the response (if required)

- Statistical significance is tested using **ANOVA**, and the prediction model is obtained using **regression analysis.**

Applications of DOE in Engineering Design

- Experiments are conducted in the field of engineering to:
  - evaluate and compare basic design configurations
  - evaluate different materials
  - select design parameters so that the design will work well under a wide variety of field conditions (robust design)
  - determine key design parameters that impact performance

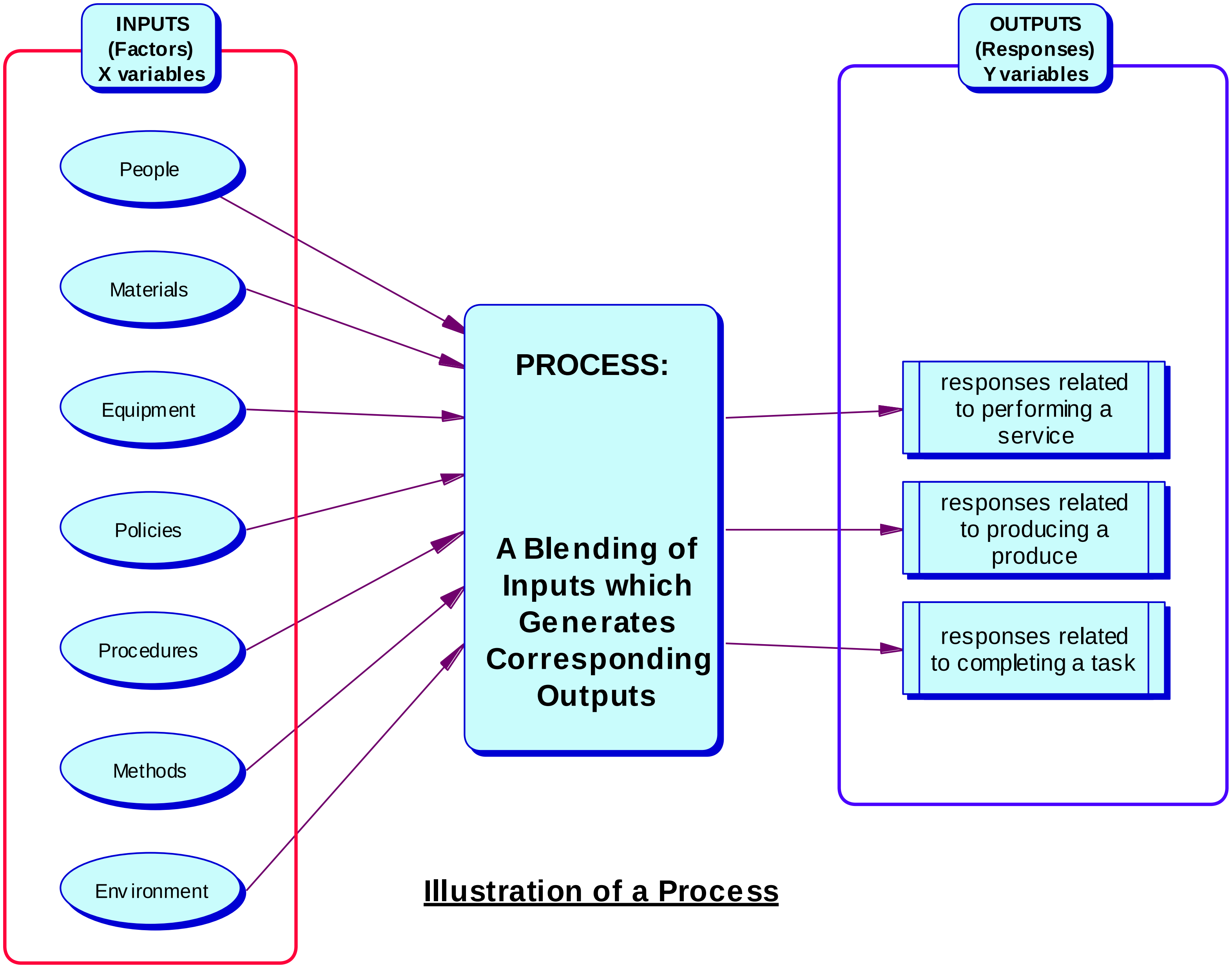
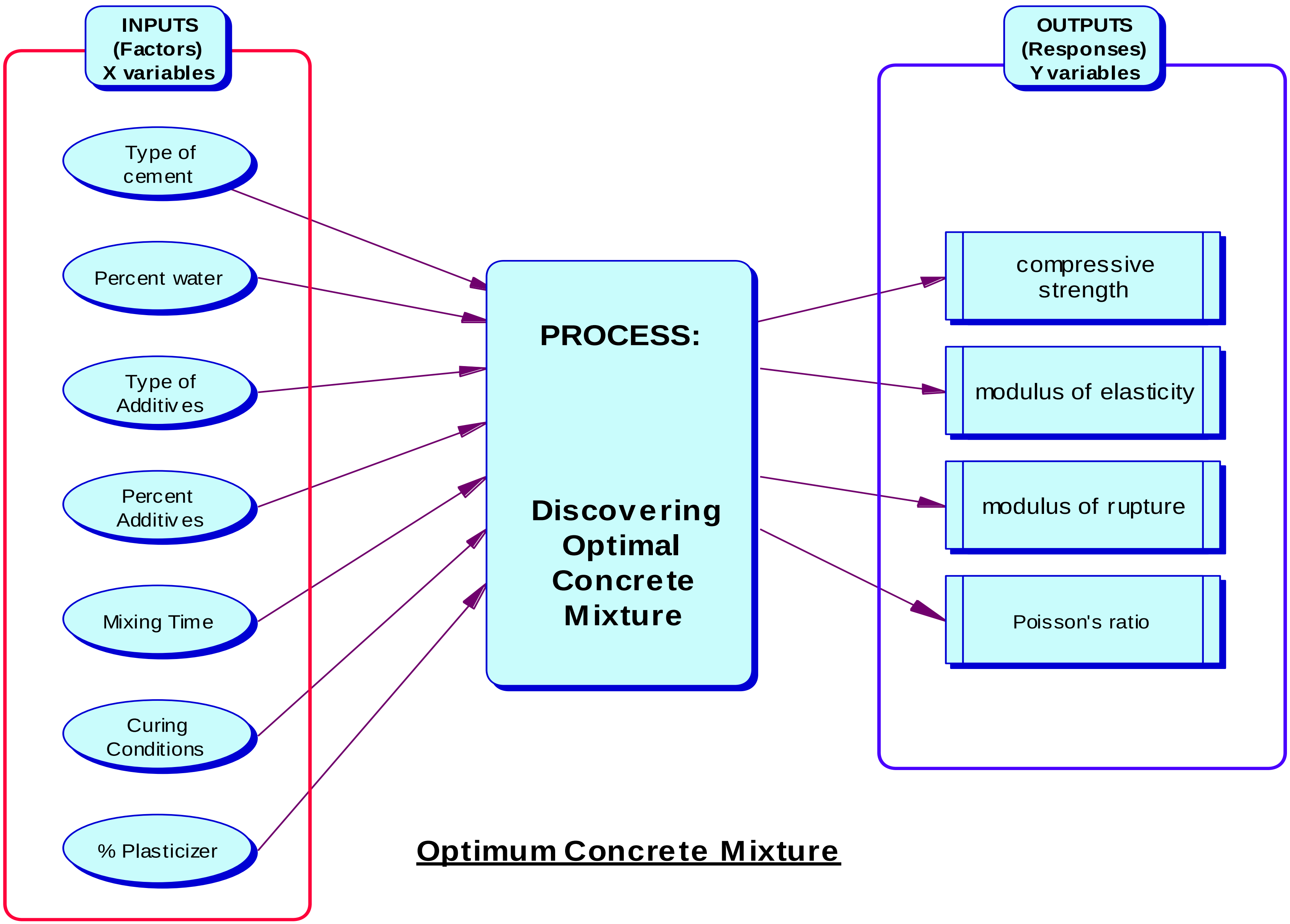
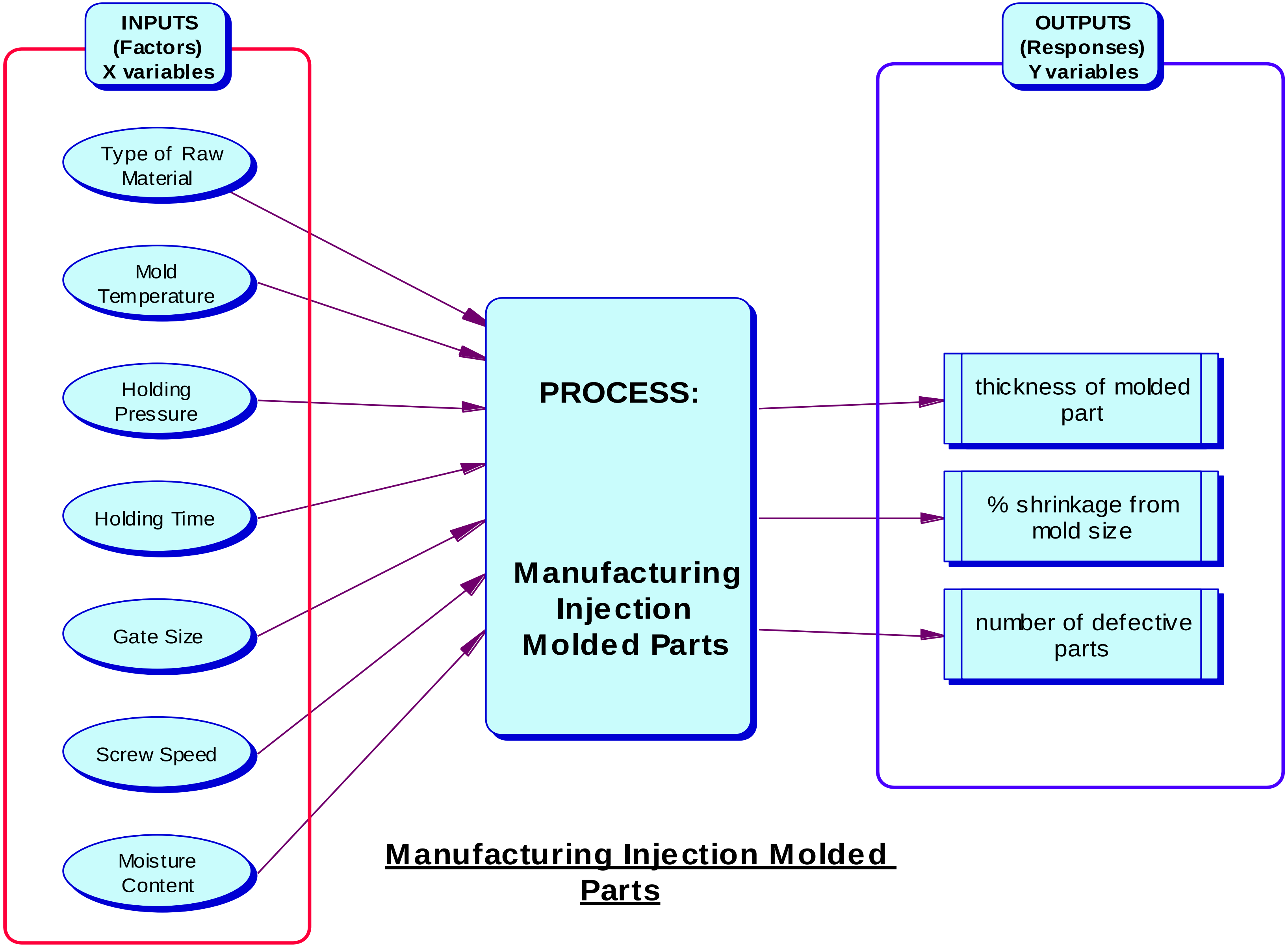


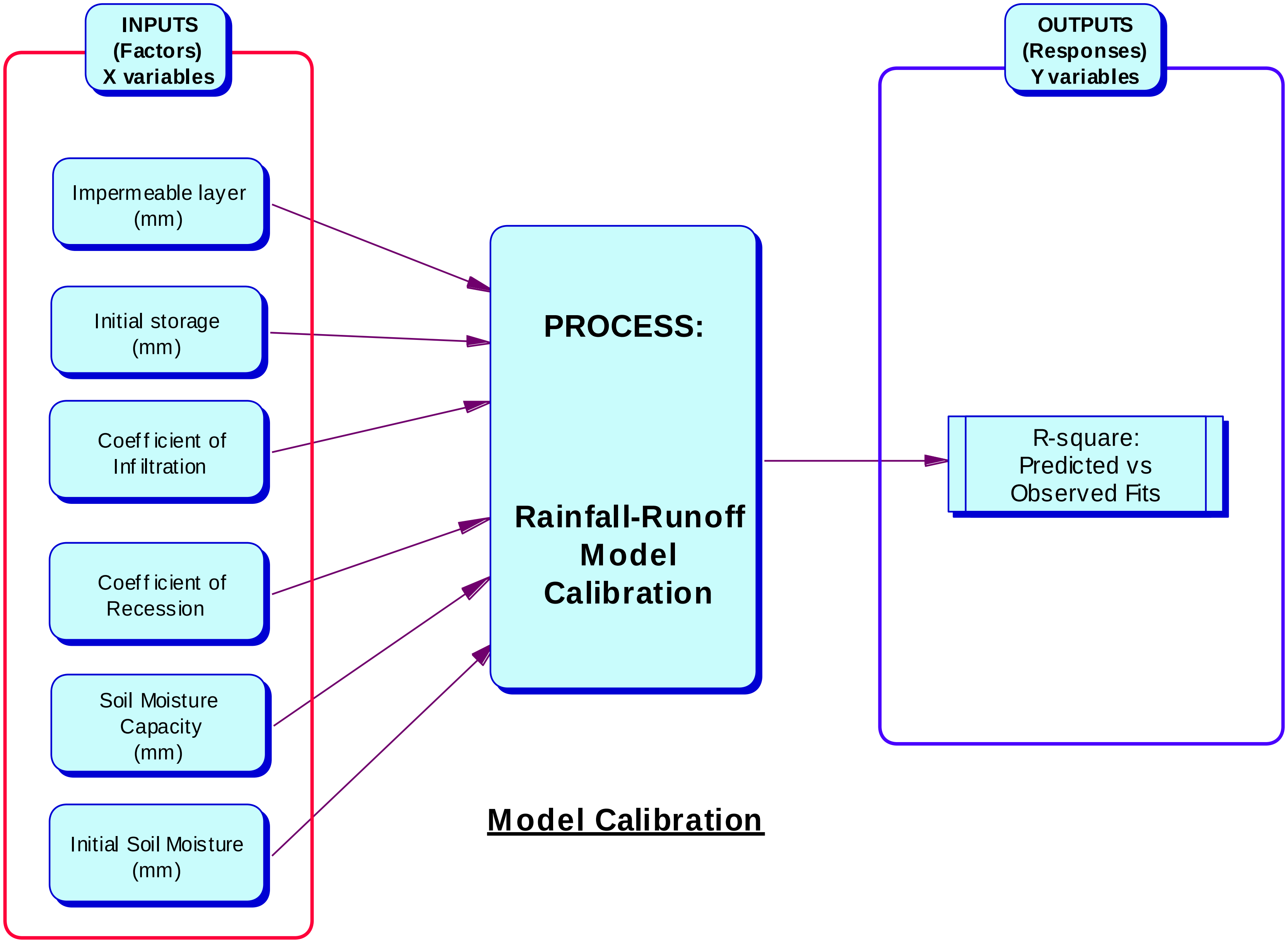
Illustration of a Process



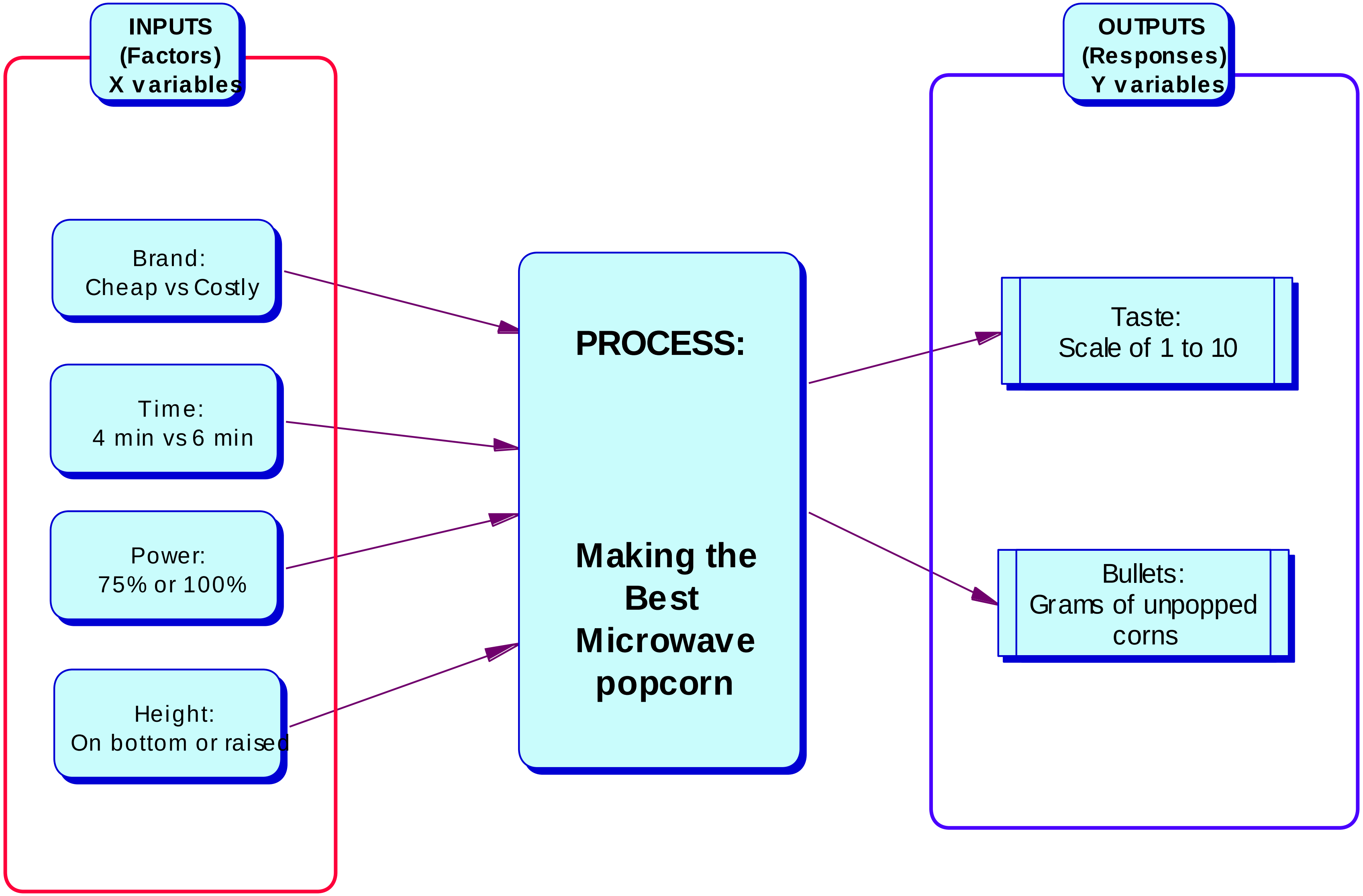
Optimum Concrete Mixture



Manufacturing Injection Molded Parts



Model Calibration



Making microwave popcorn

Examples of experiments from daily life

- Photography
  - Factors: speed of film, lighting, shutter speed
  - Response: quality of slides made close up with flash attachment
- Boiling water
  - Factors: Pan type, burner size, cover
  - Response: Time to boil water
- D-day
  - Factors: Type of drink, number of drinks, rate of drinking, time after last meal
  - Response: Time to get a steel ball through a maze
- Mailing
  - Factors: stamp, area code, time of day when letter mailed
  - Response: Number of days required for letter to be delivered

More examples

- Cooking
  - Factors: amount of cooking wine, oyster sauce, sesame oil
  - Response: Taste of stewed chicken
- Basketball
  - Factors: Distance from basket, type of shot, location on floor
  - Response: Number of shots made (out of 10) with basketball
- Skiing
  - Factors: Ski type, temperature, type of wax
  - Response: Time to go down ski slope

Basic Principles

- Statistical design of experiments (DOE)
  - the process of planning experiments so that appropriate data can be analyzed by statistical methods that results in valid, objective, and meaningful conclusions from the data
  - involves two aspects: design and statistical analysis

- Every experiment involves a sequence of activities:
  - Conjecture - hypothesis that motivates the experiment
  - Experiment - the test performed to investigate the conjecture
  - Analysis - the statistical analysis of the data from the experiment
  - Conclusion - what has been learned about the original conjecture from the experiment.

Three basic principles of Statistical DOE

- Replication
  - allows an estimate of experimental error
  - allows for a more precise estimate of the sample mean value
- Randomization
  - cornerstone of all statistical methods
  - “average out” effects of extraneous factors
  - reduce bias and systematic errors
- Blocking
  - increases precision of experiment
  - “factor out” variable not studied

Guidelines for Designing Experiments

- Recognition of of and statement of the problem
  - need to develop all ideas about the objectives of the experiment - get input from everybody - use team approach.
- Choice of factors, levels, ranges, and response variables.
  - Need to use engineering judgment or prior test results.
- Choice of experimental design
  - sample size, replicates, run order, randomization, software to use, design of data collection forms.

- Performing the experiment
  - vital to monitor the process carefully. Easy to underestimate logistical and planning aspects in a complex R and D environment.
- Statistical analysis of data
  - provides objective conclusions - use simple graphics whenever possible.
- Conclusion and recommendations
  - follow-up test runs and confirmation testing to validate the conclusions from the experiment.
- Do we need to add or drop factors, change ranges, levels, new responses, etc.. ???

Using Statistical Techniques in Experimentation - things to keep in mind

- Use non-statistical knowledge of the problem
  - physical laws, background knowledge
- Keep the design and analysis as simple as possible
  - Don't use complex, sophisticated statistical techniques
  - If design is good, analysis is relatively straightforward
  - If design is bad - even the most complex and elegant statistics cannot save the situation
- Recognize the difference between practical and statistical significance
  - statistical significance $\neq$ practically significance

- Experiments are usually iterative
  - unwise to design a comprehensive experiment at the start of the study
  - may need modification of factor levels, factors, responses, etc.. - too early to know whether experiment would work
  - use a sequential or iterative approach
  - should not invest more than 25% of resources in the initial design.
  - Use initial design as learning experiences to accomplish the final objectives of the experiment.

DOE (II)

Factorial vs OFAT

Factorial v.s. OFAT

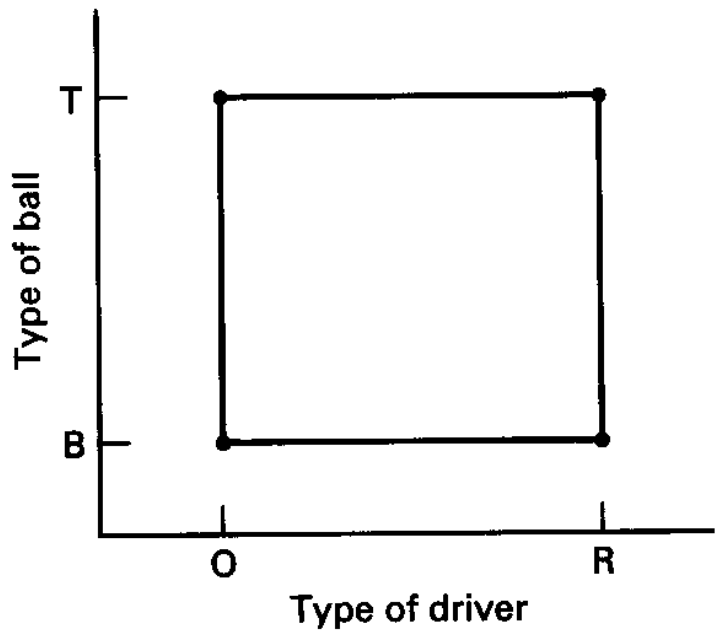
- Factorial design - experimental trials or runs are performed at all possible combinations of factor levels in contrast to OFAT experiments.
- Factorial and fractional factorial experiments are among the most useful multi-factor experiments for engineering and scientific investigations.

- The ability to gain competitive advantage requires extreme care in the design and conduct of experiments. Special attention must be paid to joint effects and estimates of variability that are provided by factorial experiments.

- Full and fractional experiments can be conducted using a variety of statistical designs. The design selected can be chosen according to specific requirements and restrictions of the investigation.

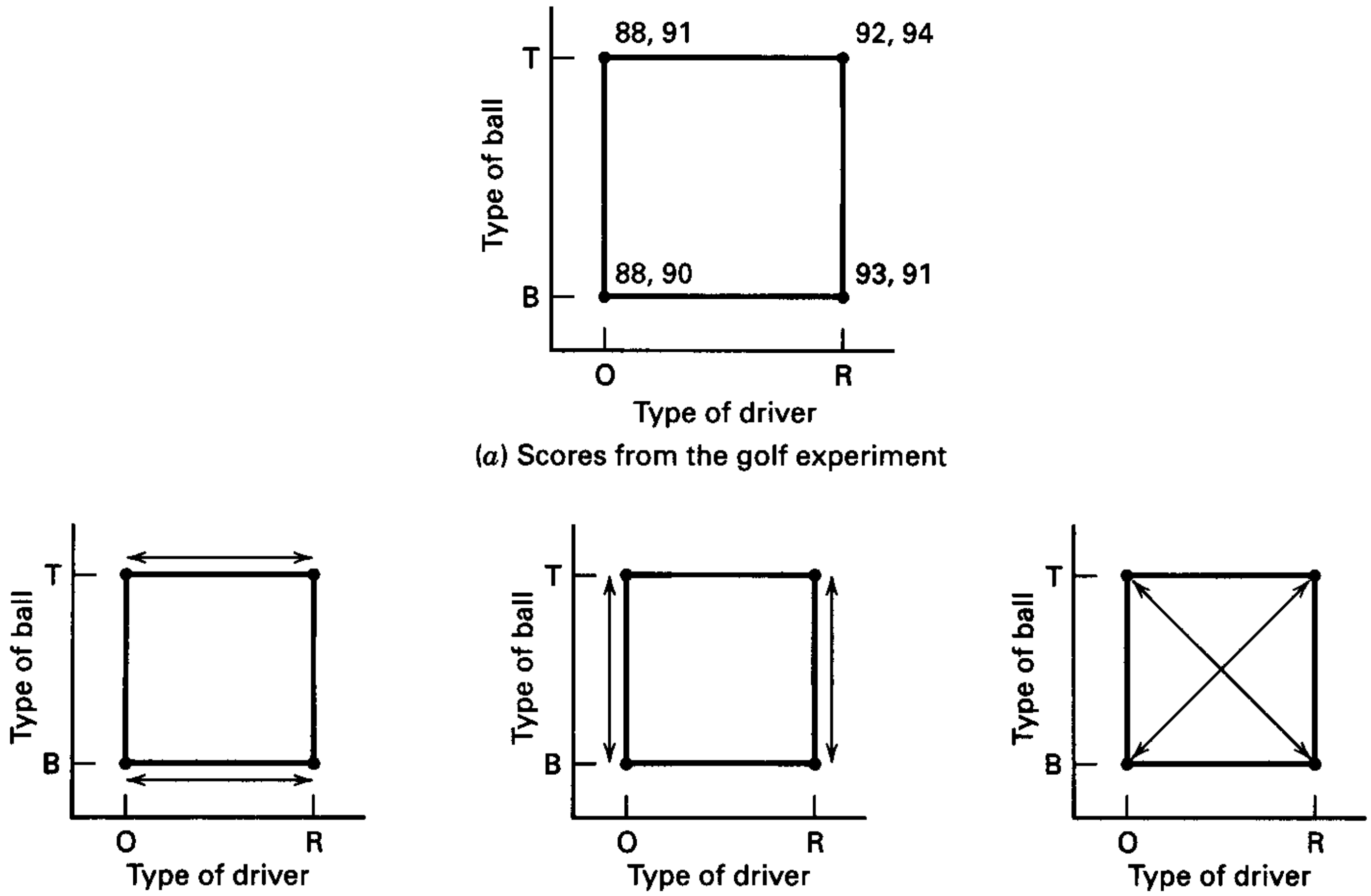
Factorial Designs

- In a factorial experiment, **all possible combinations** of factor levels are tested
- The golf experiment:
  - Type of driver (over or regular)
  - Type of ball (balata or 3-piece)
  - Walking vs. riding a cart
  - Type of beverage (Beer vs water)
  - Time of round (am or pm)
  - Weather
  - Type of golf spike
  - Etc, etc, etc…



A two-factor factorial experiment involving type of driver and type of ball.

Factorial Design



(*a*) Scores from the golf experiment

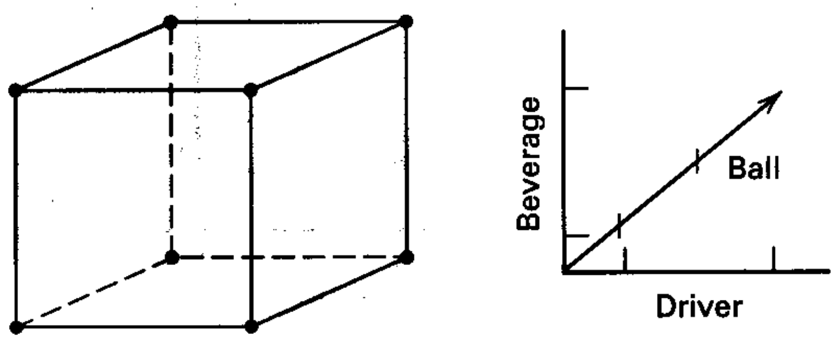
(*b*) Comparison of scores leading to the driver effect

(*c*) Comparison of scores leading to the ball effect

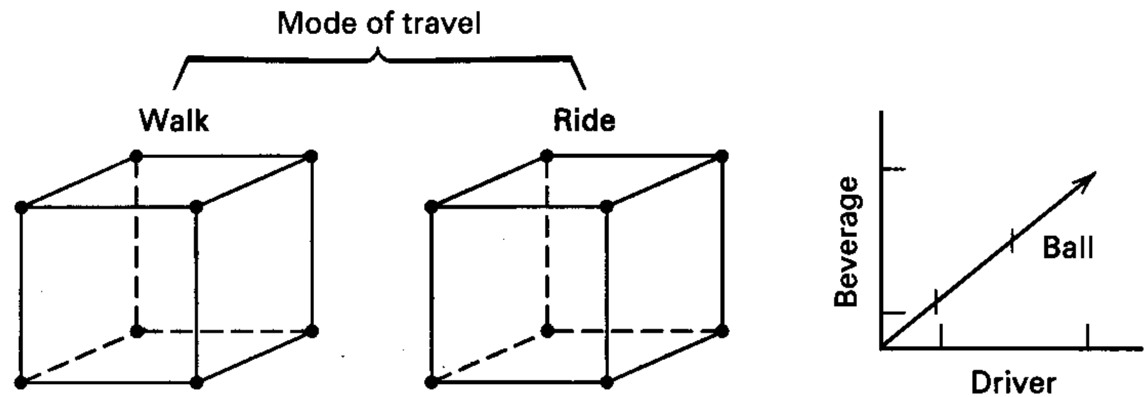
(*d*) Comparison of scores leading to the ball-driver interaction effect

Scores from the golf experiment

Factorial Designs with Several Factors



A three-factor factorial experiment involving type of driver, type of ball, and type of beverage.



A four-factor factorial experiment involving type of driver, type of ball, type of beverage, and mode of travel.

Erroneous Impressions About Factorial Experiments

- Wasteful and do not compensate the extra effort with additional useful information - this folklore presumes that one knows (not assumes) that factors independently influence the responses (i.e. there are no factor interactions) and that each factor has a linear effect on the response - almost any reasonable type of experimentation will identify optimum levels of the factors
- Information on the factor effects becomes available only after the entire experiment is completed. Takes too long. Actually, factorial experiments can be blocked and conducted sequentially so that data from each block can be analyzed as they are obtained.

One-factor-at-a-time experiments (OFAT)

- OFAT is a prevalent, but potentially disastrous type of experimentation commonly used by many engineers and scientists in both industry and academia.
- Tests are conducted by systematically changing the levels of one factor while holding the levels of all other factors fixed. The "optimal" level of the first factor is then selected.
- Subsequently, each factor in turn is varied and its "optimal" level selected while the other factors are held fixed.

One-factor-at-a-time experiments (OFAT)

- OFAT experiments are regarded as easier to implement, more easily understood, and more economical than factorial experiments. Better than trial and error.
- OFAT experiments are believed to provide the optimum combinations of the factor levels.
- Unfortunately, each of these presumptions can generally be shown to be false except under very special circumstances.
- The key reasons why OFAT should not be conducted except under very special circumstances are:
  - ***Do not provide adequate information on interactions***
  - ***Do not provide efficient estimates of the effects***

Factorial vs OFAT (2-levels only)

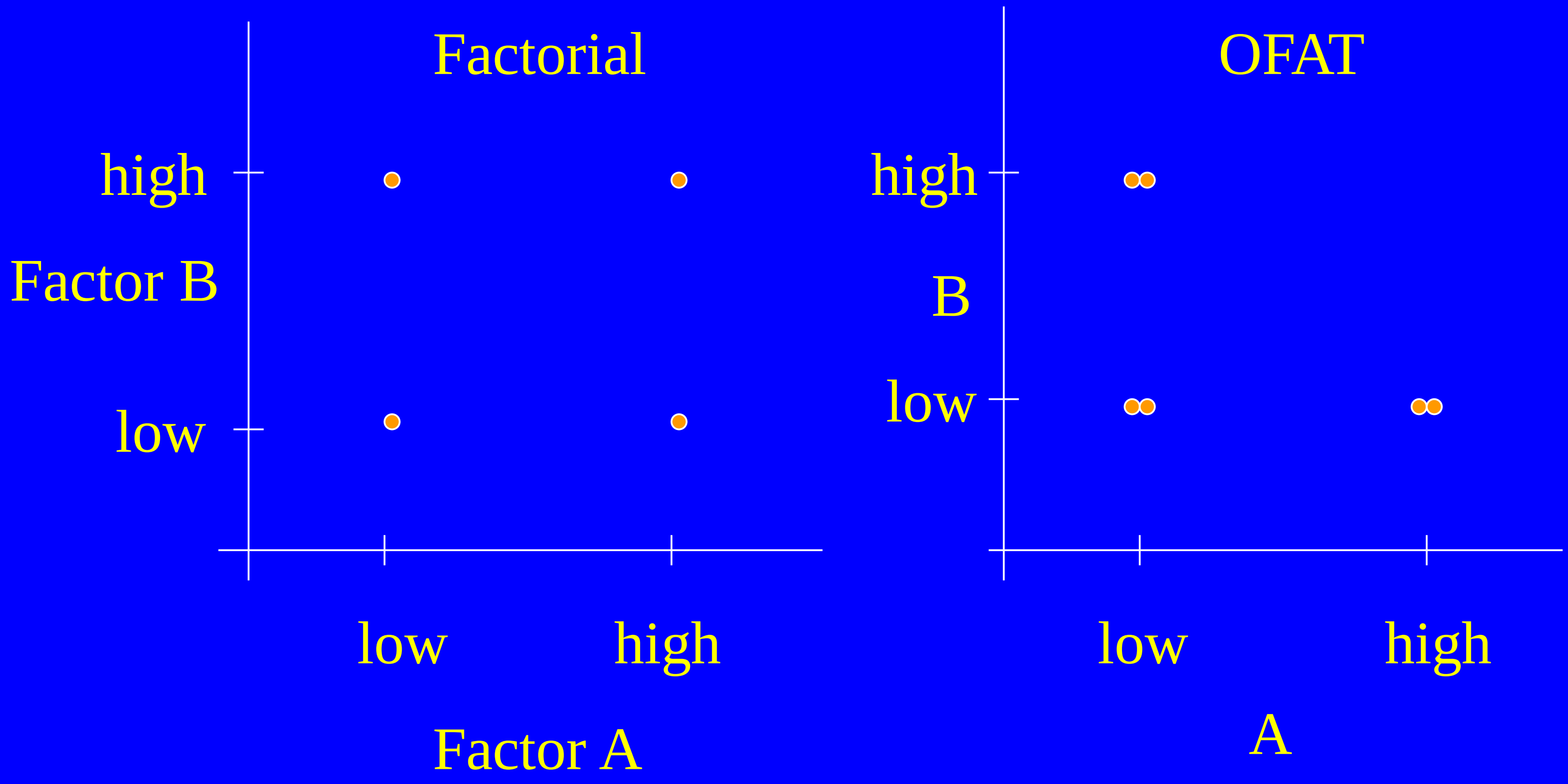
Factorial

- 2 factors: 4 runs
  - 3 effects
- 3 factors: 8 runs
  - 7 effects
- 5 factors: 32 or 16 runs
  - 31 or 15 effects
- 7 factors: 128 or 64 runs
  - 127 or 63 effects

OFAT

- 2 factors: 6 runs
  - 2 effects
- 3 factors: 16 runs
  - 3 effects
- 5 factors: 96 runs
  - 5 effects
- 7 factors: 512 runs
  - 7 effects

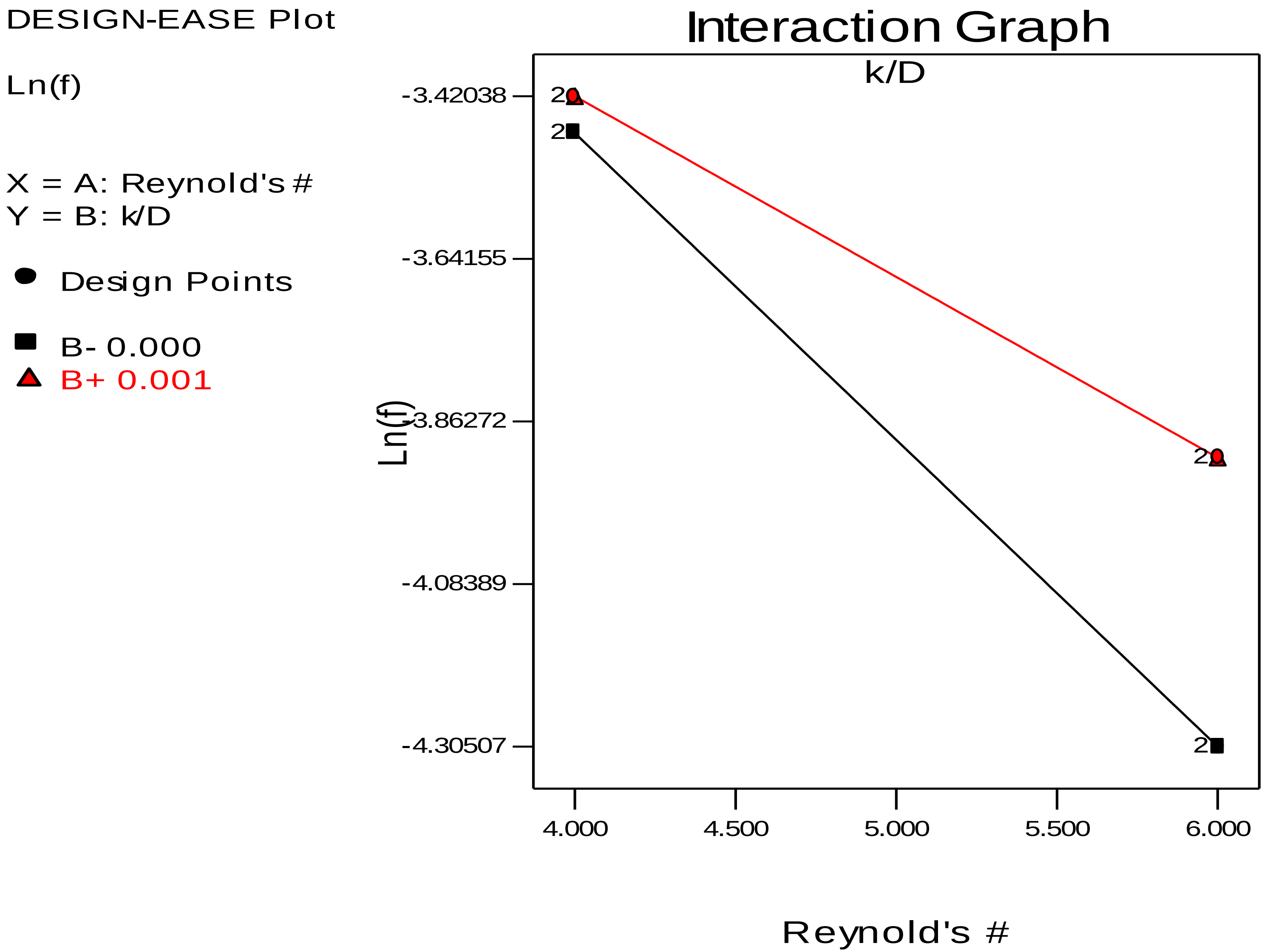
Example: Factorial vs OFAT

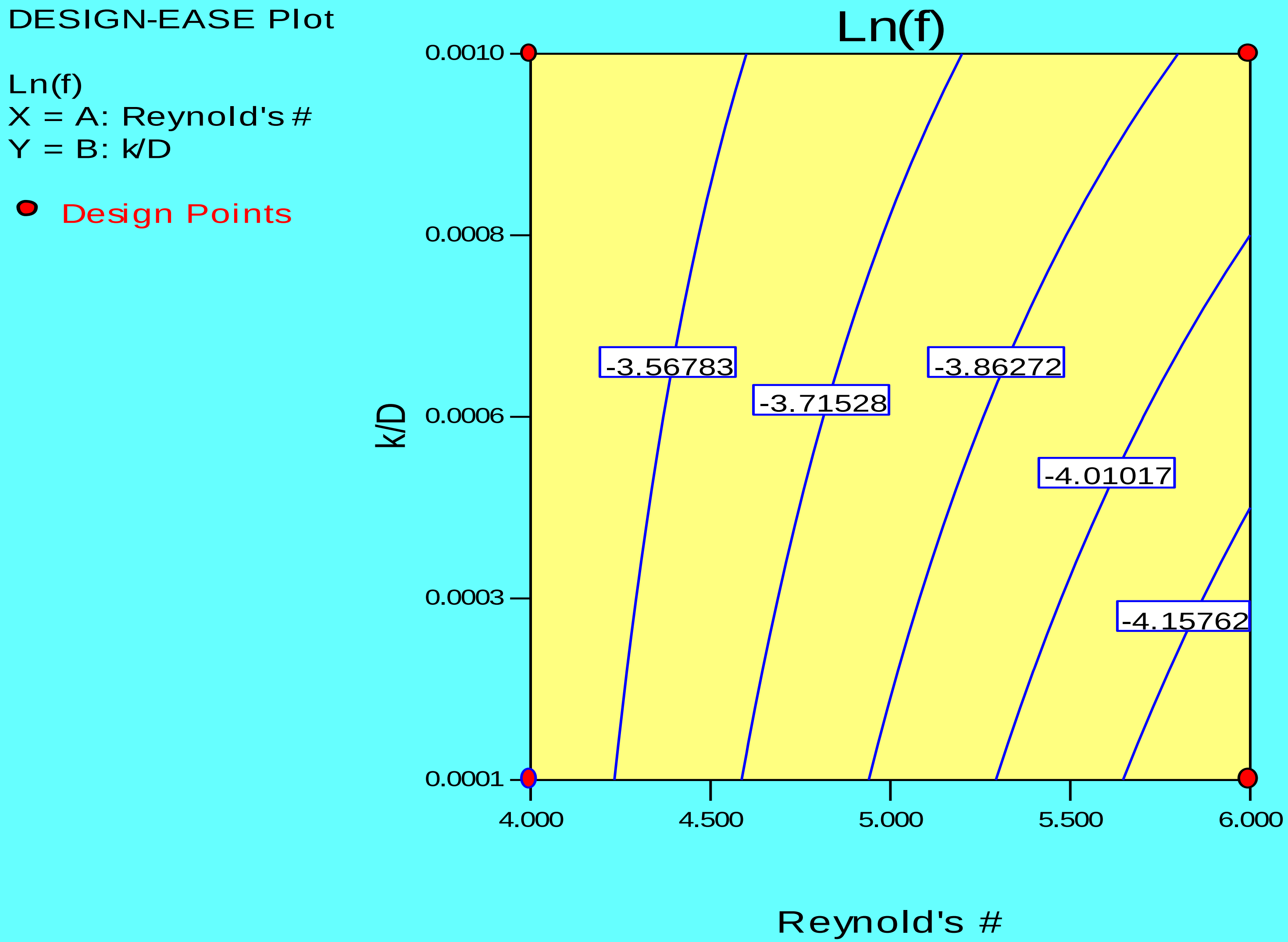


E.g. Factor A: Reynold's number, Factor B: k/D

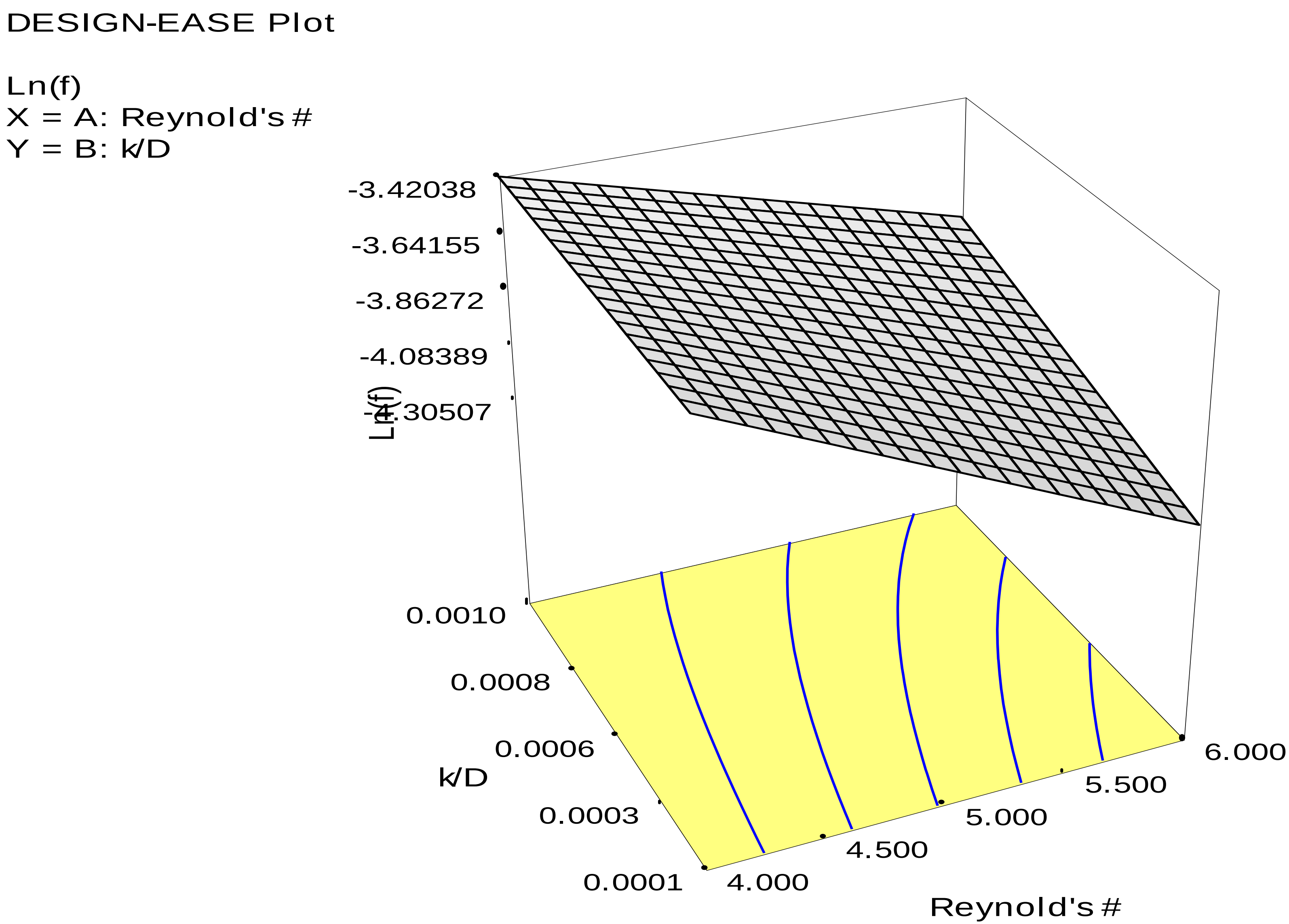
Example: Effect of Re and k/D on friction factor f

- Consider a 2-level factorial design (2^2)
- Reynold's number = Factor A; k/D = Factor B
- Levels for A: 10^4 (low) 10^6 (high)
- Levels for B: 0.0001 (low) 0.001 (high)
- Responses: (1) = 0.0311, a = 0.0135, b = 0.0327, ab = 0.0200
- Effect (A) = -0.66, Effect (B) = 0.22, Effect (AB) = 0.17
- % contribution: A = 84.85%, B = 9.48%, AB = 5.67%
- The presence of interactions implies that one cannot satisfactorily describe the effects of each factor using main effects.


DESIGN-EASE Plot
Ln(f)
X = A: Reynold's #
Y = B: k/D
Design Points
B- 0.000
B+ 0.001
Interaction Graph
k/D
Ln(f)
-3.42038
-3.64155
-3.86272
-4.08389
-4.30507
4.000
4.500
5.000
5.500
6.000
Reynold's #

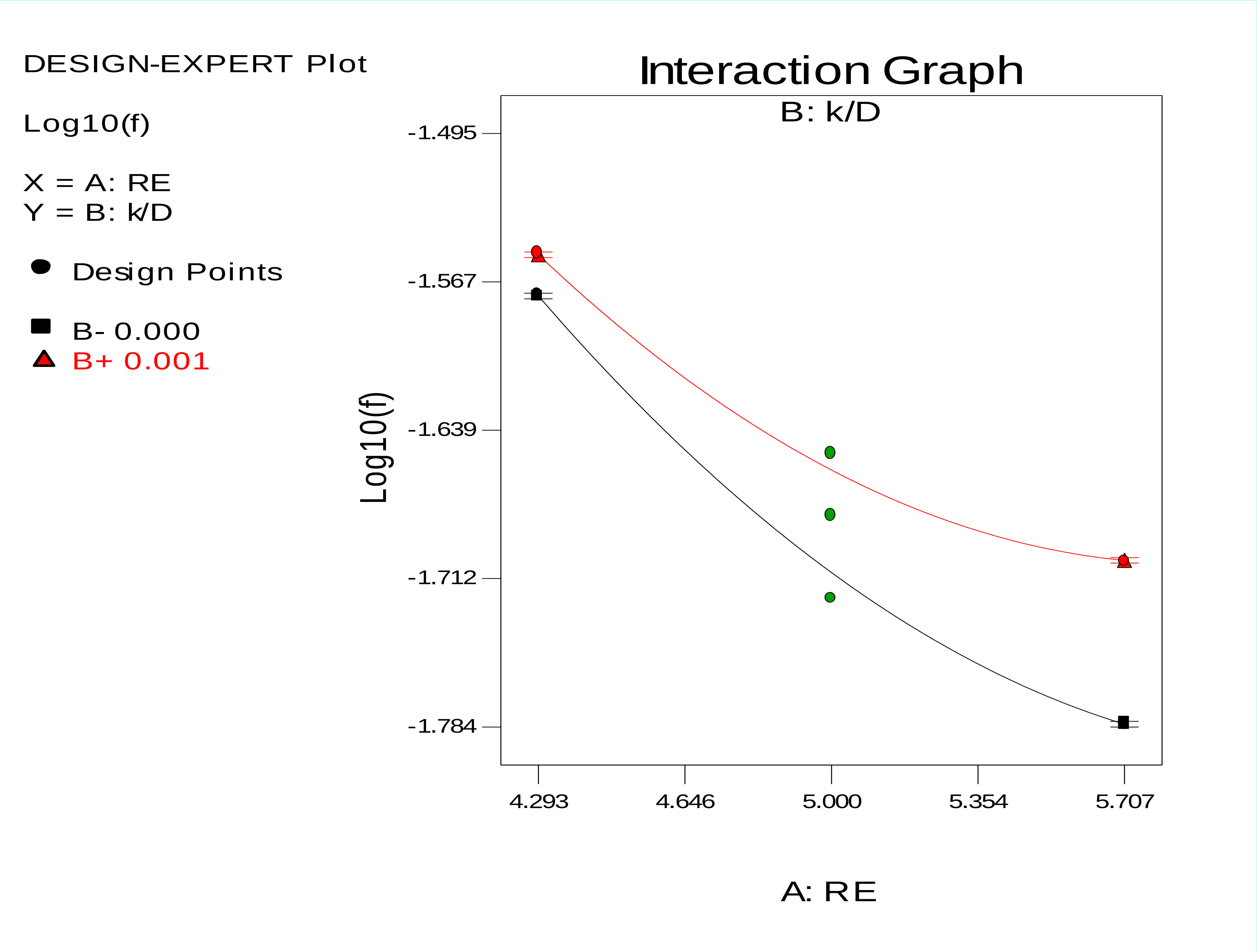


DESIGN-EASE Plot
Ln(f)
X = A: Reynold's #
Y = B: k/D
Design Points
Ln(f)
0.0010
0.0008
0.0006
0.0003
0.0001
k/D
-3.56783
-3.71528
-3.86272
-4.01017
-4.15762
4.000
4.500
5.000
5.500
6.000
Reynold's #


DESIGN-EASE Plot
Ln(f)
X = A: Reynold's #
Y = B: k/D
-3.42038
-3.64155
-3.86272
-4.08389
-4.30507
Ln(f)
0.0010
0.0008
0.0006
0.0003
0.0001
k/D
4.000
4.500
5.000
5.500
6.000
Reynold's #

With the addition of a few more points

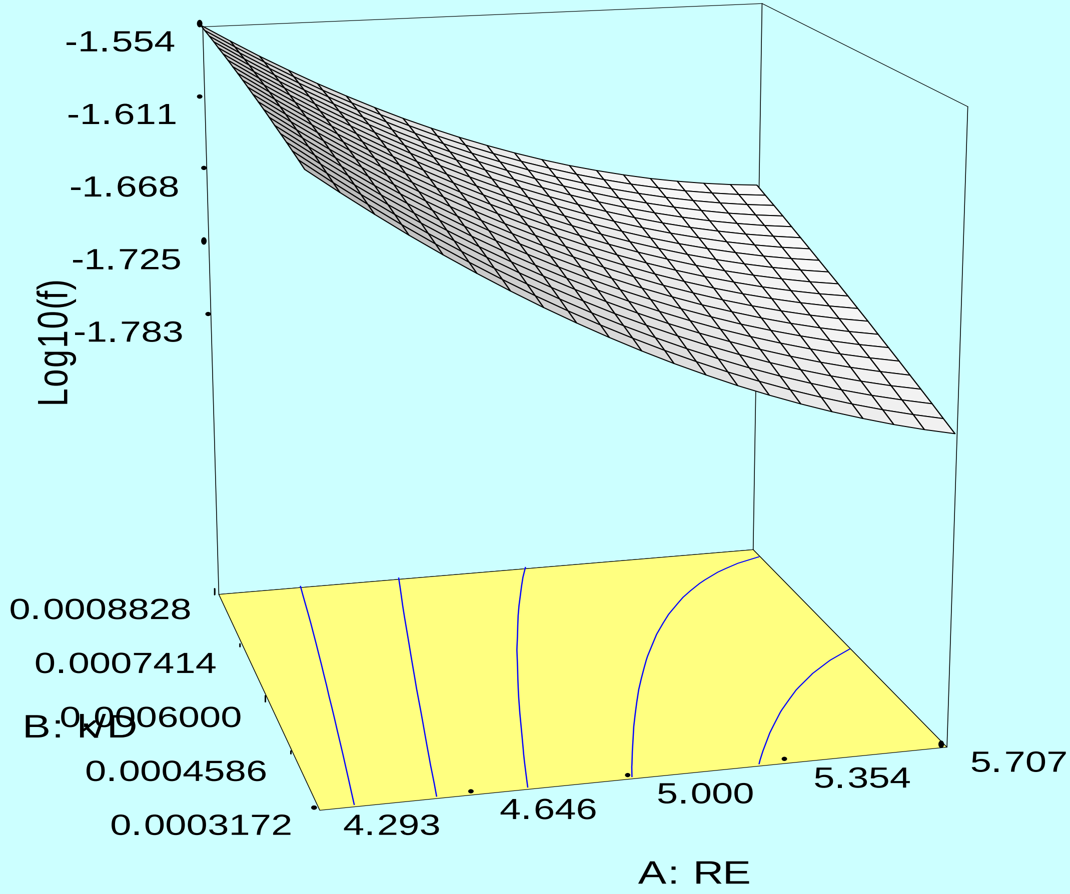
- Augmenting the basic 2^2 design with a center point and 5 axial points we get a central composite design (CCD) and a 2nd order model can be fit.
- The nonlinear nature of the relationship between Re, k/D and the friction factor f can be seen.
- If Nikuradse (1933) had used a factorial design in his pipe friction experiments, he would need far less experimental runs!!
- If the number of factors can be reduced by dimensional analysis, the problem can be made simpler for experimentation.

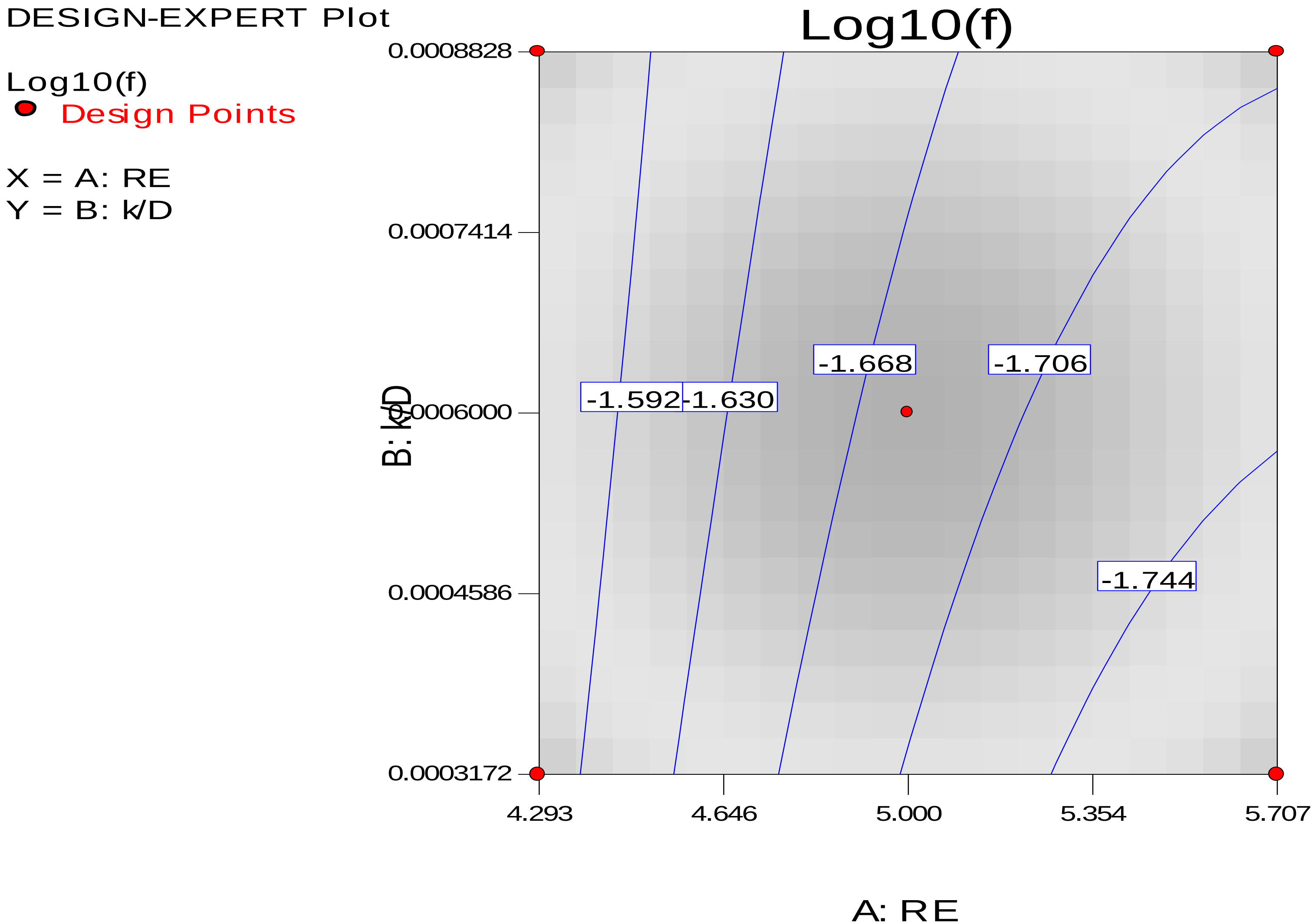


DESIGN-EXPERT Plot
Log10(f)
X = A: RE
Y = B: k/D
Design Points
B- 0.000
B+ 0.001
Interaction Graph
B: k/D
-1.495
-1.567
-1.639
-1.712
-1.784
Log10(f)
4.293
4.646
5.000
5.354
5.707
A: RE

DESIGN-EXPERT Plot

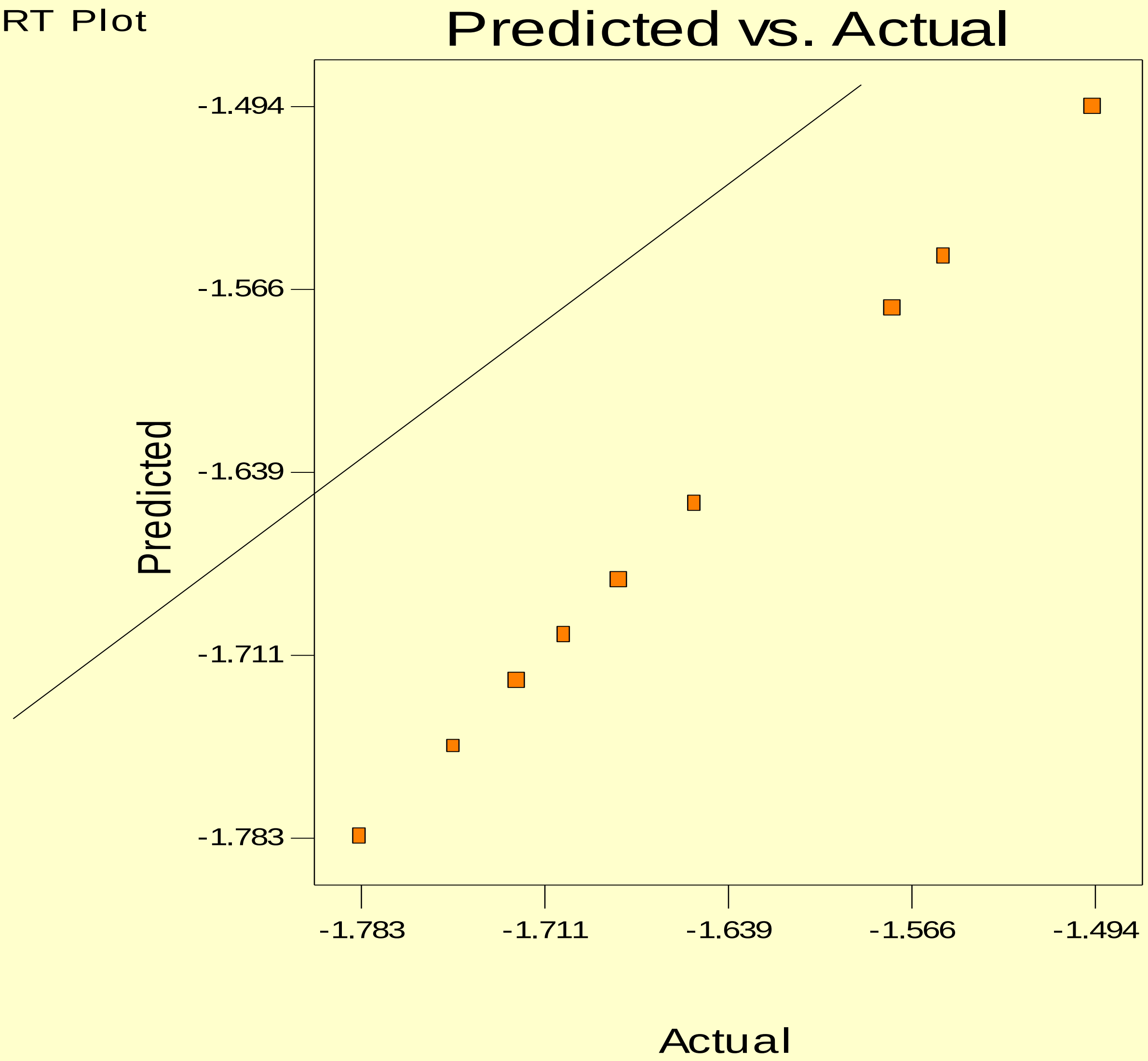
Log10(f)
X = A: RE
Y = B: k/D





DESIGN-EXPERT Plot
Log10(f)
Design Points
X = A: RE
Y = B: k/D
Log10(f)
0.0008828
0.0007414
0.0006000
0.0004586
0.0003172
B: k/D
-1.592
-1.630
-1.668
-1.706
-1.744
4.293
4.646
5.000
5.354
5.707
A: RE

DESIGN-EXPERT Plot
Log10(f)



DOE (III)

Basic Concepts

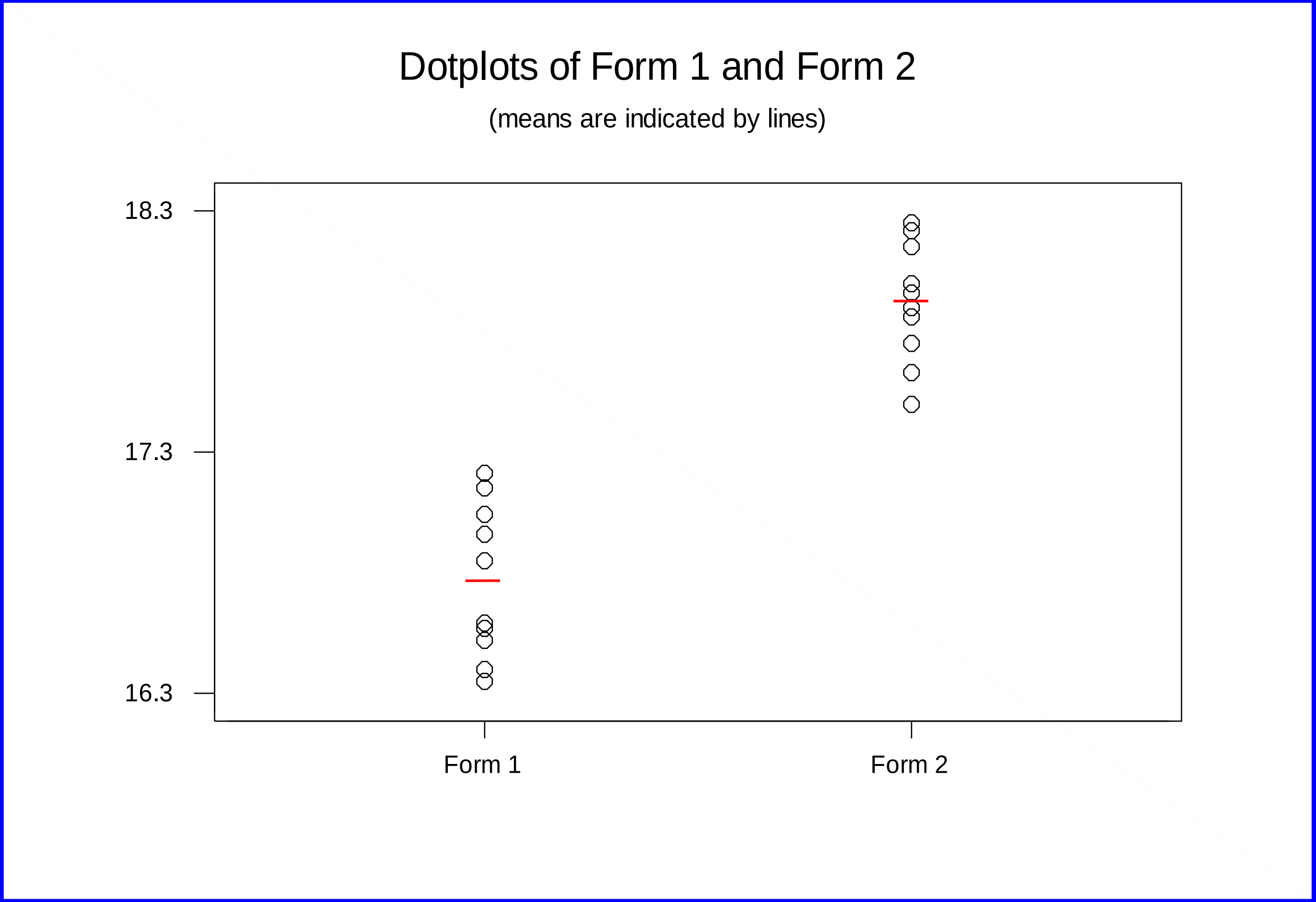
Design of Engineering Experiments Basic Statistical Concepts

- Simple **comparative** experiments
  - The hypothesis testing framework
  - The two-sample *t*-test
  - Checking assumptions, validity
- Comparing more than two factor levels…**the analysis of variance**
  - ANOVA decomposition of total variability
  - Statistical testing & analysis
  - Checking assumptions, model validity
  - Post-ANOVA testing of means

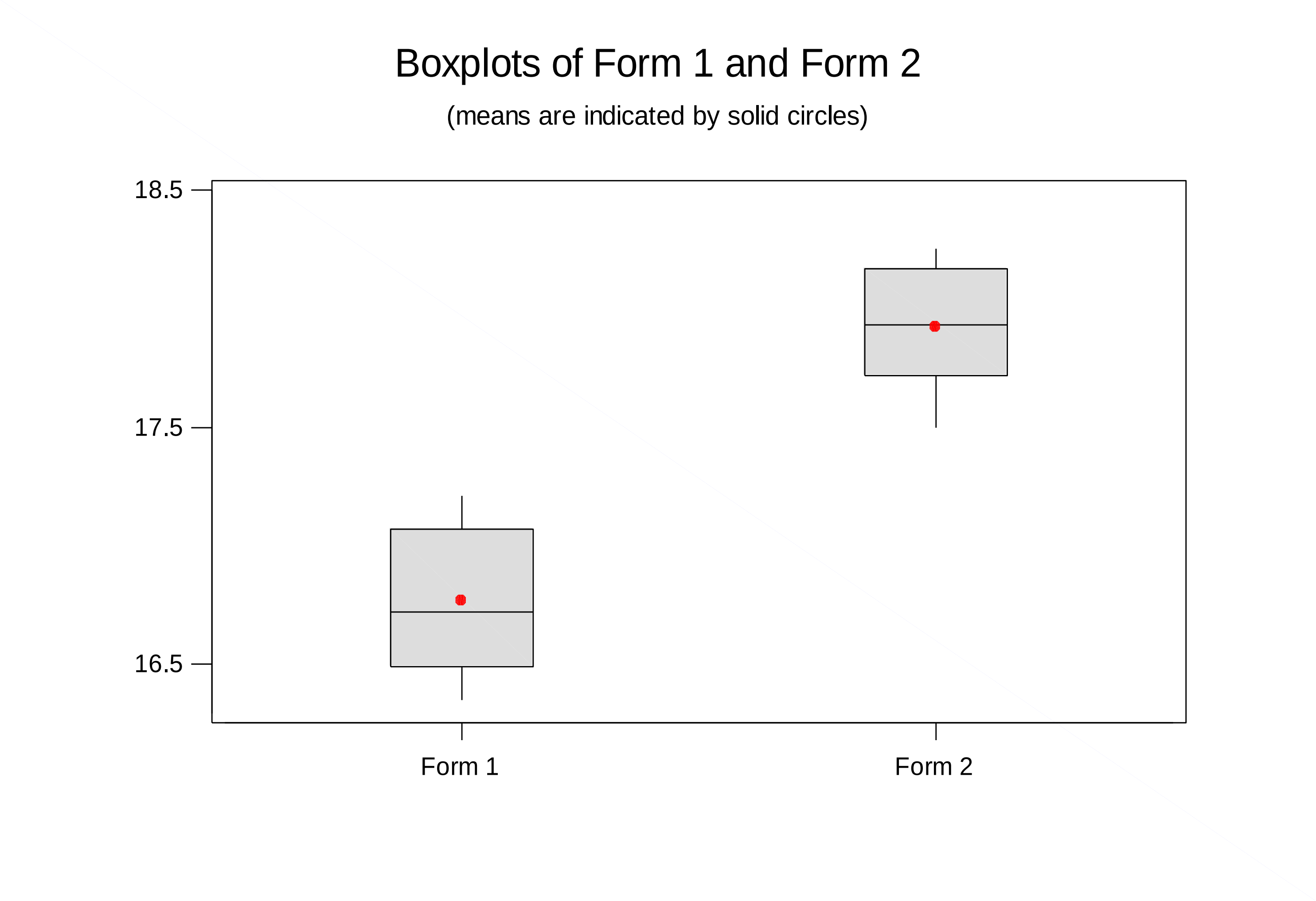
Portland Cement Formulation

Observation (sample), j	Modified Mortar (Formulation 1) y_{1j}	Unmodified Mortar (Formulation 2) y_{2j}
1	16.85	17.50
2	16.40	17.63
3	17.21	18.25
4	16.35	18.00
5	16.52	17.86
6	17.04	17.75
7	16.96	18.22
8	17.15	17.90
9	16.59	17.96
10	16.57	18.15

Graphical View of the Data
Dot Diagram



Box Plots

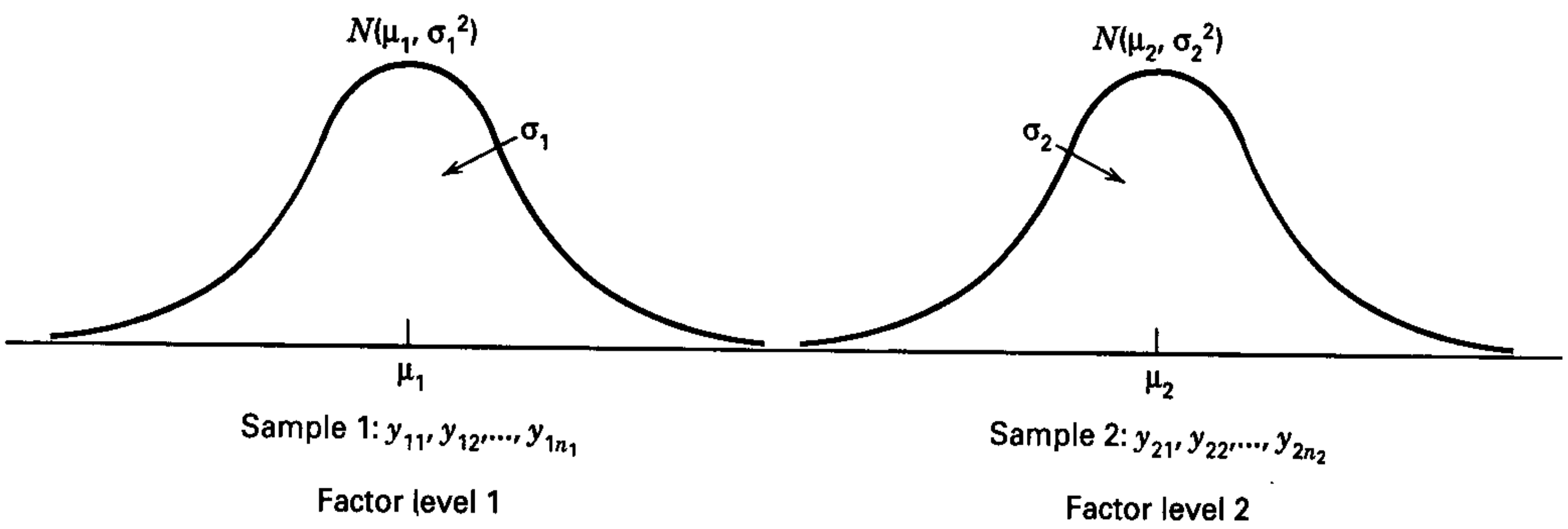


Boxplots of Form 1 and Form 2
(means are indicated by solid circles)
18.5
17.5
16.5
Form 1
Form 2

The Hypothesis Testing Framework

- **Statistical hypothesis testing** is a useful framework for many experimental situations
- Origins of the methodology date from the early 1900s
- We will use a procedure known as the **two-sample *t*-test**

The Hypothesis Testing Framework



The sampling situation for the two-sample *t*-test.

- Sampling from a **normal** distribution
- Statistical hypotheses:

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

Estimation of Parameters

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \text{ estimates the population mean } \mu$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 \text{ estimates the variance } \sigma^2$$

Summary Statistics

Formulation 1

"New recipe"

$$\bar{y}_1 = 16.76$$
$$S_1^2 = 0.100$$
$$S_1 = 0.316$$
$$n_1 = 10$$

Formulation 2

"Original recipe"

$$\bar{y}_2 = 17.92$$
$$S_2^2 = 0.061$$
$$S_2 = 0.247$$
$$n_2 = 10$$

How the Two-Sample *t*-Test Works:

Use the sample means to draw inferences about the population means

$$\bar{y}_1 - \bar{y}_2 = 16.76 - 17.92 = -1.16$$

$$\frac{\text{Difference in sample means}}{\text{Standard deviation of the difference in sample means}}$$

$$\sigma_{\bar{y}}^2 = \frac{\sigma^2}{n}$$

This suggests a statistic:

$$Z_0 = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

How the Two-Sample *t*-Test Works:

Use S_1^2 and S_2^2 to estimate σ_1^2 and σ_2^2

The previous ratio becomes $\dfrac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$

However, we have the case where $\sigma_1^2 = \sigma_2^2 = \sigma^2$

Pool the individual sample variances:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

How the Two-Sample *t*-Test Works:

The test statistic is

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- Values of t_0 that are near zero are consistent with the null hypothesis
- Values of t_0 that are very different from zero are consistent with the alternative hypothesis
- t_0 is a "distance" measure-how far apart the averages are expressed in standard deviation units
- Notice the interpretation of t_0 as a **signal-to-noise** ratio

The Two-Sample (Pooled) *t*-Test

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{9(0.100) + 9(0.061)}{10 + 10 - 2} = 0.081$$

$$S_p = 0.284$$

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{16.76 - 17.92}{0.284\sqrt{\frac{1}{10} + \frac{1}{10}}} = -9.13$$

The two sample means are about 9 standard deviations apart

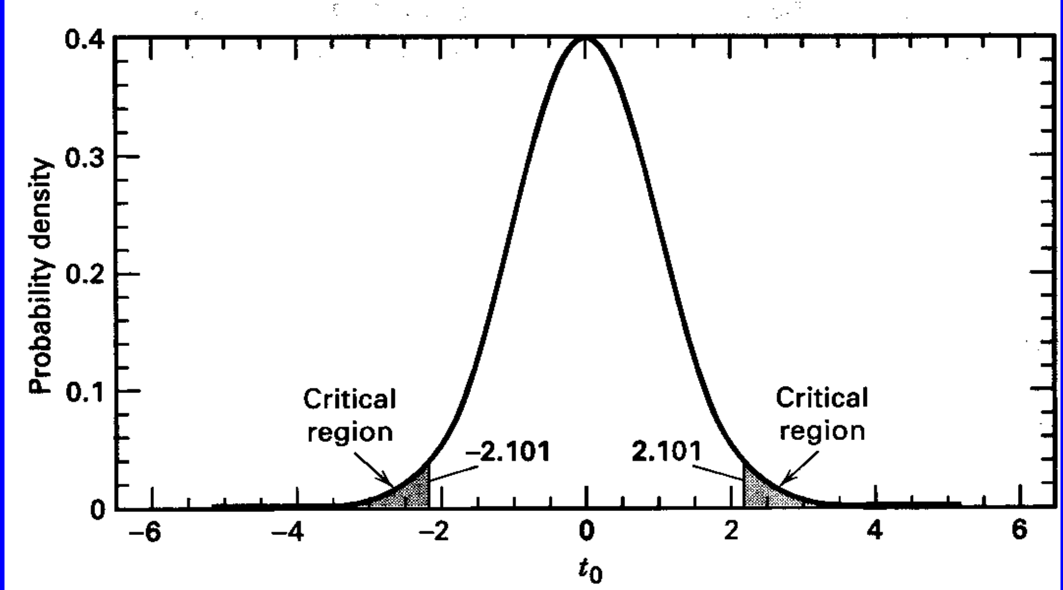
Is this a "large" difference?

The Two-Sample (Pooled) *t*-Test

- So far, we haven’t really done any “statistics”
- We need an **objective** basis for deciding how large the test statistic t_0 really is
- In 1908, W. S. Gosset derived the **reference distribution** for t_0 … called the *t* distribution
- Tables of the *t* distribution - any stats text.
- The t-distribution looks almost exactly like the normal distribution except that it is shorter and fatter when the degrees of freedom is less than about 100.
- Beyond 100, the t is practically the same as the normal.

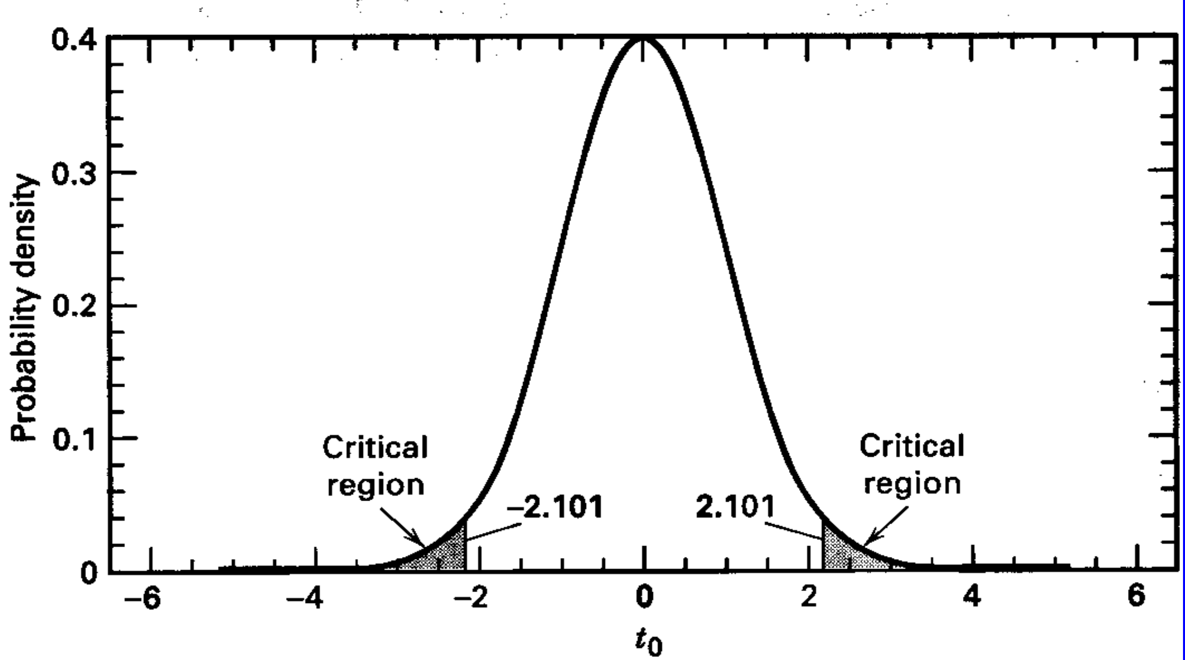
The Two-Sample (Pooled) *t*-Test

- A value of t_0 between –2.101 and 2.101 is consistent with equality of means
- It is possible for the means to be equal and t_0 to exceed either 2.101 or –2.101, but it would be a "**rare event**" … leads to the conclusion that the means are different
- Could also use the ***P*-value** approach



The t distribution with 18 degrees of freedom with the critical region $\pm t_{0.025,18} = \pm 2.101$.

The Two-Sample (Pooled) *t*-Test



The t distribution with 18 degrees of freedom with the critical region $\pm t_{0.025,18} = \pm 2.101$.

- The ***P*-value is the risk of wrongly rejecting** the null hypothesis of equal means (it measures rareness of the event)
- The *P*-value in our problem is $P = 0.00000038$

Minitab Two-Sample *t*-Test Results

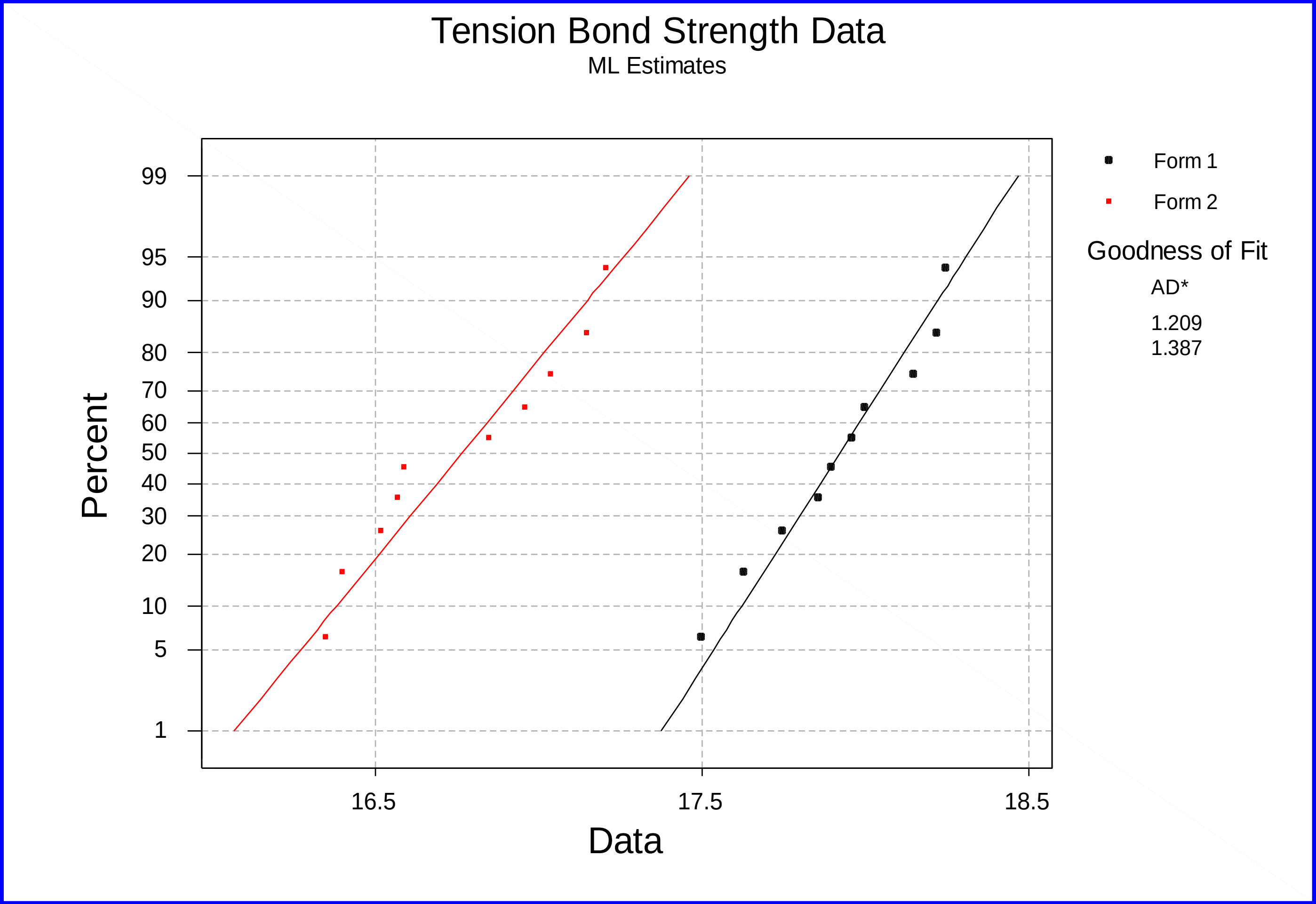
Two-Sample T-Test and CI: Form 1, Form 2

```
Two-sample T for Form 1 vs Form 2


          N      Mean      StDev   SE Mean
Form 1   10    16.764      0.316      0.10
Form 2   10    17.922      0.248     0.078


Difference = mu Form 1 - mu Form 2
Estimate for difference:  -1.158
95% CI for difference: (-1.425, -0.891)
T-Test of difference = 0 (vs not =): T-Value = -9.11
P-Value = 0.000  DF = 18
Both use Pooled StDev = 0.284
```

Checking Assumptions – The Normal Probability Plot



Importance of the *t*-Test

- Provides an **objective** framework for simple comparative experiments
- Could be used to test all relevant hypotheses in a two-level factorial design, because all of these hypotheses involve the mean response at one "side" of the cube versus the mean response at the opposite "side" of the cube

What If There Are More Than Two Factor Levels?

- The *t*-test does not directly apply
- There are lots of practical situations where there are either more than two levels of interest, or there are several factors of simultaneous interest
- The **analysis of variance** (ANOVA) is the appropriate analysis "engine" for these types of experiments
- The ANOVA was developed by Fisher in the early 1920s, and initially applied to agricultural experiments
- Used extensively today for industrial experiments

An Example

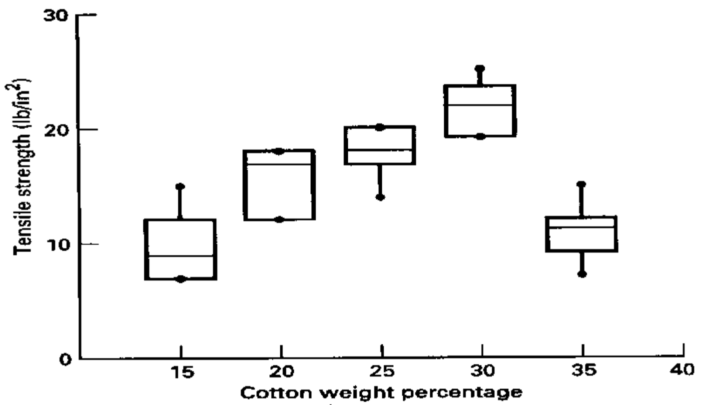
- Consider an investigation into the formulation of a new “synthetic” fiber that will be used to make ropes
- The response variable is tensile strength
- The experimenter wants to determine the “best” level of cotton (in wt %) to combine with the synthetics
- Cotton content can vary between 10 – 40 wt %; some non-linearity in the response is anticipated
- The experimenter chooses 5 **levels** of cotton “content”;   15, 20, 25, 30, and 35 wt %
- The experiment is **replicated** 5 times – runs made in random order

An Example

Data (in lb/in^2) from the Tensile Strength Experiment

Cotton Weight Percentage	Observations 1	2	3	4	5	Total	Average
15	7	7	15	11	9	49	9.8
20	12	17	12	18	18	77	15.4
25	14	18	18	19	19	88	17.6
30	19	25	22	19	23	108	21.6
35	7	10	11	15	11	54	10.8
						376	15.04

- Does **changing** the cotton weight percent change the mean tensile strength?
- Is there an **optimum** level for cotton content?



Box plots of tensile strength versus cotton weight

The Analysis of Variance

Table 3-2 Typical Data for a Single-Factor Experiment

Treatment (level)	Observations				Totals	Averages
1	y_{11}	y_{12}	$\cdots$	y_{1n}	$y_{1.}$	$\bar{y}_{1.}$
2	y_{21}	y_{22}	$\cdots$	y_{2n}	$y_{2.}$	$\bar{y}_{2.}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
a	y_{a1}	y_{a2}	$\cdots$	y_{an}	$y_{a.}$	$\bar{y}_{a.}$
					$y_{..}$	$\bar{y}_{..}$

- In general, there will be *a* **levels** of the factor, or *a* **treatments, and** *n* **replicates** of the experiment, run in **random order**…a completely randomized design **(CRD)**
- $N = an$ total runs
- We consider the **fixed effects** case only
- Objective is to test hypotheses about the equality of the *a* treatment means

The Analysis of Variance

- The name "analysis of variance" stems from a **partitioning** of the total variability in the response variable into components that are consistent with a **model** for the experiment
- The basic single-factor ANOVA model is

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}, \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, n \end{cases}$$

$\mu =$ an overall mean, $\tau_i = ith$ treatment effect,

$\varepsilon_{ij} =$ experimental error, $NID(0, \sigma^2)$

Models for the Data

There are several ways to write a model for the data:

$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$ is called the effects model

Let $\mu_i = \mu + \tau_i$, then

$y_{ij} = \mu_i + \varepsilon_{ij}$ is called the means model

Regression models can also be employed

The Analysis of Variance

- **Total variability** is measured by the total sum of squares:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2$$

- The basic ANOVA partitioning is:

$$\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^{a}\sum_{j=1}^{n}[(\bar{y}_{i.} - \bar{y}_{..}) + (y_{ij} - \bar{y}_{i.})]^2$$

$$= n\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{y}_{..})^2 + \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{i.})^2$$

$$SS_T = SS_{Treatments} + SS_E$$

The Analysis of Variance

$$SS_T = SS_{Treatments} + SS_E$$

- A large value of $SS_{Treatments}$ reflects large differences in treatment means
- A small value of $SS_{Treatments}$ likely indicates no differences in treatment means
- Formal statistical hypotheses are:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_a$$
$$H_1 : \text{ At least one mean is different}$$

The Analysis of Variance

- While sums of squares cannot be directly compared to test the hypothesis of equal means, **mean squares** can be compared.
- A mean square is a sum of squares divided by its degrees of freedom:

$$df_{Total} = df_{Treatments} + df_{Error}$$

$$an - 1 = a - 1 + a(n-1)$$

$$MS_{Treatments} = \frac{SS_{Treatments}}{a-1}, MS_E = \frac{SS_E}{a(n-1)}$$

- If the treatment means are equal, the treatment and error mean squares will be (theoretically) equal.
- If treatment means differ, the treatment mean square will be larger than the error mean square.

The Analysis of Variance is Summarized in a Table

The Analysis of Variance Table for the Single-Factor, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between treatments	$SS_{\text{Treatments}} = n \sum_{i=1}^{a} (\bar{y}_{i.} - \bar{y}_{..})^2$	$a - 1$	$MS_{\text{Treatments}}$	$F_0 = \frac{MS_{\text{Treatments}}}{MS_E}$
Error (within treatments)	$SS_E = SS_T - SS_{\text{Treatments}}$	$N - a$	MS_E	
Total	$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{..})^2$	$N - 1$		

- The **reference distribution** for F_0 is the $F_{a-1,\, a(n-1)}$ distribution
- **Reject** the null hypothesis (equal treatment means) if

$$F_0 > F_{\alpha, a-1, a(n-1)}$$

ANOVA Computer Output (Design-Expert)

Response: Strength

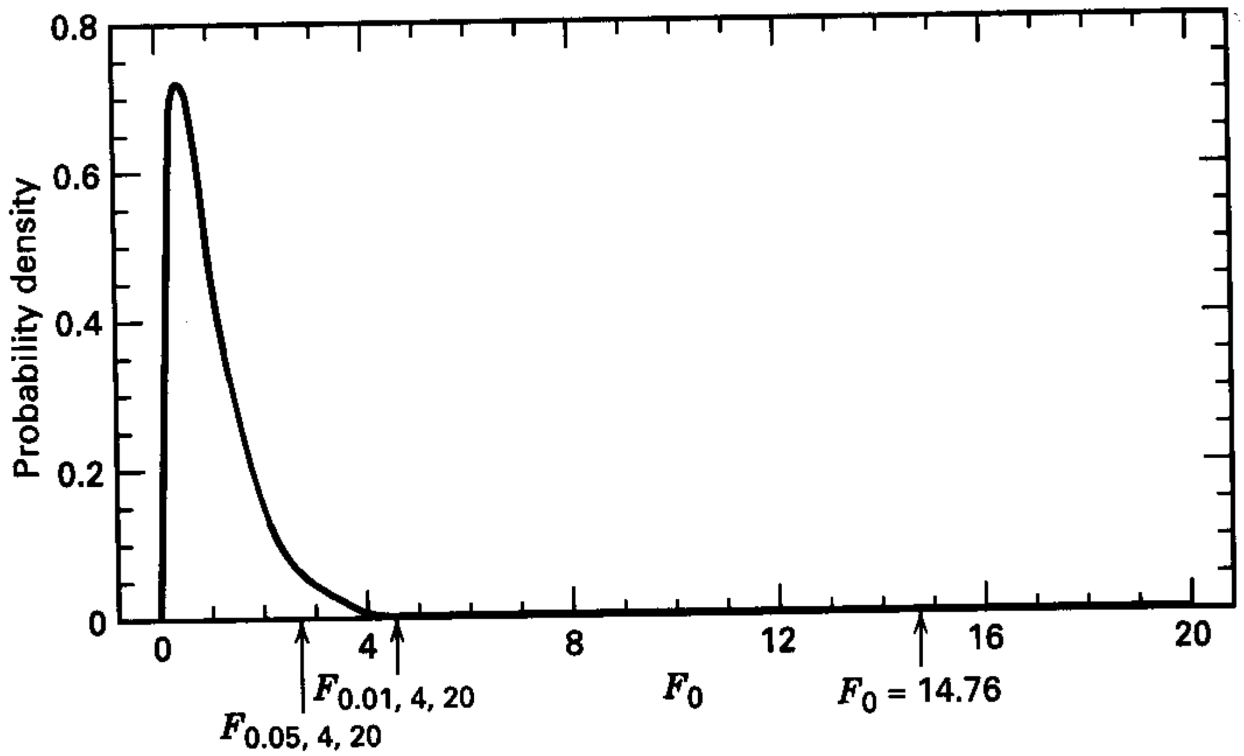
ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**
Model	475.76	4	118.94	14.76	< 0.0001
A	*475.76*	*4*	*118.94*	*14.76*	*< 0.0001*
Pure Error	161.20	20	8.06		
Cor Total	636.96	24			

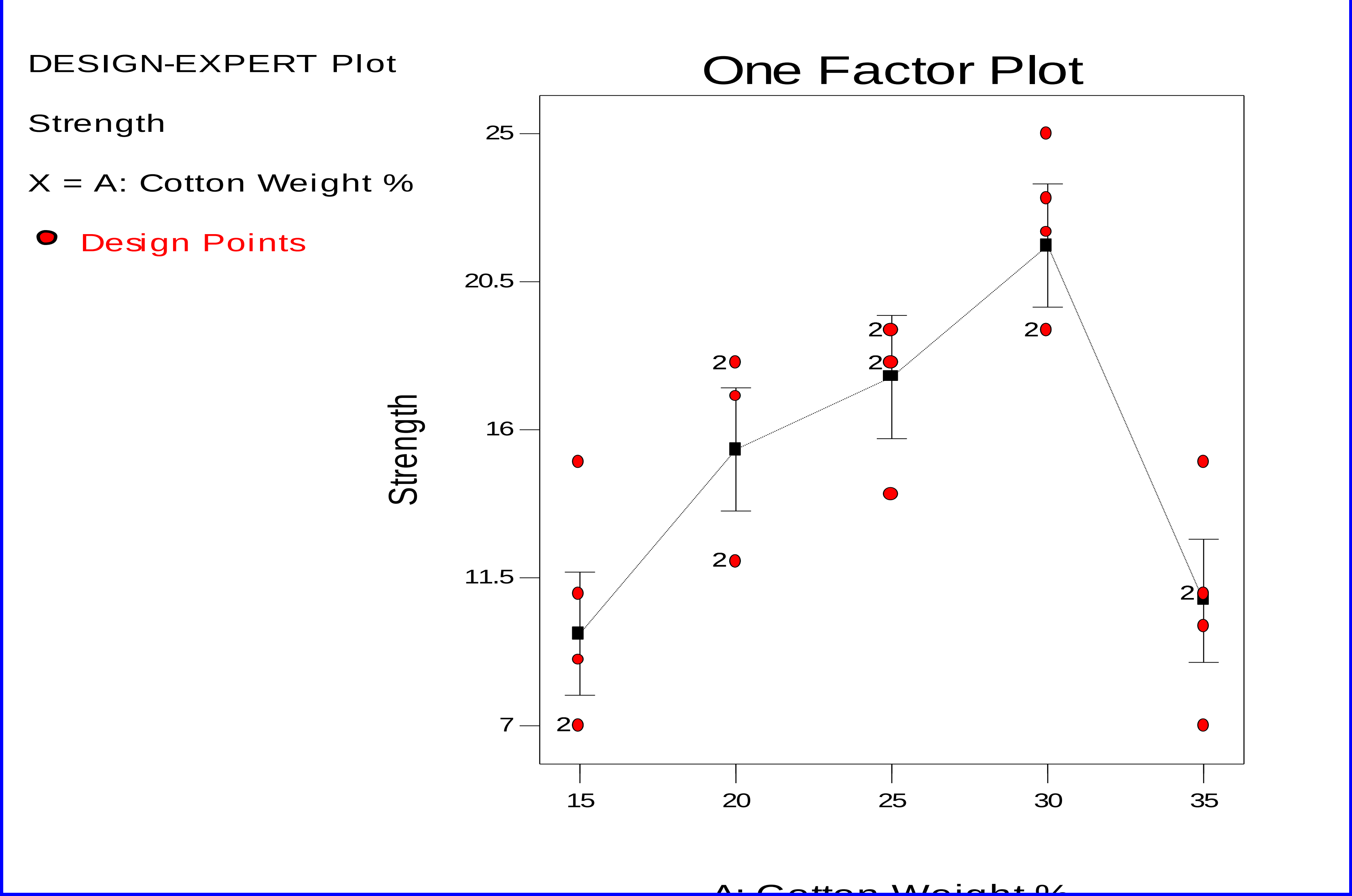
Std. Dev.	2.84	R-Squared	0.7469
Mean	15.04	Adj R-Squared	0.6963
C.V.	18.88	Pred R-Squared	0.6046
PRESS	251.88	Adeq Precision	9.294

The Reference Distribution:



The reference distribution ($F_{4,20}$) for the test statistic F_0 i

Graphical View of the Results



Model Adequacy Checking in the ANOVA

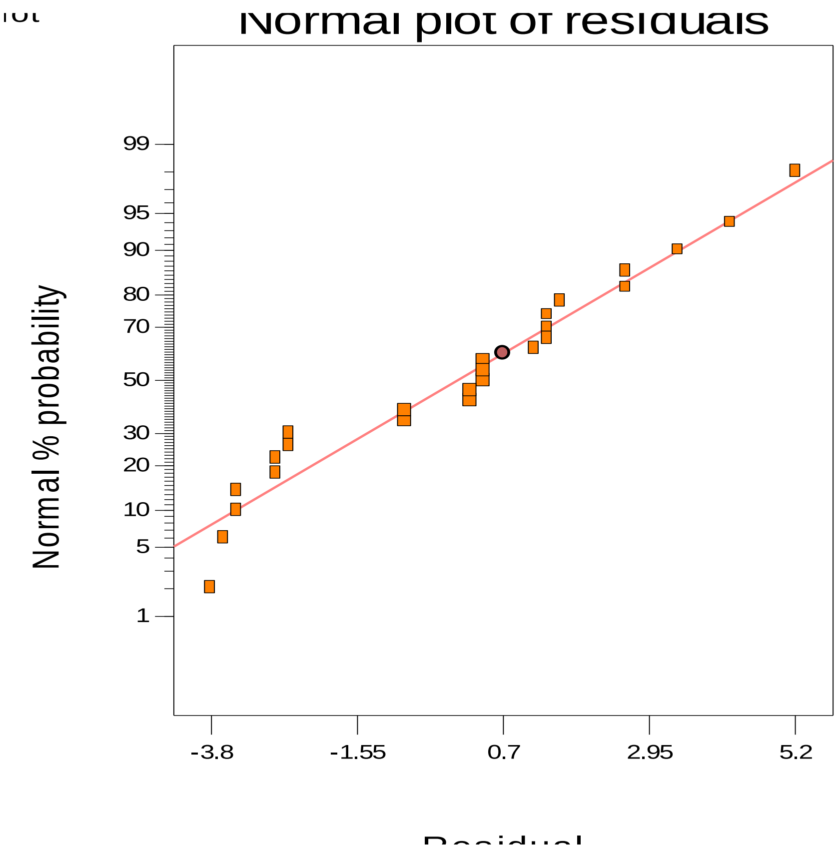
- **Checking assumptions** is important
- Normality
- Constant variance
- Independence
- Have we fit the right model?
- Later we will talk about what to do if some of these assumptions are **violated**

Model Adequacy Checking in the ANOVA

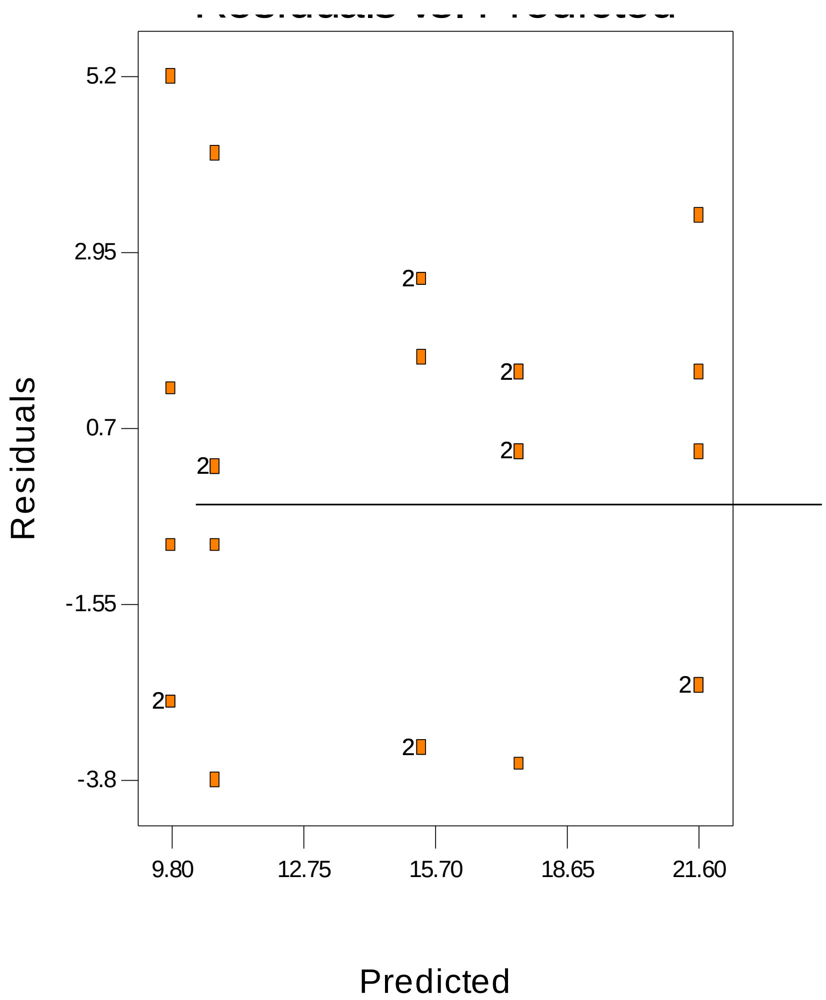
- Examination of **residuals**

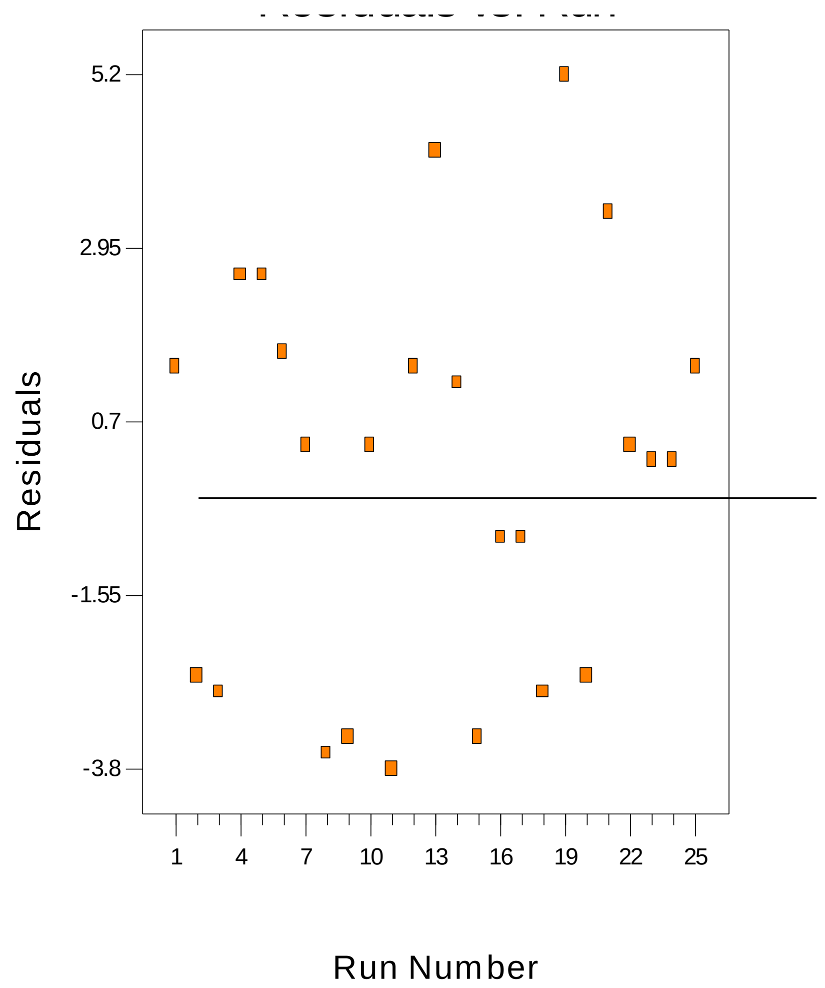
$$e_{ij} = y_{ij} - \hat{y}_{ij}$$
$$= y_{ij} - \bar{y}_{i.}$$

- Design-Expert generates the residuals
- **Residual plots** are very useful
- **Normal probability plot** of residuals



Other Important Residual Plots





Post-ANOVA Comparison of Means

- The analysis of variance tests the hypothesis of equal treatment means
- Assume that residual analysis is satisfactory
- If that hypothesis is rejected, we don't know **which specific means** are different
- Determining which specific means differ following an ANOVA is called the **multiple comparisons problem**
- There are **lots** of ways to do this
- We will use pairwise *t*-tests on means…sometimes called Fisher's Least Significant Difference (or Fisher's **LSD**) Method

Design-Expert Output

Treatment Means (Adjusted, If Necessary)

	Estimated Mean	**Standard Error**
1-15	9.80	1.27
2-20	15.40	1.27
3-25	17.60	1.27
4-30	21.60	1.27
5-35	10.80	1.27

Treatment	**Mean Difference**	**DF**	**Standard Error**	**t for H0 Coeff=0**	**Prob > |t|**
1 vs 2	-5.60	1	1.80	-3.12	0.0054
1 vs 3	-7.80	1	1.80	-4.34	0.0003
1 vs 4	-11.80	1	1.80	-6.57	< 0.0001
1 vs 5	-1.00	1	1.80	-0.56	0.5838
2 vs 3	-2.20	1	1.80	-1.23	0.2347
2 vs 4	-6.20	1	1.80	-3.45	0.0025
2 vs 5	4.60	1	1.80	2.56	0.0186
3 vs 4	-4.00	1	1.80	-2.23	0.0375
3 vs 5	6.80	1	1.80	3.79	0.0012
4 vs 5	10.80	1	1.80	6.01	< 0.0001

For the Case of Quantitative Factors, a Regression Model is often Useful

Response:Strength

ANOVA for Response Surface Cubic Model

Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**
Model	441.81	3	147.27	15.85	< 0.0001
A	*90.84*	*1*	*90.84*	*9.78*	*0.0051*
A^2	*343.21*	*1*	*343.21*	*36.93*	*< 0.0001*
A^3	*64.98*	*1*	*64.98*	*6.99*	*0.0152*
Residual	195.15	21	9.29		
Lack of Fit	*33.95*	*1*	*33.95*	*4.21*	*0.0535*
Pure Error	*161.20*	*20*	*8.06*		
Cor Total	636.96	24			

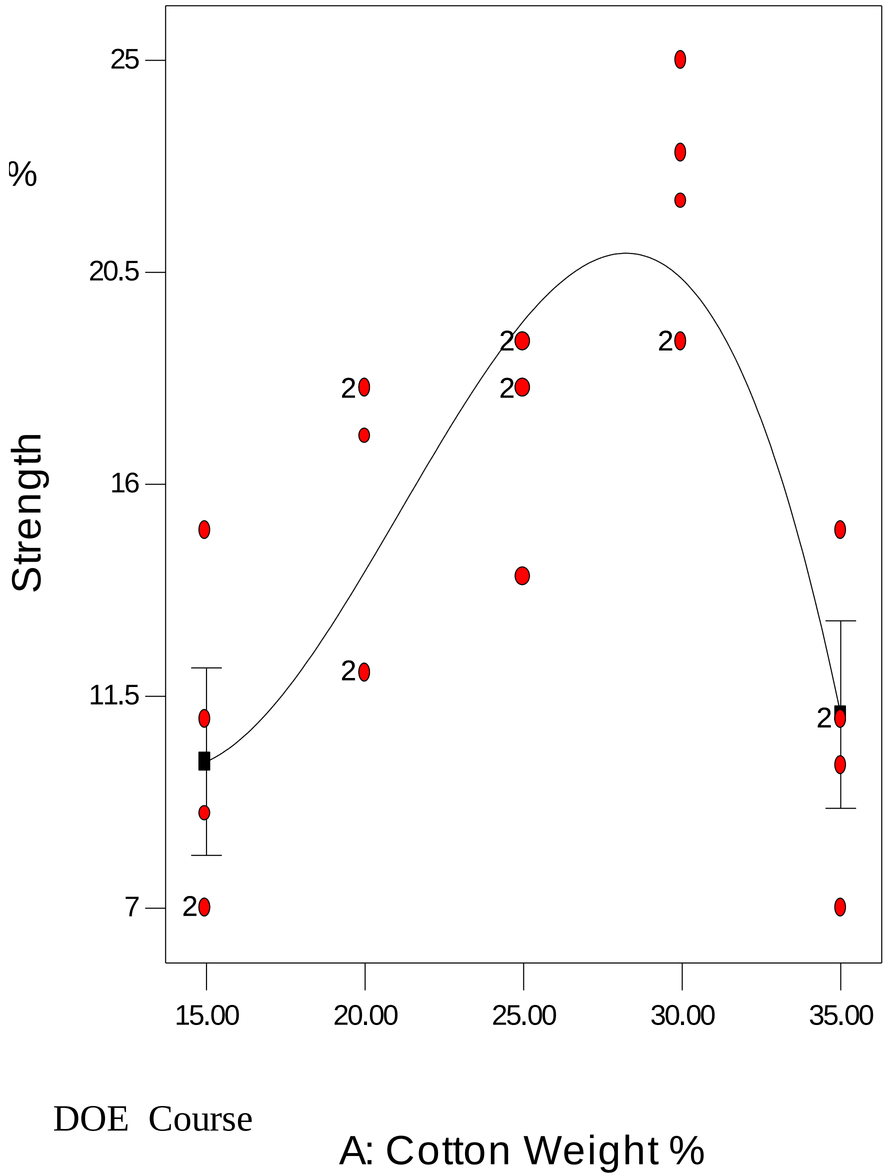
Factor	**Coefficient Estimate**	**DF**	**Standard Error**	**95% CI Low**	**95% CI High**	**VIF**
Intercept	19.47	1	0.95	17.49	21.44	
A-Cotton %	8.10	1	2.59	2.71	13.49	9.03
A^2	-8.86	1	1.46	-11.89	-5.83	1.00
A^3	-7.60	1	2.87	-13.58	-1.62	9.03

The Regression Model

Final Equation in Terms of Actual Factors:

Strength = 62.611 - 9.011* Wt % + 0.481* Wt %^2 - 7.600E-003 * Wt %^3

This is an **empirical model** of the experimental results

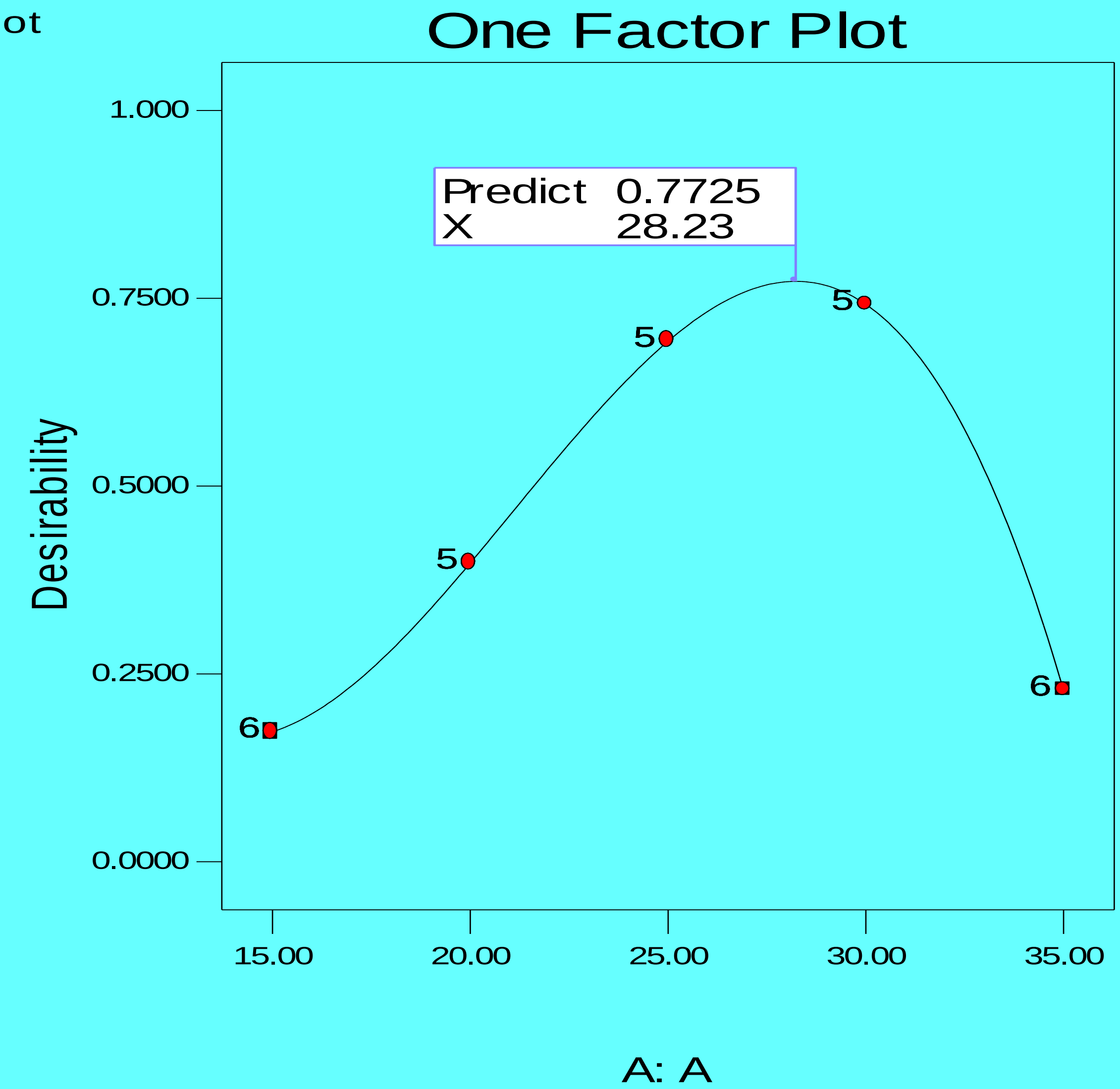


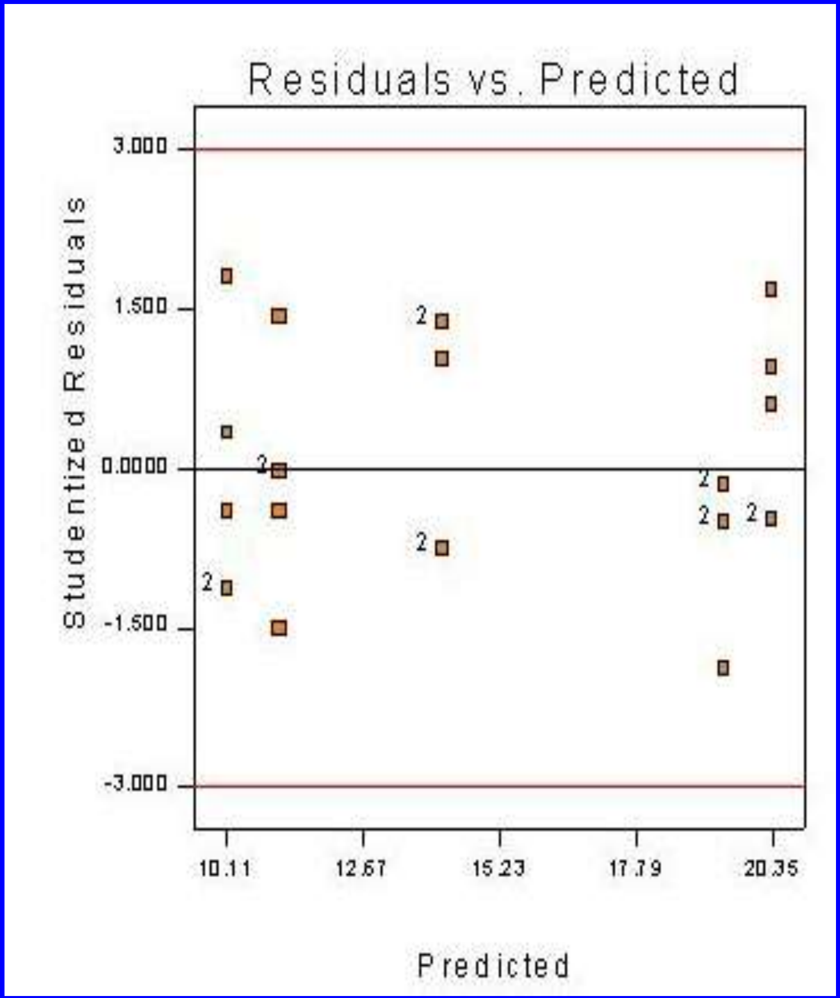
DESIGN-EXPERT Plot

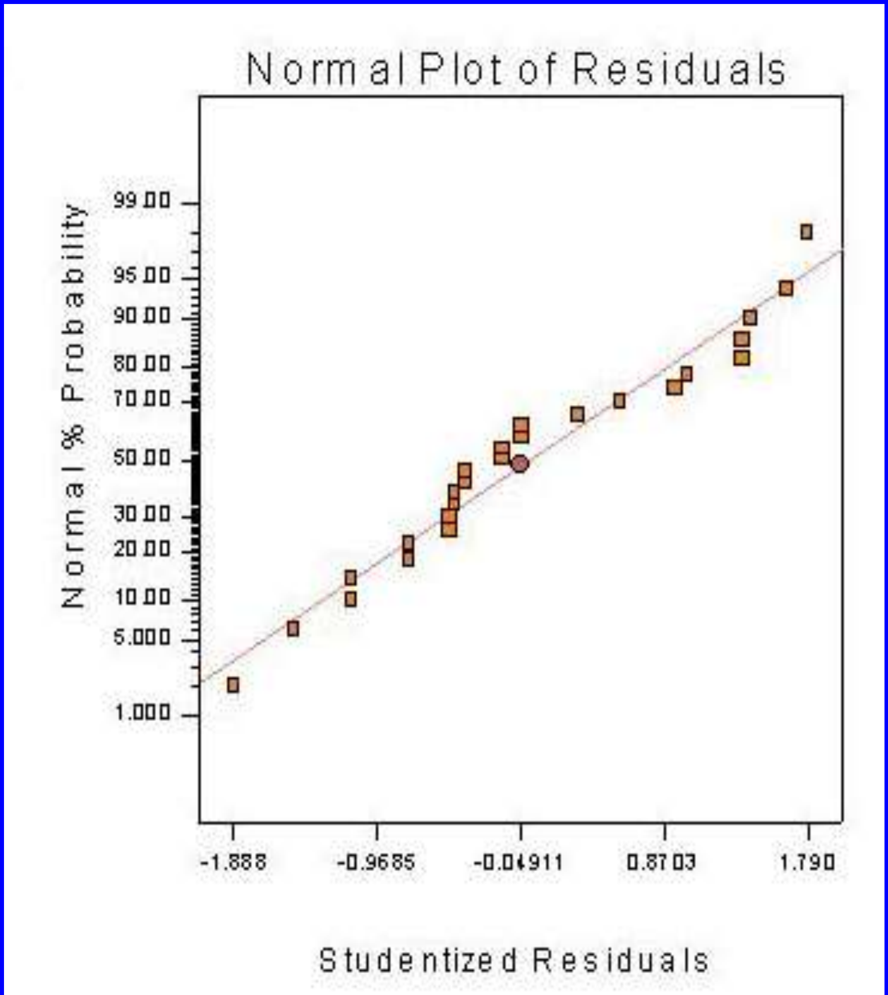
Desirability

X = A: A

Design Points




Residuals vs. Predicted
Studentized Residuals
3.000
1.500
0.0000
-1.500
-3.000
10.11
12.67
15.23
17.79
20.35
Predicted


Normal Plot of Residuals
Normal % Probability
99.00
95.00
90.00
80.00
70.00
50.00
30.00
20.00
10.00
5.000
1.000
-1.888
-0.9685
-0.04911
0.8703
1.790
Studentized Residuals

Sample Size Determination

- **FAQ** in designed experiments
- Answer depends on lots of things; including what type of experiment is being contemplated, how it will be conducted, resources, and desired **sensitivity**
- Sensitivity refers to the **difference in means** that the experimenter wishes to detect
- Generally, **increasing** the number of **replications increases** the **sensitivity** or it makes it easier to detect small differences in means

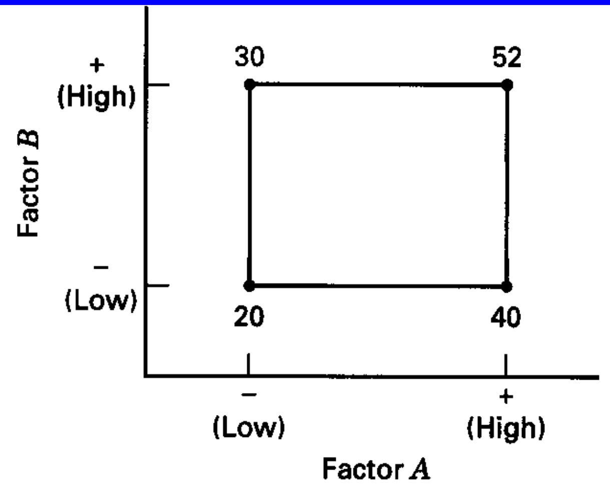
DOE (IV)

General Factorials

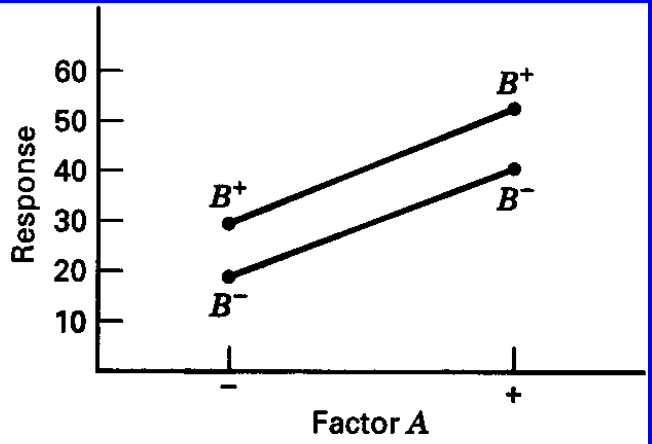
Design of Engineering Experiments Introduction to General Factorials

- **General principles** of factorial experiments
- The **two-factor factorial** with fixed effects
- The **ANOVA** for factorials
- Extensions to more than two factors
- **Quantitative** and **qualitative** factors – response curves and surfaces

Some Basic Definitions



A two-factor factorial experiment, with the response (y) shown at the corners.



A factorial experiment without interaction.

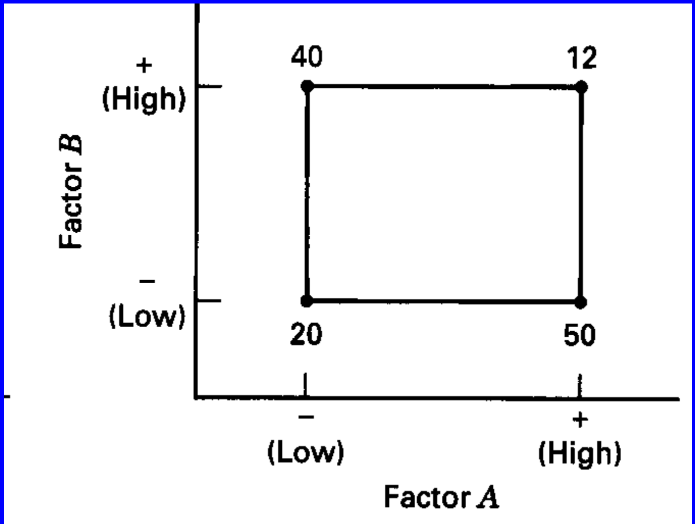
Definition of a factor effect: The change in the mean response when the factor is changed from low to high

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{40+52}{2} - \frac{20+30}{2} = 21$$

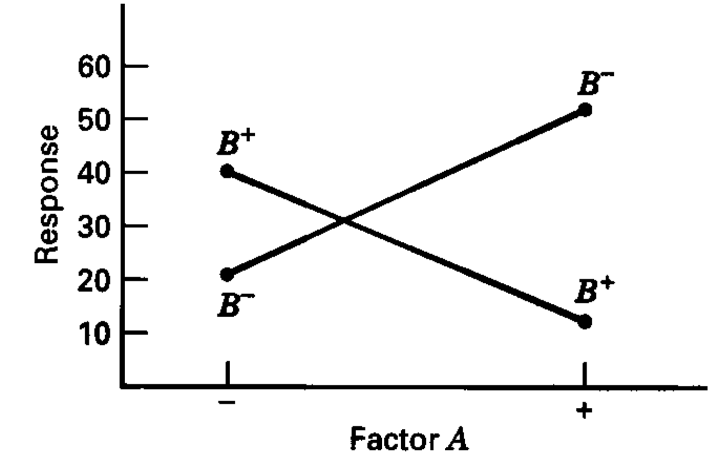
$$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{30+52}{2} - \frac{20+40}{2} = 11$$

$$AB = \frac{52+20}{2} - \frac{30+40}{2} = -1$$

The Case of Interaction:



A two-factor factorial experiment with interaction.



A factorial experiment with interaction.

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{50+12}{2} - \frac{20+40}{2} = 1$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{40+12}{2} - \frac{20+50}{2} = -9$$

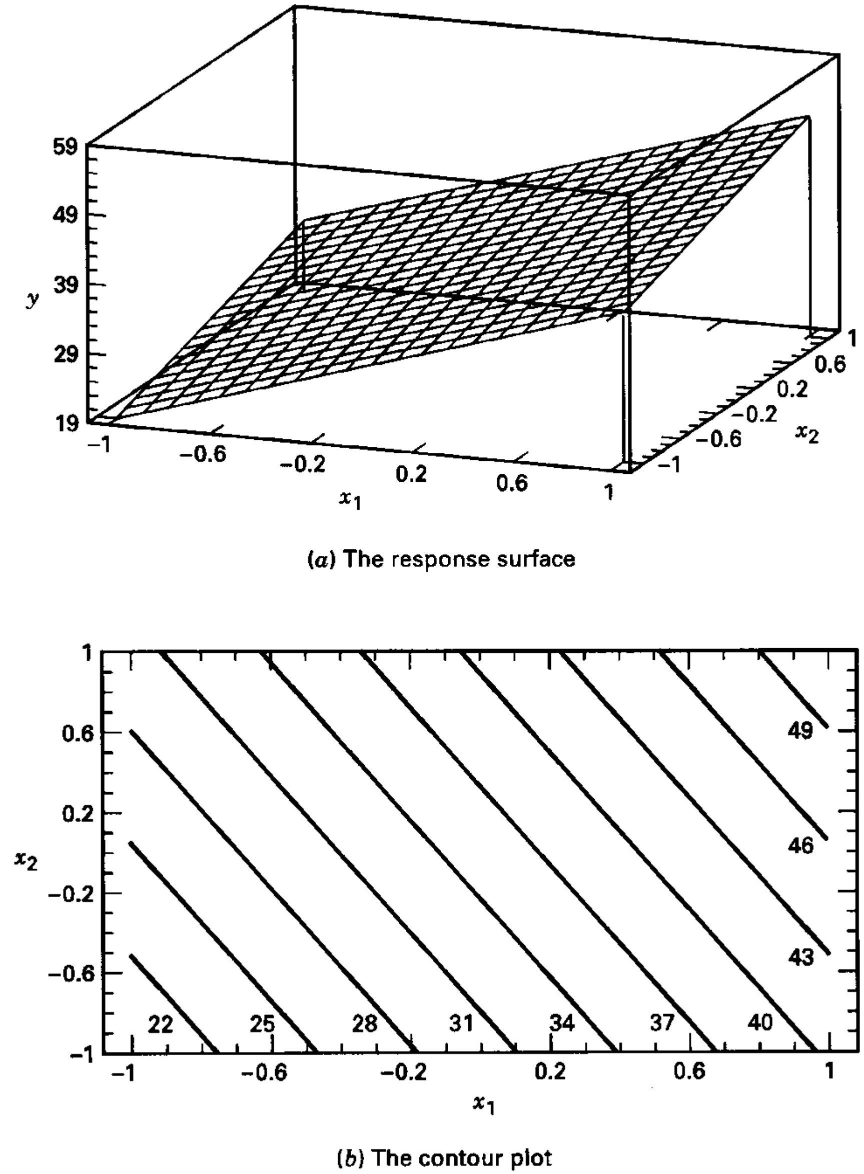
$$AB = \frac{12+20}{2} - \frac{40+50}{2} = -29$$

Regression Model & The Associated Response Surface

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$$

The least squares fit is

$$\hat{y} = 35.5 + 10.5x_1 + 5.5x_2 + 0.5x_1 x_2 \cong 35.5 + 10.5x_1 + 5.5x_2$$



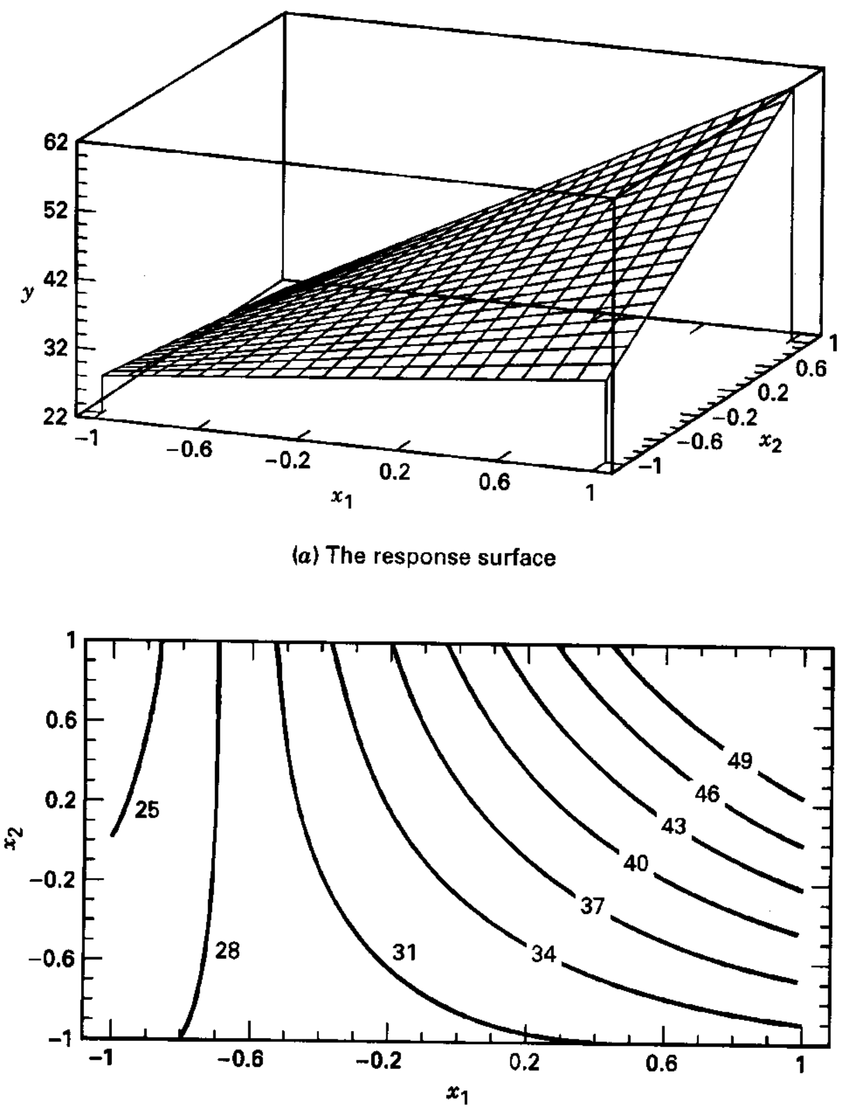
Response surface and contour plot for the model $\hat{y} = 35.5 + 10.5x_1 + 5.5x_2$.

The Effect of Interaction on the Response Surface

Suppose that we add an interaction term to the model:

$$\hat{y} = 35.5 + 10.5x_1 + 5.5x_2 + 8x_1x_2$$

Interaction is actually a form of **curvature**



(a) The response surface

(b) The contour plot

Response surface and contour plot for the model $\hat{y} = 35.5 + 10.5x_1 + 5.5x_2 + 8x_1x_2$.

Example: Battery Life Experiment

Life (in hours) Data for the Battery Design Example

Material Type	Temperature (°F) 15		70		125	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

A = Material type; *B* = Temperature (A **quantitative** variable)

1. What **effects** do material type & temperature have on life?
2. Is there a choice of material that would give long life ***regardless of temperature*** (a **robust** product)?

The General Two-Factor Factorial Experiment

General Arrangement for a Two-Factor Factorial Design

		Factor B			
		1	2	···	b
Factor A	1	$y_{111}, y_{112}, \ldots, y_{11n}$	$y_{121}, y_{122}, \ldots, y_{12n}$		$y_{1b1}, y_{1b2}, \ldots, y_{1bn}$
	2	$y_{211}, y_{212}, \ldots, y_{21n}$	$y_{221}, y_{222}, \ldots, y_{22n}$		$y_{2b1}, y_{2b2}, \ldots, y_{2bn}$
	⋮				
	a	$y_{a11}, y_{a12}, \ldots, y_{a1n}$	$y_{a21}, y_{a22}, \ldots, y_{a2n}$		$y_{ab1}, y_{ab2}, \ldots, y_{abn}$

a levels of factor *A*; *b* levels of factor *B*; *n* replicates

This is a **completely randomized design**

Statistical (effects) model:

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1, 2, ..., a \\ j = 1, 2, ..., b \\ k = 1, 2, ..., n \end{cases}$$

Other models (means model, regression models) can be useful

Regression model allows for prediction of responses when we have quantitative factors. ANOVA model does not allow for prediction of responses - treats all factors as qualitative.

Extension of the ANOVA to Factorials (Fixed Effects Case)

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{...})^2 = bn\sum_{i=1}^{a}(\bar{y}_{i..}-\bar{y}_{...})^2 + an\sum_{j=1}^{b}(\bar{y}_{.j.}-\bar{y}_{...})^2$$

$$+n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{ij.})^2$$

$$SS_T = SS_A + SS_B + SS_{AB} + SS_E$$

df breakdown:

$$abn-1 = a-1+b-1+(a-1)(b-1)+ab(n-1)$$

ANOVA Table – Fixed Effects Case

The Analysis of Variance Table for the Two-Factor Factorial, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	SS_A	$a - 1$	$MS_A = \frac{SS_A}{a - 1}$	$F_0 = \frac{MS_A}{MS_E}$
B treatments	SS_B	$b - 1$	$MS_B = \frac{SS_B}{b - 1}$	$F_0 = \frac{MS_B}{MS_E}$
Interaction	SS_{AB}	$(a - 1)(b - 1)$	$MS_{AB} = \frac{SS_{AB}}{(a - 1)(b - 1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
Error	SS_E	$ab(n - 1)$	$MS_E = \frac{SS_E}{ab(n - 1)}$	
Total	SS_T	$abn - 1$		

Design-Expert will perform the computations

Most text gives details of **manual computing** (ugh!)

Design-Expert Output

Response: Life

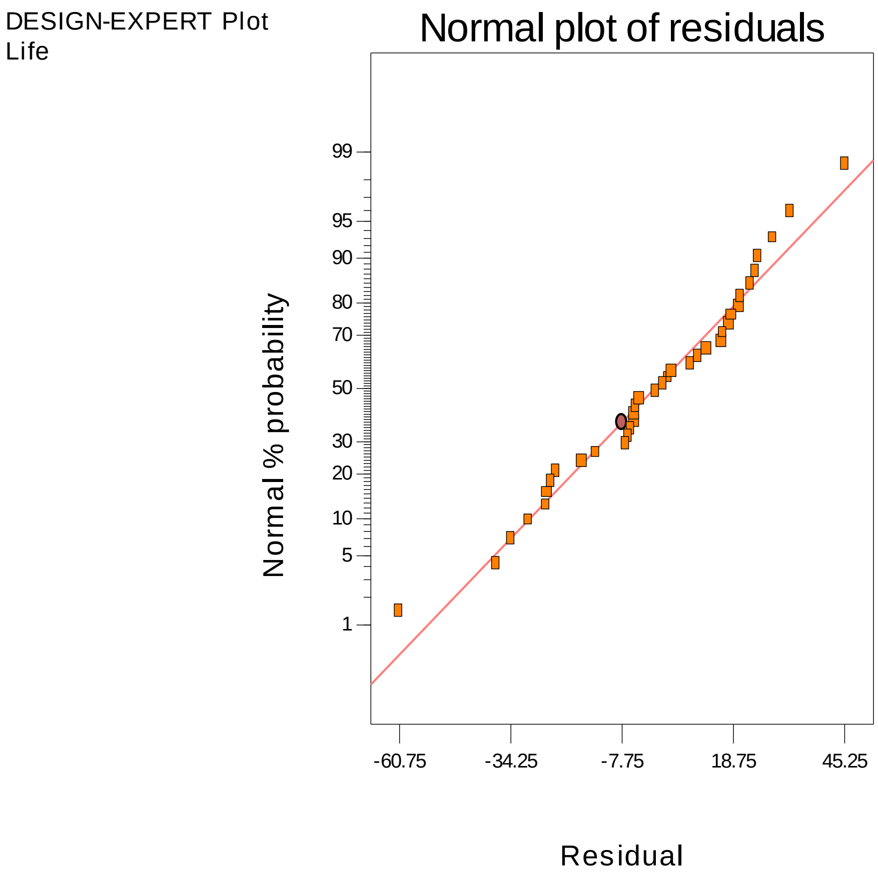
ANOVA for Selected Factorial Model

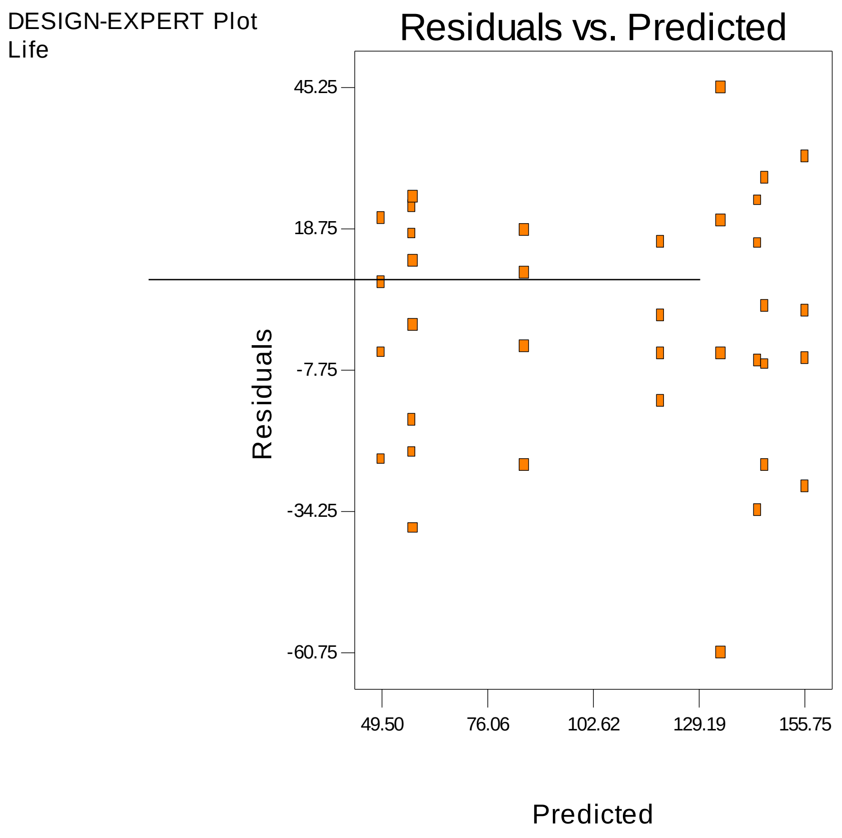
Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**
Model	59416.22	8	7427.03	11.00	< 0.0001
A	*10683.72*	*2*	*5341.86*	*7.91*	*0.0020*
B	*39118.72*	*2*	*19559.36*	*28.97*	*< 0.0001*
AB	*9613.78*	*4*	*2403.44*	*3.56*	*0.0186*
Pure E	18230.75	27	675.21		
C Total	77646.97	35			

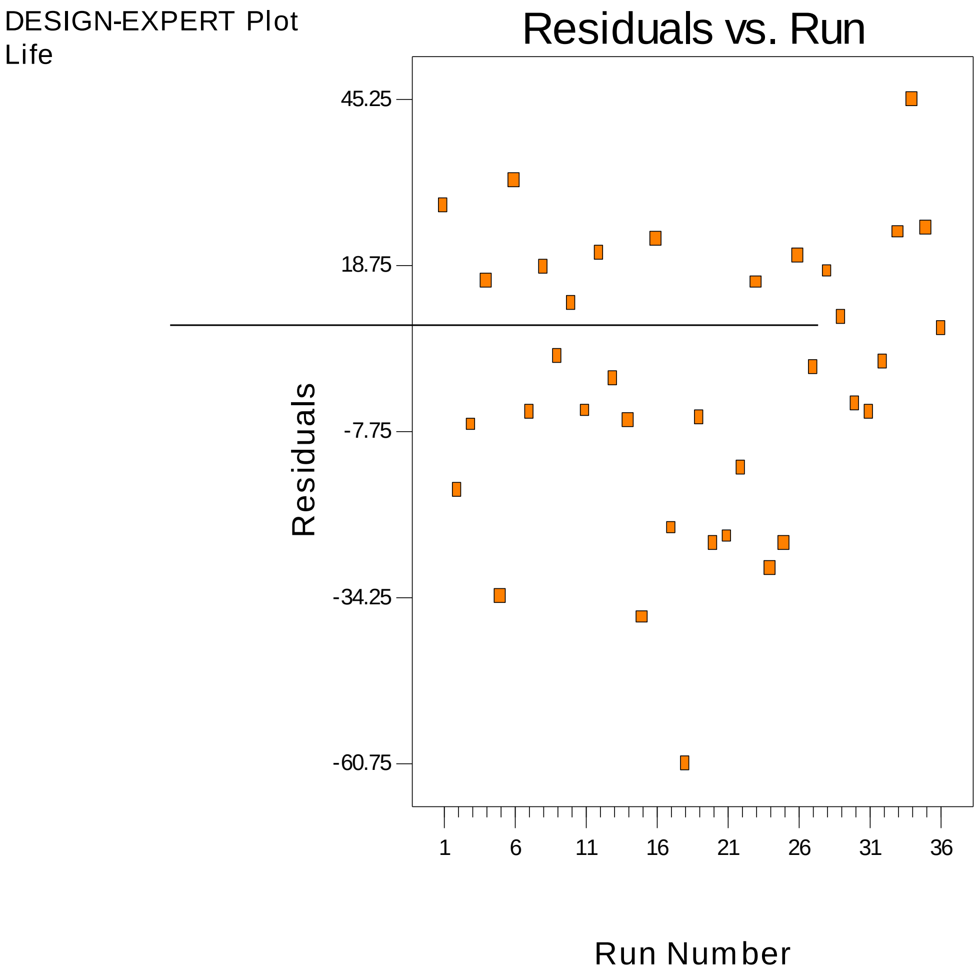
Std. Dev.	25.98	R-Squared	0.7652
Mean	105.53	Adj R-Squared	0.6956
C.V.	24.62	Pred R-Squared	0.5826
PRESS	32410.22	Adeq Precision	8.178

Residual Analysis

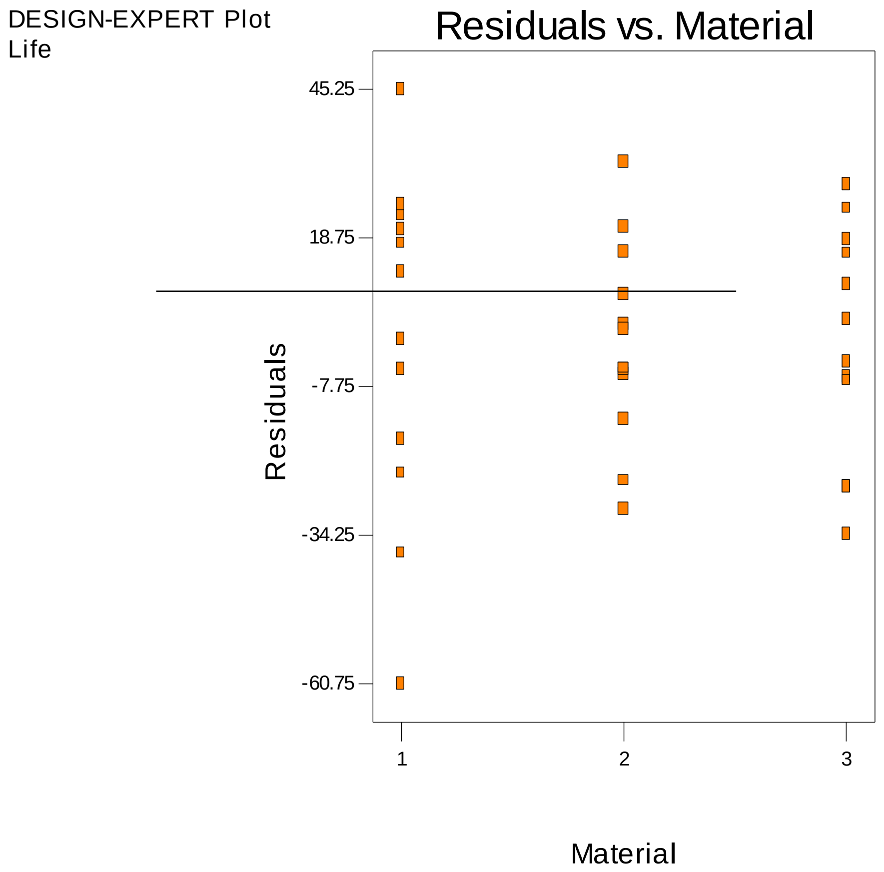


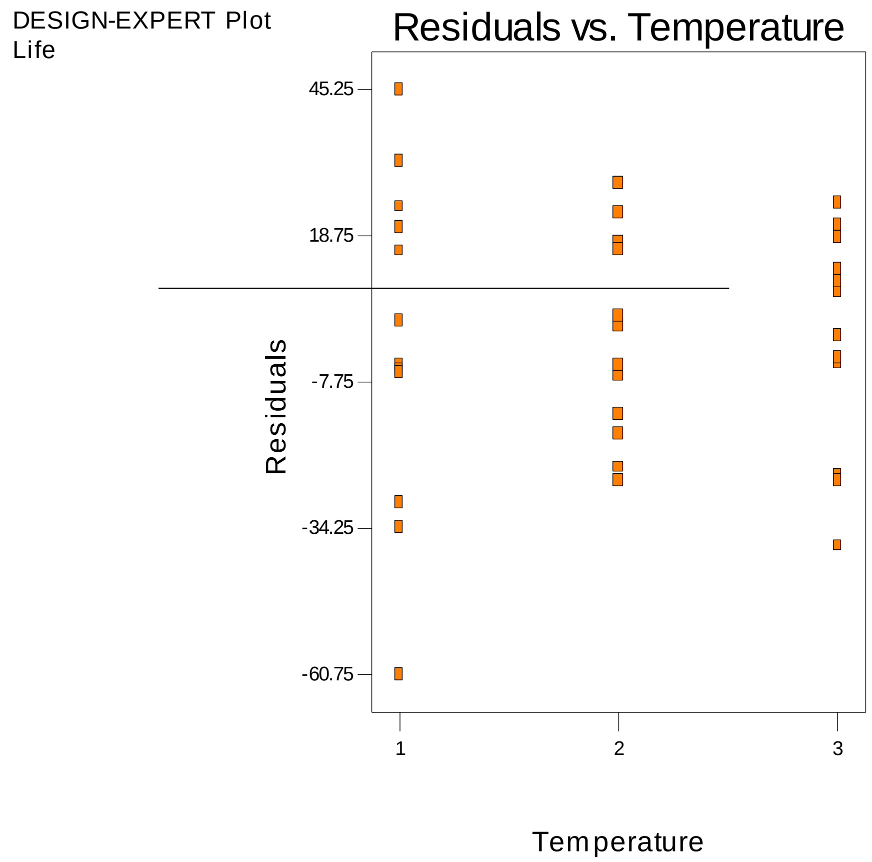


Residual Analysis

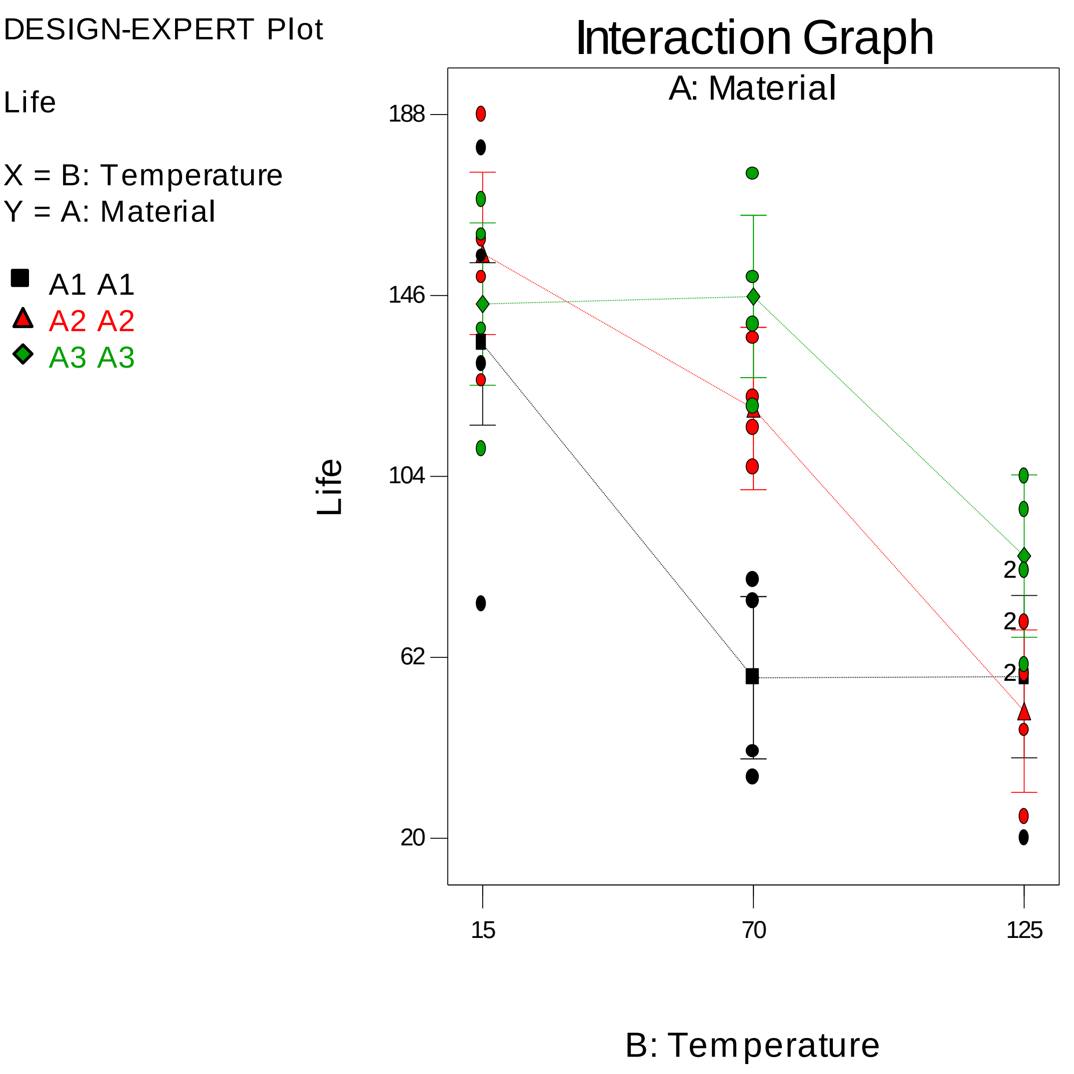


Residual Analysis





Interaction Plot



Quantitative and Qualitative Factors

- The basic ANOVA procedure treats every factor as if it were **qualitative**
- Sometimes an experiment will involve both **quantitative** and **qualitative** factors, such as in the example
- This can be accounted for in the analysis to produce **regression models** for the quantitative factors at each level (or combination of levels) of the qualitative factors
- These **response curves** and/or **response surfaces** are often a considerable aid in practical interpretation of the results

Quantitative and Qualitative Factors

Response: Life

***** WARNING: The Cubic Model is Aliased! *****

Sequential Model Sum of Squares

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Mean	4.009E+005	1	4009E+005			
Linear	49726.39	3	16575.46	19.00	< 0.0001	Suggested
2FI	2315.08	2	1157.54	1.36	0.2730	
Quadratic	76.06	1	76.06	0.086	0.7709	
Cubic	7298.69	2	3649.35	5.40	0.0106	Aliased
Residual	18230.75	27	675.21			
Total	4.785E+005	36	13292.97			

"Sequential Model Sum of Squares": Select the highest order polynomial where the additional terms are significant.

Quantitative and Qualitative Factors

A = Material type

B = Linear effect of Temperature

B^2 = Quadratic effect of Temperature

AB = Material type – $Temp_{Linear}$

AB^2 = Material type - $Temp_{Quad}$

B^3 = Cubic effect of Temperature (Aliased)

Candidate model terms from Design-Expert:

Intercept

A

B

B^2

AB

B^3

AB^2

Quantitative and Qualitative Factors

Lack of Fit Tests

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Linear	9689.83	5	1937.97	2.87	0.0333	Suggested
2FI	7374.75	3	2458.25	3.64	0.0252	
Quadratic	7298.69	2	3649.35	5.40	0.0106	
Cubic	0.00	0				Aliased
Pure Error	18230.75	27	675.21			

"Lack of Fit Tests": Want the selected model to have insignificant lack-of-fit.

Quantitative and Qualitative Factors

Model Summary Statistics

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	29.54	0.6404	0.6067	0.5432	35470.60	Suggested
2FI	29.22	0.6702	0.6153	0.5187	37371.08	
Quadratic	29.67	0.6712	0.6032	0.4900	39600.97	
Cubic	25.98	0.7652	0.6956	0.5826	32410.22	Aliased

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

Quantitative and Qualitative Factors

Response: Life

ANOVA for Response Surface Reduced Cubic Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	59416.22	8	7427.03	11.00	< 0.0001
A	*10683.72*	*2*	*5341.86*	*7.91*	*0.0020*
B	*39042.67*	*1*	*39042.67*	*57.82*	*< 0.0001*
B^2	*76.06*	*1*	*76.06*	*0.11*	*0.7398*
AB	*2315.08*	*2*	*1157.54*	*1.71*	*0.1991*
AB^2	*7298.69*	*2*	*3649.35*	*5.40*	*0.0106*
Pure E	18230.75	27	675.21		
C Total	77646.97	35			

Std. Dev.	25.98	R-Squared	0.7652
Mean	105.53	Adj R-Squared	0.6956
C.V.	24.62	Pred R-Squared	0.5826
PRESS	32410.22	Adeq Precision	8.178

Regression Model Summary of Results

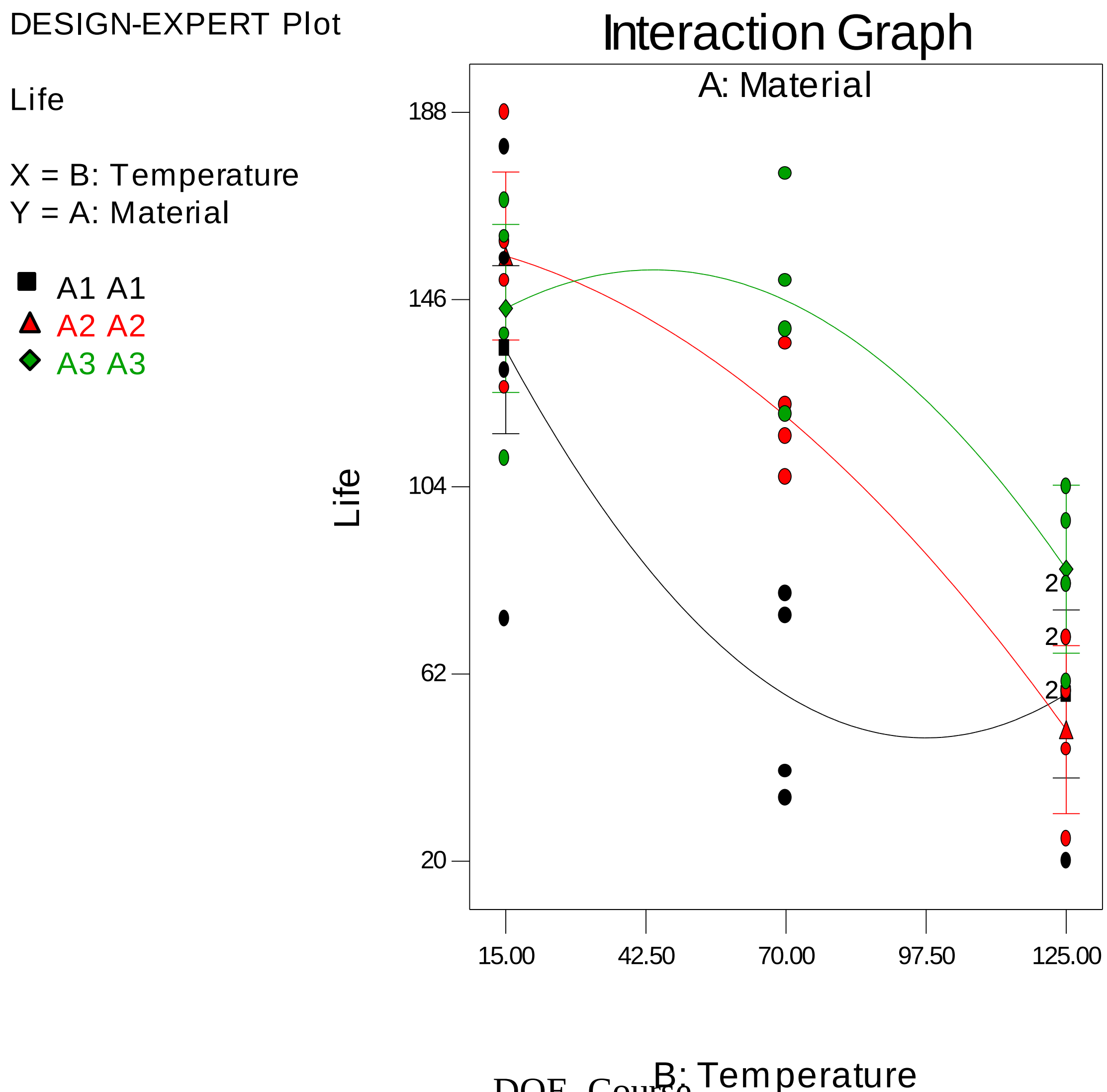
Final Equation in Terms of Actual Factors:

Material A1
Life =
+169.38017
-2.50145 * Temperature
+0.012851 * $Temperature^2$

Material A2
Life =
+159.62397
-0.17335 * Temperature
-5.66116E-003 * $Temperature^2$

Material A3
Life =
+132.76240
+0.90289 * Temperature
-0.010248 * $Temperature^2$

Regression Model Summary of Results



Factorials with More Than Two Factors

- Basic procedure is similar to the two-factor case; all *abc…kn* treatment combinations are run in random order
- ANOVA identity is also similar:

$$SS_T = SS_A + SS_B + \cdots + SS_{AB} + SS_{AC} + \cdots$$
$$+ SS_{ABC} + \cdots + SS_{AB\cdots K} + SS_E$$

More than 2 factors

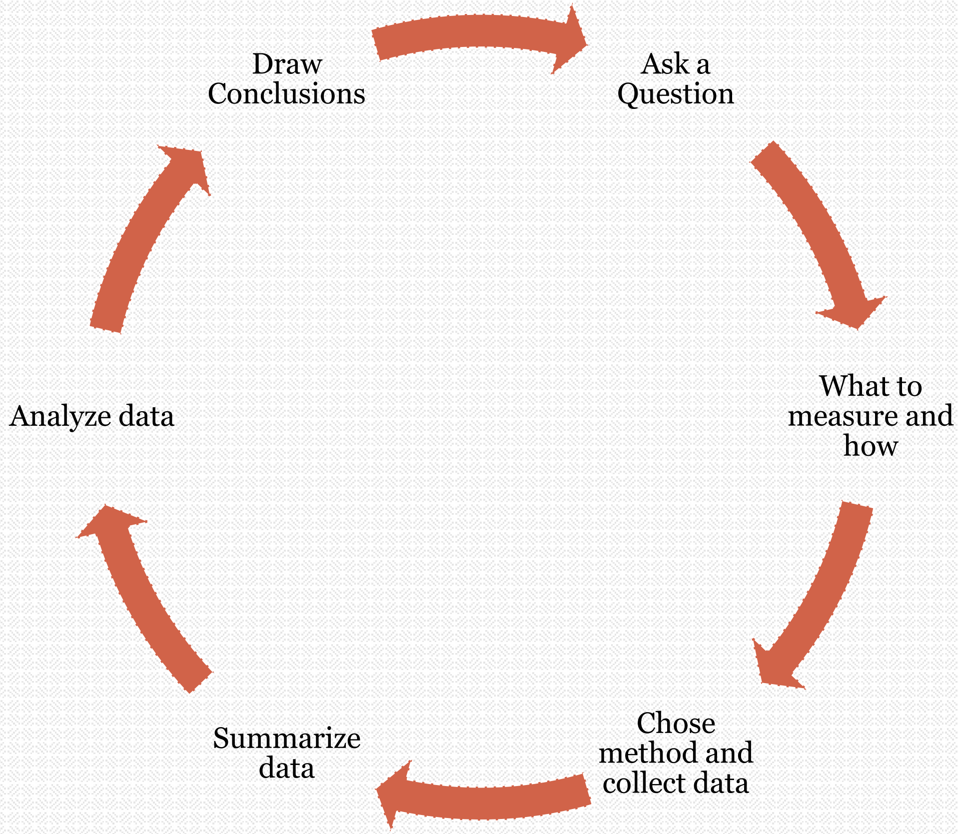
- With more than 2 factors, the most useful type of experiment is the 2-level factorial experiment.
- Most efficient design (least runs)
- Can add additional levels only if required
- Can be done sequentially
- That will be the next topic of discussion

Design of Experiments

DR. SHASHANK SHEKHAR
MSE, IIT KANPUR
FEB 19TH 2016

TEQIP (IIT KANPUR)

Data Analysis



 https://www.bcps.org/offices/lis/researchcourse/statistics_role.html

List of Topics

- Objective of experiment
- Strategy of Experimentation
- Replication, Repetition and Randomization
- Various approaches of experimentation
- Guidelines for designing experiments

Objective of Experiment

- Data collection is not the sole objective. Objective are usually :
  - Determining which variables are most influential on the response 'y'
  - Determining where to set the influential 'x's so that 'y' is as close to desired value as possible
  - Determining where to set the influential 'x's so that variability in 'y' is small (eg. thermal instability)
  - Determining where to set the influential 'x's so that the effects of uncontrollable variables 'z's are minimized (eg. avoiding formation of deleterious phases)

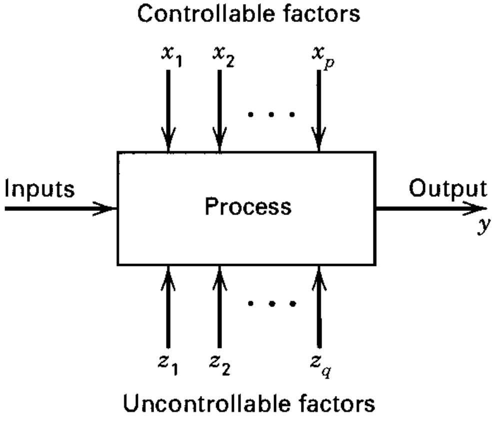


Figure 1-1 General model of a process or system.

Objective of the experimenter is to determine the influence of factors on output response

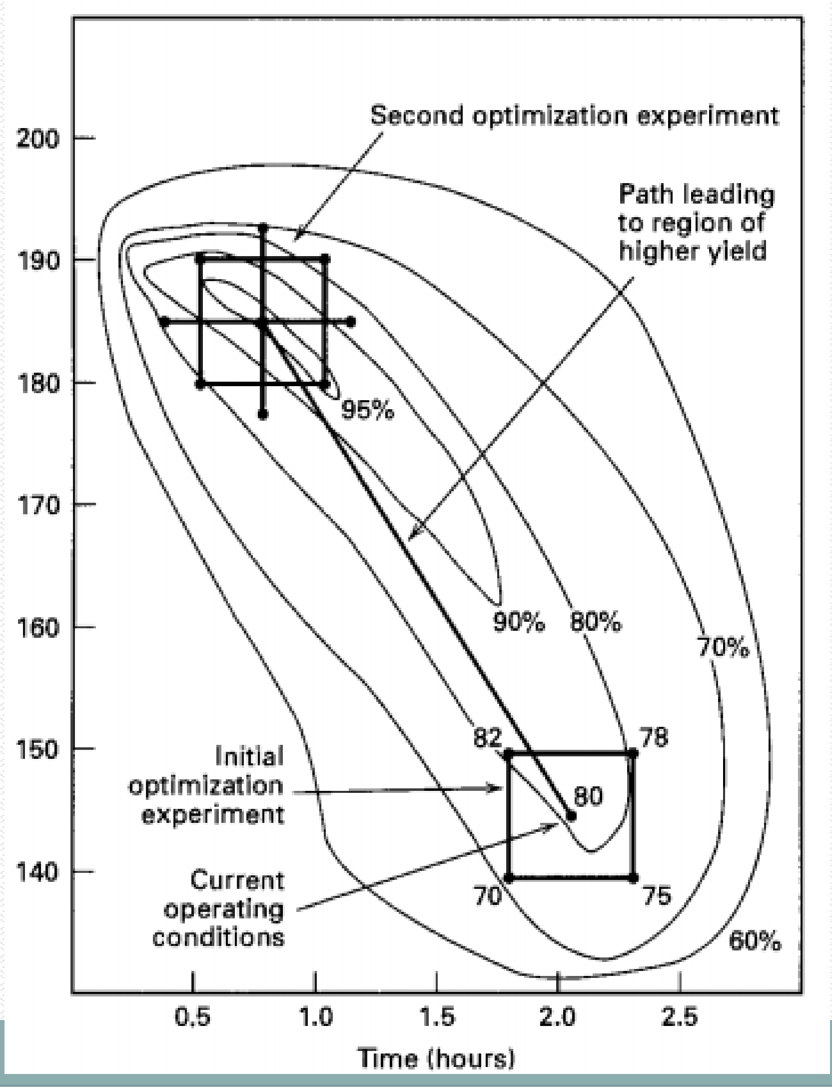
Strategy of Experimentation

- Experiment may be defined as a test or a series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and *identify the reasons* for changes that may be observed in the output response

An Example

- A food processor may be interested in studying the effect of cooking medium (viz. with butter and with ghee) on quality of these cooked popcorns. His objective can be to determine which medium produces the best quality popcorn.
- He may conduct tests on a number of collected samples in two different mediums and cook them and measure the quality to compare the effect of source. The quality may be determined, by say, the fraction of pop-corns that fracture under certain pressure. The average fraction of the properly cooked popcorns in the two mediums will be used to determine if there is a difference and which one produces better quality.

Objective of Experiment



Example: Questions to ponder

- Are there any other factors that might affect quality that should be investigated (eg. Electrical Power of cooking system, time of cooking, moisture, room temperature, room humidity)
- How many samples are required for each condition
- In what order should the data be collected (eg. what if there is a drift in measurement values)
- What method of data analysis should be used
- What difference in average fraction between the two cooking media will be considered important (eg. ANOVA)

Example: Data Collection

Method of data collection is also important

- Suppose that the food scientist in the above experiment used specimens from one batch in the butter and specimens from a second batch in ghee
- Engineer measures fractured fraction of all the samples cooked in one medium and then the fractured fractions cooked in the other medium
- So what is the right method?

Completely randomized design is required

Components of an Experiment

- A good experimental design must:
  - Avoid systematic error: it can lead to bias in comparison
  - Be precise: Random errors need to be reduced
  - Allow estimation of error: Permits statistical inference of confidence interval etc.
  - Have broad validity: sample should be good representation to be valid for the whole population

Basic Principles

- Randomization – Random allocation and order
  - Averaging out
- Blocking – to improve precision in comparisons
- Replication
  - Replication vs repeated measurements
- Proper selection of sample (where should the corn samples be picked from)

Haphazard is not randomized

- Lets say you are given 16 paper clips and you are to treat them in 4 different ways (A,B,C and D)
  - (1) You mark 16 identical slips of paper, marked A,B,C and D for 4 different treatments and mix them. Every time you take one paper clip, you draw a slip of paper and use the treatment marked on the slip
  - (2) Treatment A is given to first 4 units, then treatment B is given to next 4 units and so on
  - (3) Each unit is given treatment A, B, C or D based on whether the “seconds” reading on the clock is first, second, third or fourth quadrant.

Approaches

- Lets say that there are four factors that need to be considered to understand the response. Lets says quality, (in terms of percentage cracked) is the response that you are interested in maximizing, and the factors are:
  - time of cooking and (t=5mins or t = 30mins)
  - cooking medium (butter or ghee)
  - Power of equipment (P= 0.5Pm or P = 0.75 Pm)
  - Moisture fraction (strain = 0.25 or strain = 0.5)
- How will you sort through each and every factor and its effect on quality? For simplicity only two states of each factor are taken and it is given that you have only 8 samples.

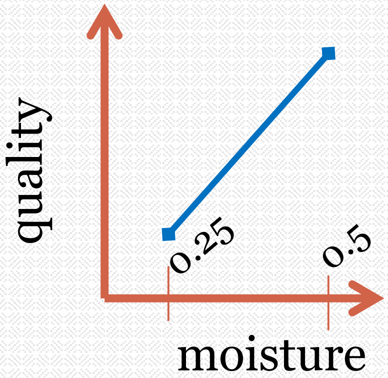
Approaches

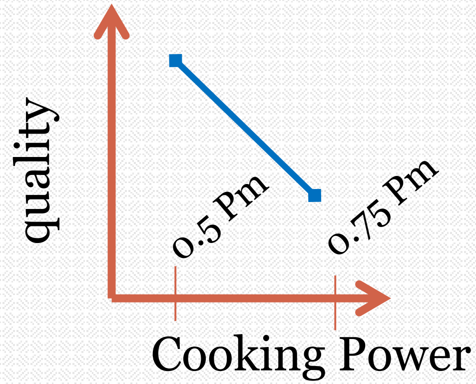
- **Best-guess approach**: Test for arbitrary combination and see the outcome. During the test however you noticed that all high power conditions conditions resulted in lower quality and so you may decide to use lower power and keep other factors same as earlier. This process can go on until all the factors are optimized
- Disadvantages
  - One has to keep trying combinations, without any guarantee of success
  - If the initial combination produces acceptable result, one may be tempted to stop testing

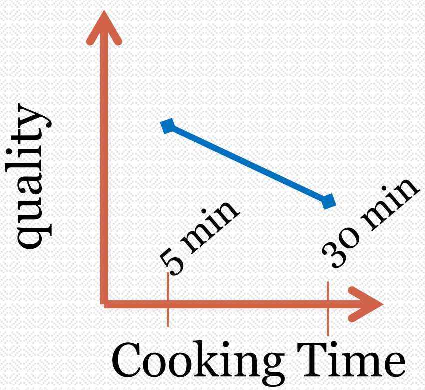
Approaches

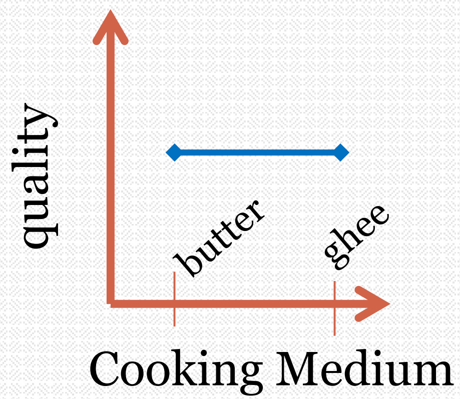
- **One-factor-at-a-time:** Select a baseline set of levels, for each factor, then successively vary each factor over its range with other factors held constant at the baseline level. A series of graphs can represent the output as a response to the change in these factors
  - Interpretation is simple and straight forward, however interaction between the factors is not highlighted (An interaction is the failure of the one factor to produce the same effect on the response at different levels of another factor)
  - One-factor-at-a-time experiments are always less efficient that the other methods based on a statistical approach to design

One-factor-at-a-time

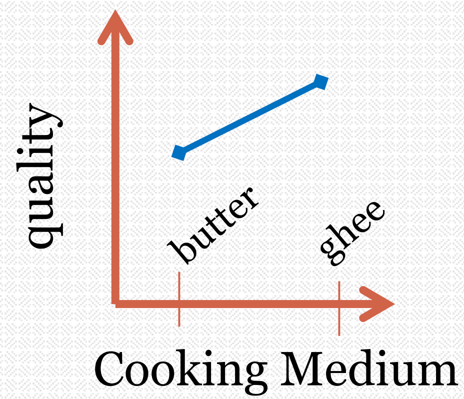








0.5 Pm

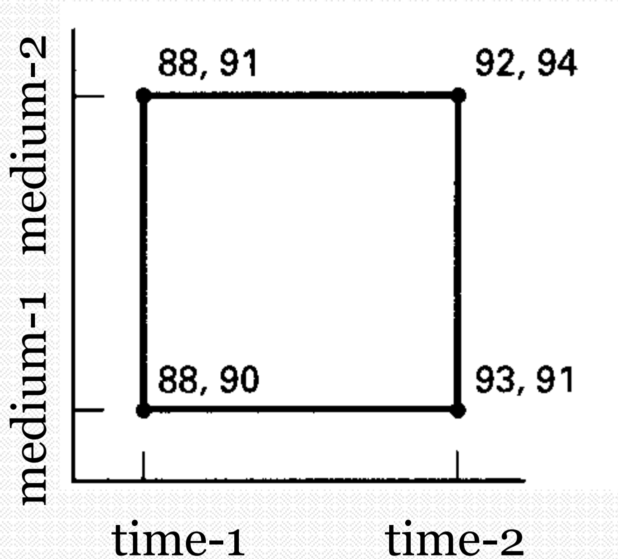


0.75 Pm

Factorial Approach

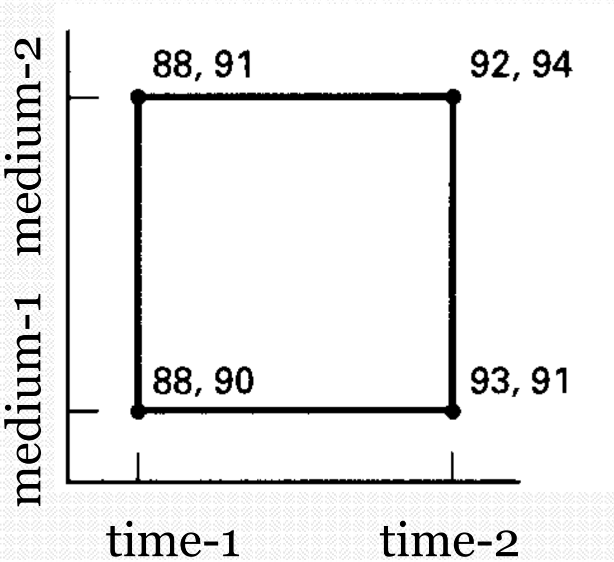
- This is an extremely important and useful approach
- Factors are varied together, instead of one at a time.

- To begin with, lets assume only two factors are important (time and medium)
- We have 2 factors at 2 levels $\Rightarrow$ 2^2 factorial design



Effects, basically describe the response in terms of a simple model using linear combinations

Factorial Approach



A= Effect of time =
(92+94+93+91)/4 – (88+91+88+90)/4 = 3.25

B = Effect of medium =
(88+91+92+94)/4 – (88+90+93+91)/4 =0.75

AB = Measure of interaction =
(92+94+88+90)/4 – (88+91+93+91)/4 =0.25

Average = 90.875

A fitted regression model to express the response in terms of the two parameters:

y= 90.875 + A/2*x1 + B/2*x2 +AB/2* x1x2
y = 90.875 + 1.625x1 + 0.375x2 + 0.125x1x2

Statistical testing is required to determine whether any of these effects differ from zero

Interaction Effect

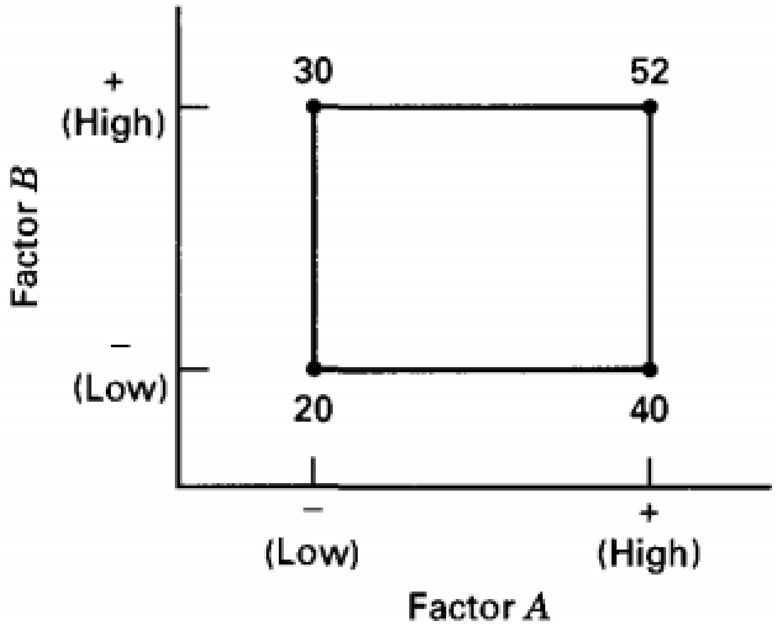


Figure 5-1 A two-factor factorial experiment, with the response (y) shown at the corners.

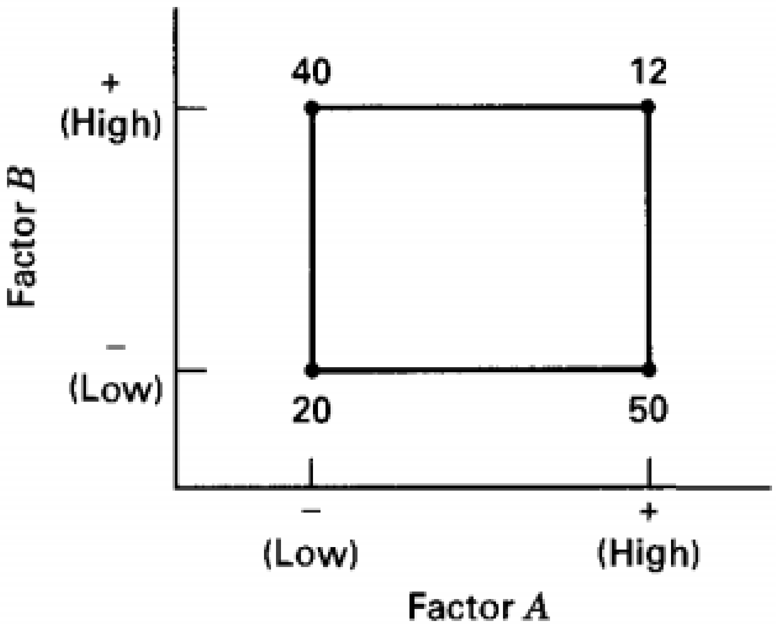


Figure 5-2 A two-factor factorial experiment with interaction.

Interaction Effect

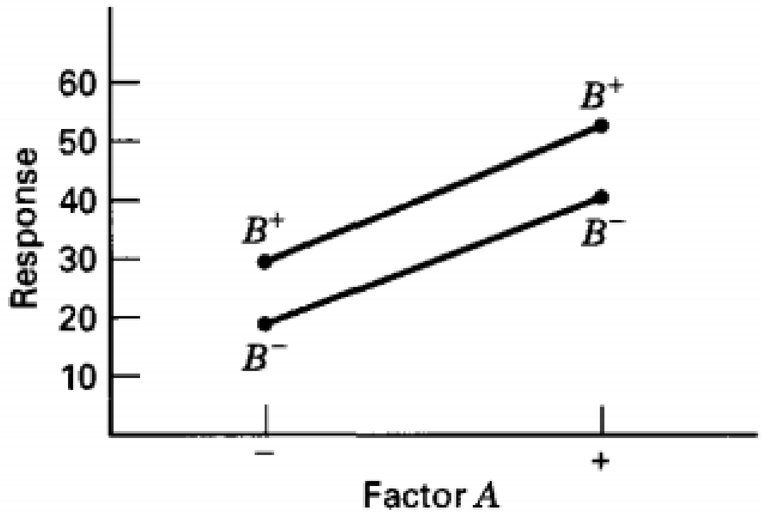


Figure 5-3 A factorial experiment without interaction.

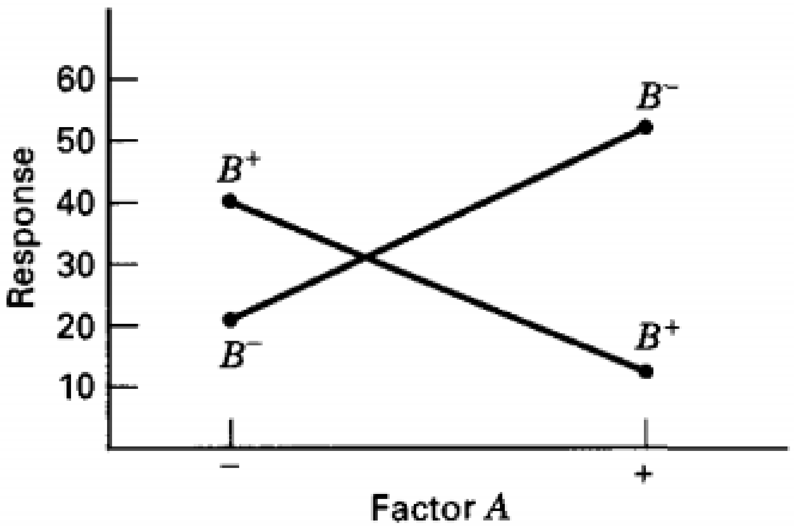
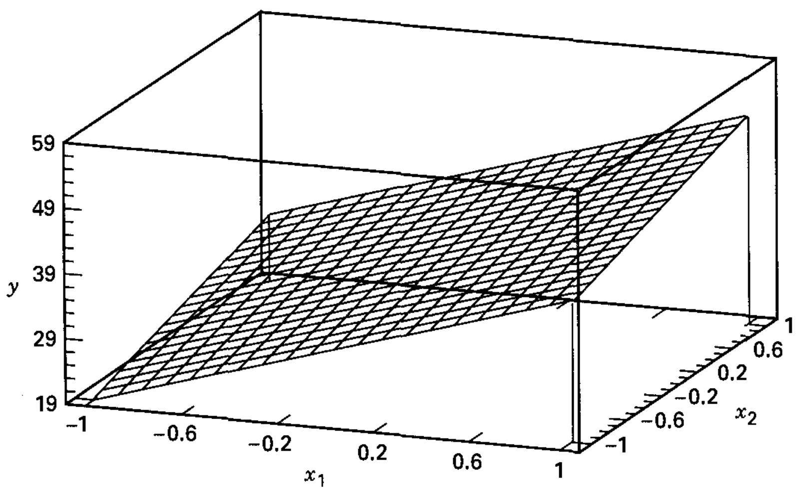


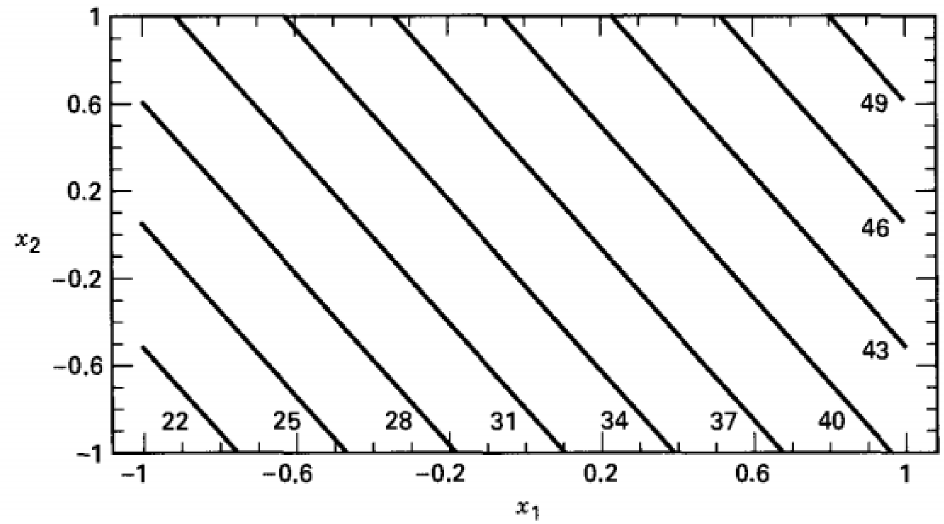
Figure 5-4 A factorial experiment with interaction.

Weak interaction

$$y = 35.5 + 10.5 * x1 + 5.5 * x2 + 0.5 * x1x2$$
$$\approx 35.5 + 10.5 * x1 + 5.5 * x2$$



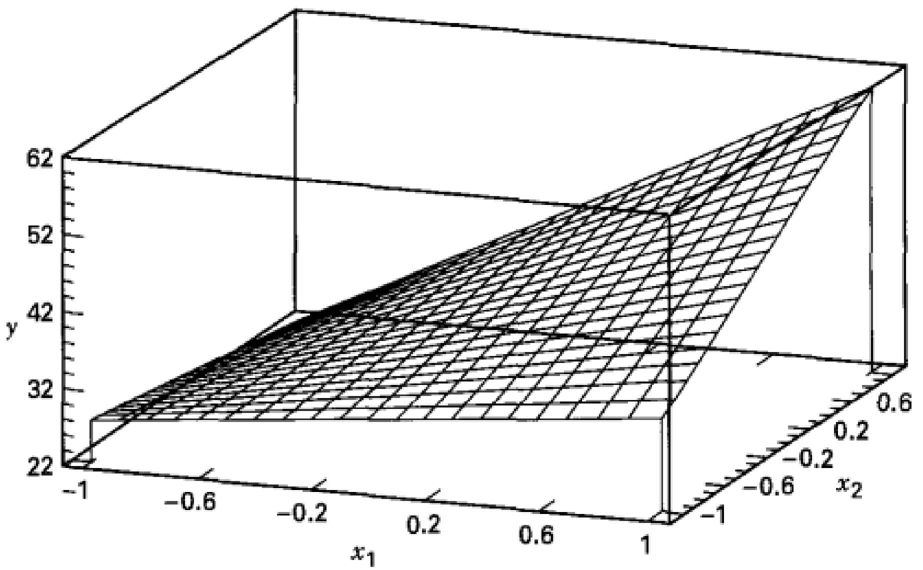
(a) The response surface



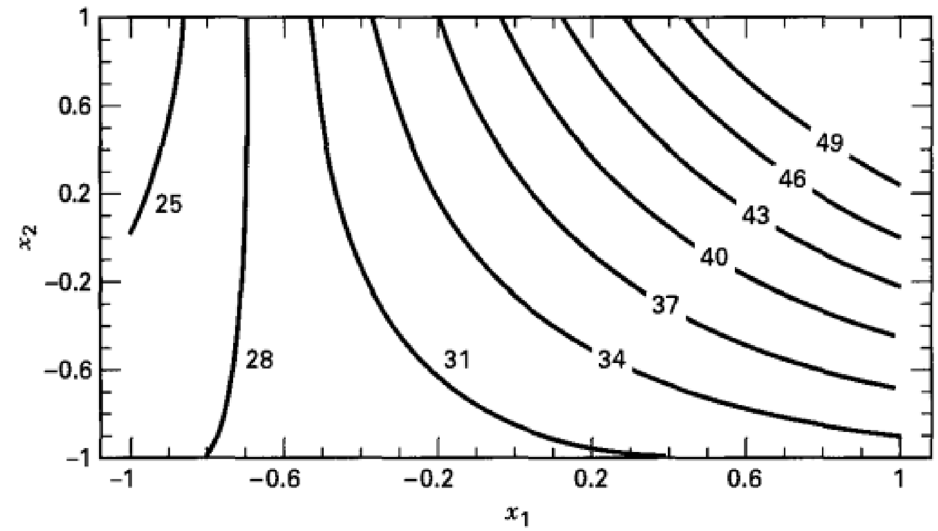
(a) Response Surface (b) Contour Plot

Strong interaction

$y = 30.5 + 0.5*x1 - 4.5*x2 - 14.5*x1x2$



(a) The response surface

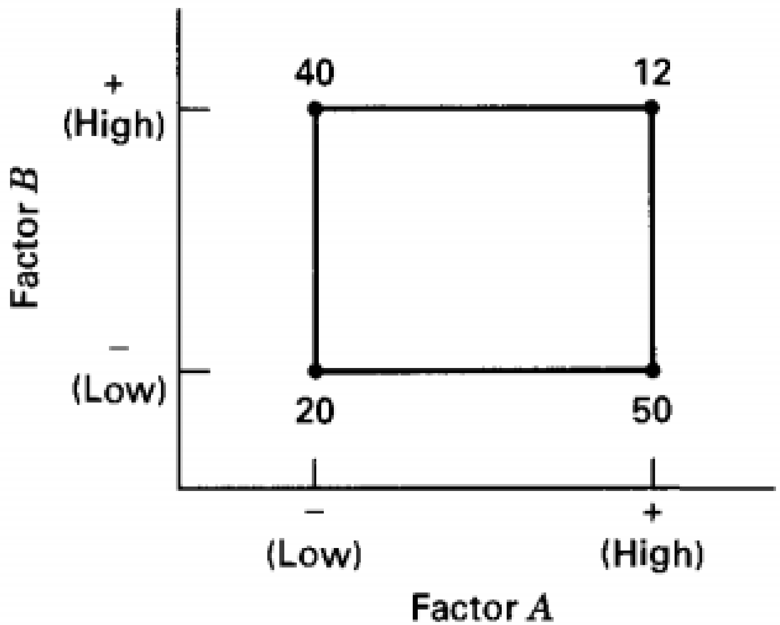


(b) The contour plot

(a) Response Surface (b) Contour Plot

Interaction is a form of curvature in the underlying response surface model of the experiment

Interaction Effect



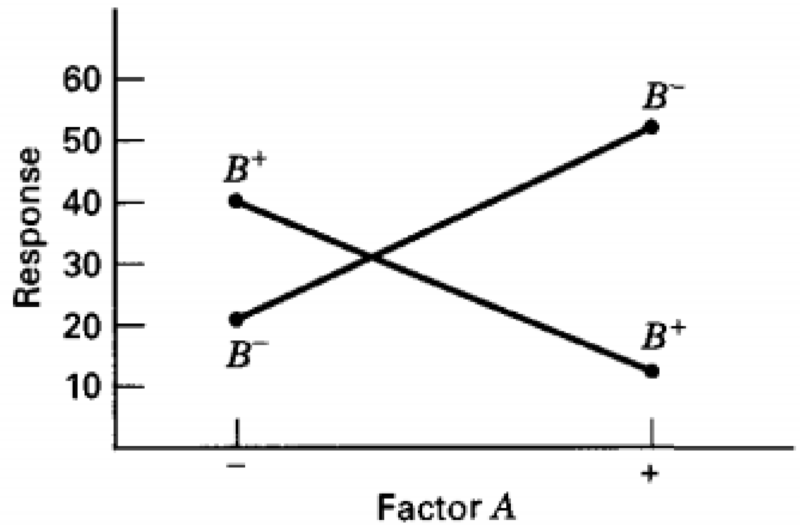


Figure 5-4 A factorial experiment with interaction.

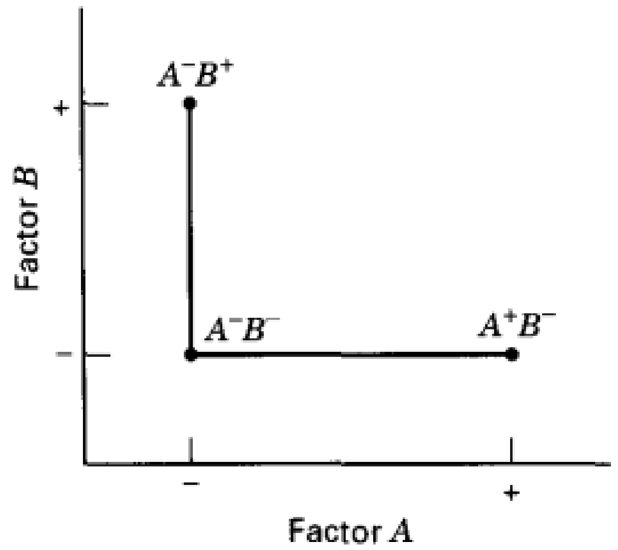
Generally when interaction effect is large, corresponding main effects have little practical meaning.

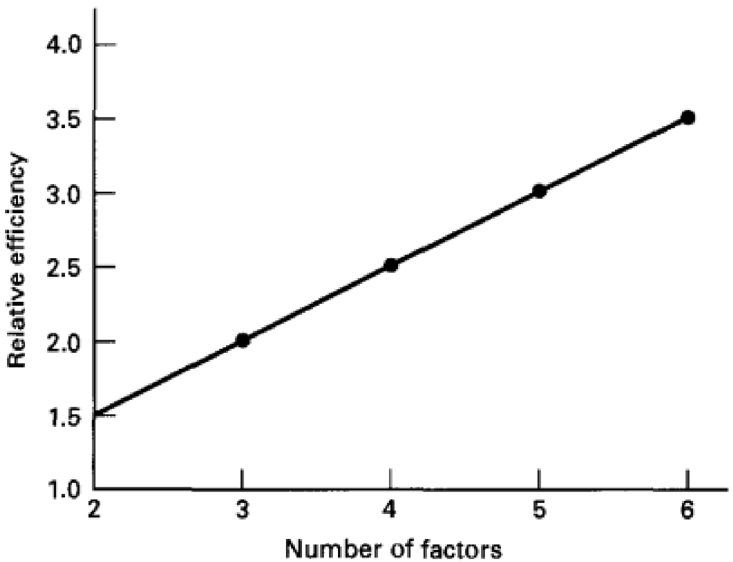
A = (50+12)/2 – (40+20)/2 = 1 $\Rightarrow$ No effect of A?

A has strong effect, but it depends on level of B

 Design Analysis Of Experiments, Douglas C. Montgomery

Advantages of Factorial



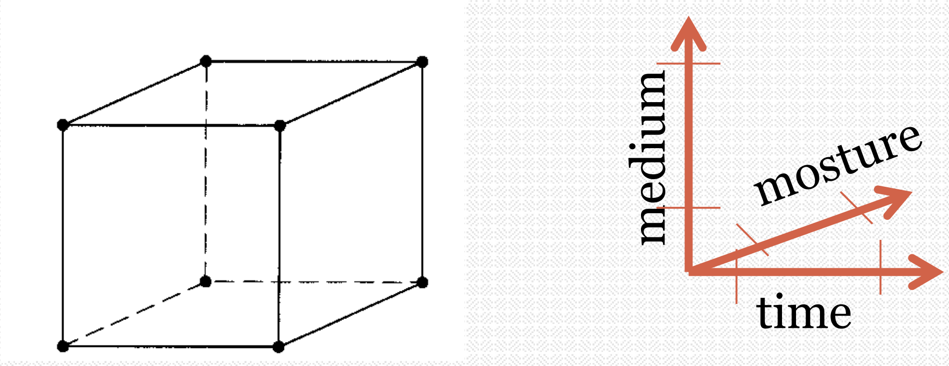


- Lets again look at two factors with two levels
- No. of experiments for one-factor-approach = 6
- No. of experiments for factorial approach = 4
- Efficiency of factorial approach = 6/4 = 1.5
- If A^-B^+ and A^+B^- gave a better response, then what about A^+B^+?

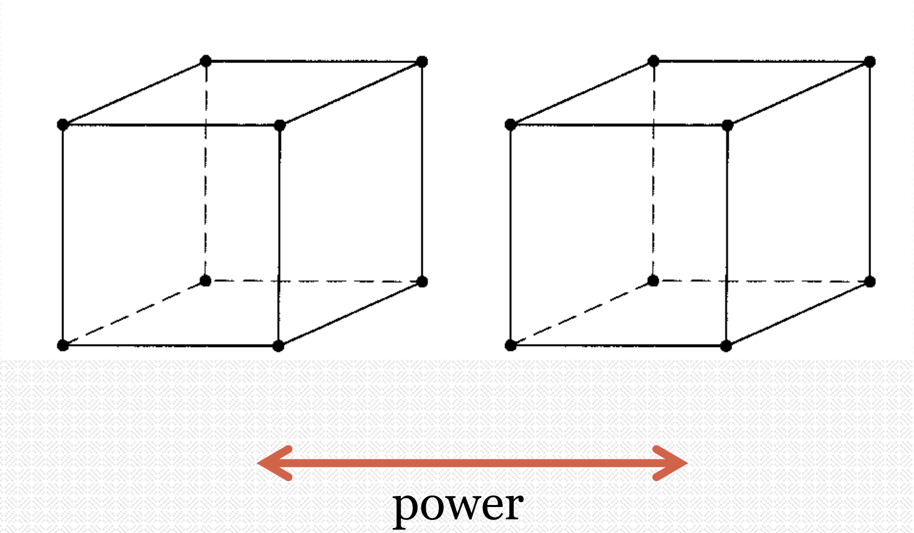
Design Analysis Of Experiments, Douglas C. Montgomery

Factorial Approach

Similarly 2^3 factorial design requires 8 tests



and 2^4 factorial design requires 16 tests

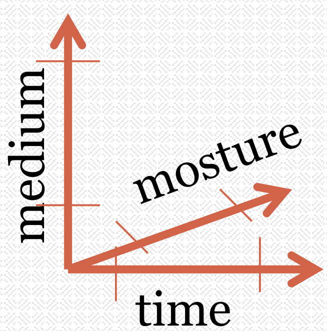


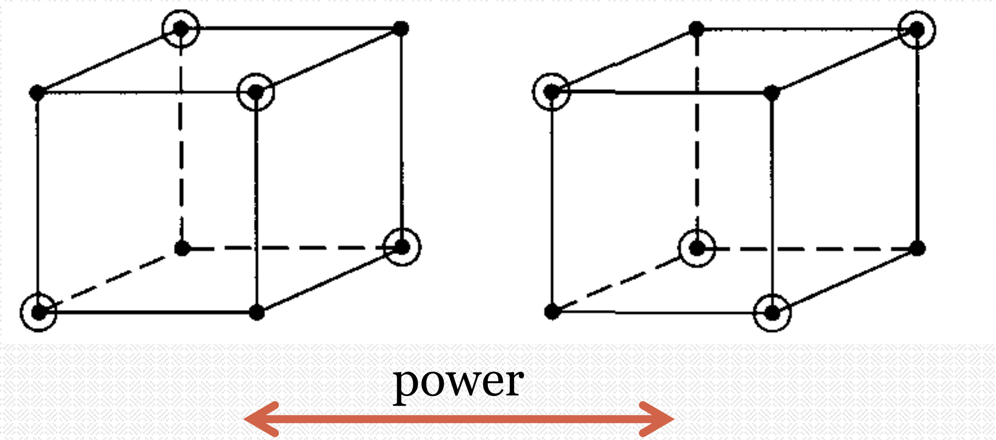
Factorial Approach

- If there are k factors, each at two levels, the factorial design would require 2^k tests
- 4 factors with 2 levels require 16 tests
- 10 factors with 2 levels require 1024 tests!!
- This is clearly infeasible from time and resource point of view
- Fractional factorial design can be used

Fractional Factorial Design

- Only a subset of the tests of basic factorial design is required
- Modified design requires only 8 tests instead of 16 and would be called a ‘one-half factorial’
- Will provide good information about the main effects of the four factors as well as some information about how these factors interact

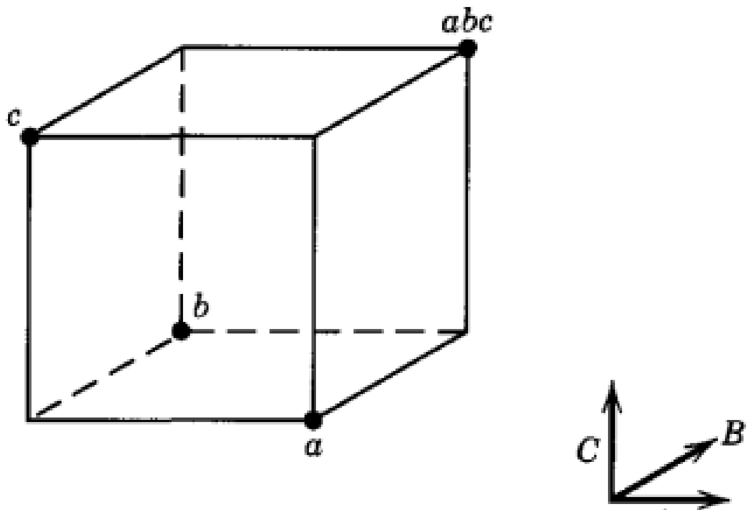




Fractional Factorial Designs

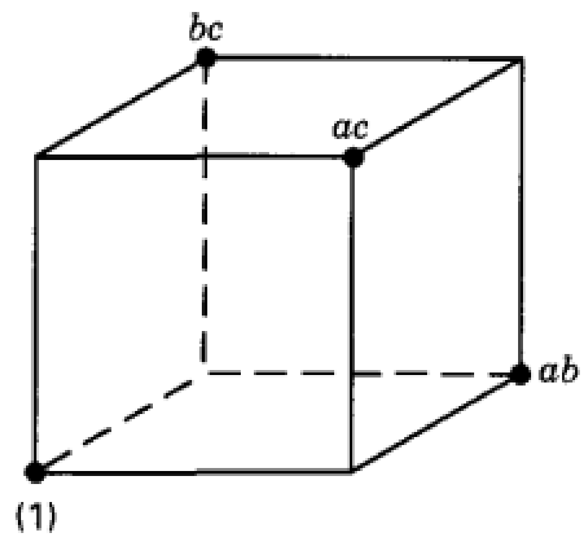
- If reasonable assumptions can be made that certain high-order interactions are negligible, then fractional factorial designs prove to be very effective
- A major use of fractional factorial is in “screening experiments” (eg to identify those factors that have large effects)
- It is based on the principle that when there are several variables, the system or process is likely to be driven primarily by some of the main effects and low-order interactions
- It is possible to combine the runs of two or more fractional factorial to assemble sequentially a larger design to estimate the factor effects and interactions of interests

Fractional Factorial Approach



(a) The principal fraction, $I = +ABC$

What are the effect of A, B, C?



(b) The alternate fraction, $I = -ABC$

What are the combined effects of AB, BC, CA?

Fractional Factorial Designs: Selecting experiments

Table 8-1 Plus and Minus Signs for the 2^3 Factorial Design

Treatment Combination	Factorial Effect							
	I	*A*	*B*	*C*	*AB*	*AC*	*BC*	*ABC*
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
c	+	−	−	+	+	−	−	+
abc	+	+	+	+	+	+	+	+
ab	+	+	+	−	+	−	−	−
ac	+	+	−	+	−	+	−	−
bc	+	−	+	+	−	−	+	−
(1)	+	−	−	−	+	+	+	−

Fractional Factorial Designs: Selecting experiments

Treatment Combination
(1)
ad
bd
ab
cd
ac
bc
abcd

Alias Structure
$\ell_A \rightarrow A + BCD$
$\ell_B \rightarrow B + ACD$
$\ell_C \rightarrow C + ABD$
$\ell_D \rightarrow D + ABC$
$\ell_{AB} \rightarrow AB + CD$
$\ell_{AC} \rightarrow AC + BD$
$\ell_{AD} \rightarrow AD + BC$

Guidelines for Designing Experiments

- Recognition of and statement of the problem (eg. is the objective to characterize response or is it understood well enough to be optimized. Or, is the objective to confirm a discovery, stability)
- Choice of factors, levels, and range (eg. are there fixed no. of levels or if there is a range, how many levels to select and how to select so as to represent the whole range)
- Selection of the response variable (eg. Measurement of hardness is a better variable but not easy to measure on each popcorn; On the other hand fraction of fractured popcorn is easy to measure, but not a good representation)

Guidelines for Designing Experiments

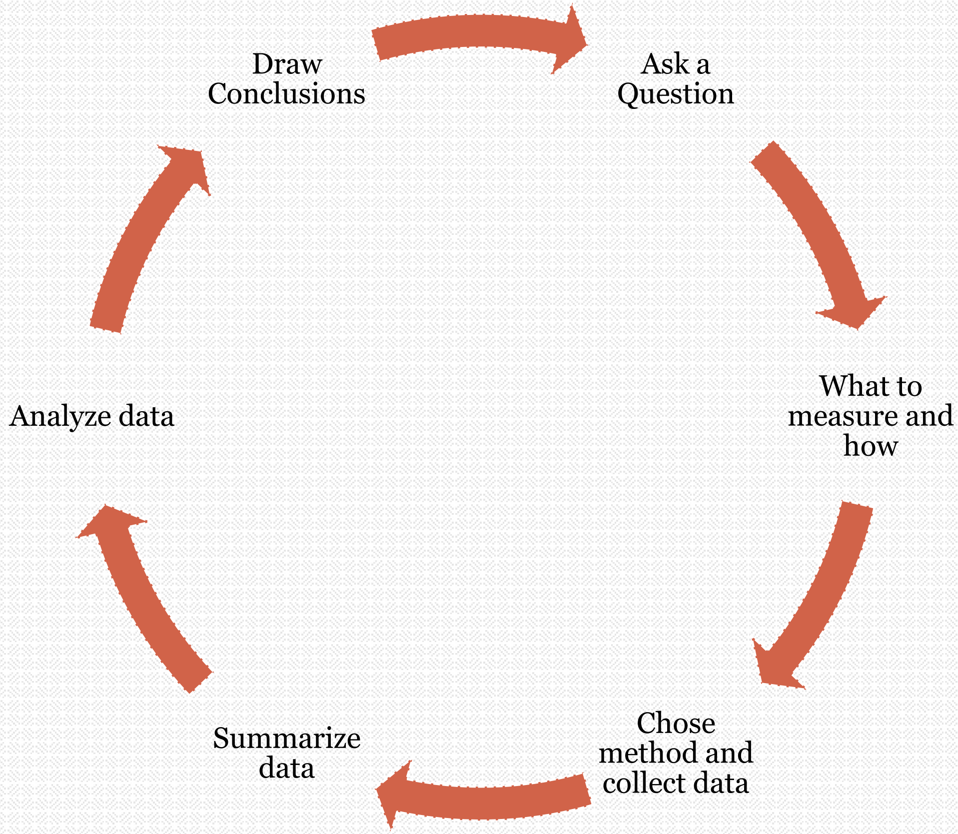
- Choice of experimental design (eg. consideration of sample size, selection of suitable order for experiments, selecting the methodology based on the objectives)
- Performing the experiment (be aware of uncontrollable parameters, sources of errors and other factors that might have been missed earlier. Eg drift in the values of the equipment being used)
- Statistical analysis of the data (what does the data mean. How statistically significant or insignificant is a particular factor)

Data Presentation

DR. SHASHANK SHEKHAR
MSE, IIT KANPUR
FEB 19^{TH} 2016

TEQIP (IIT KANPUR)

Data Analysis

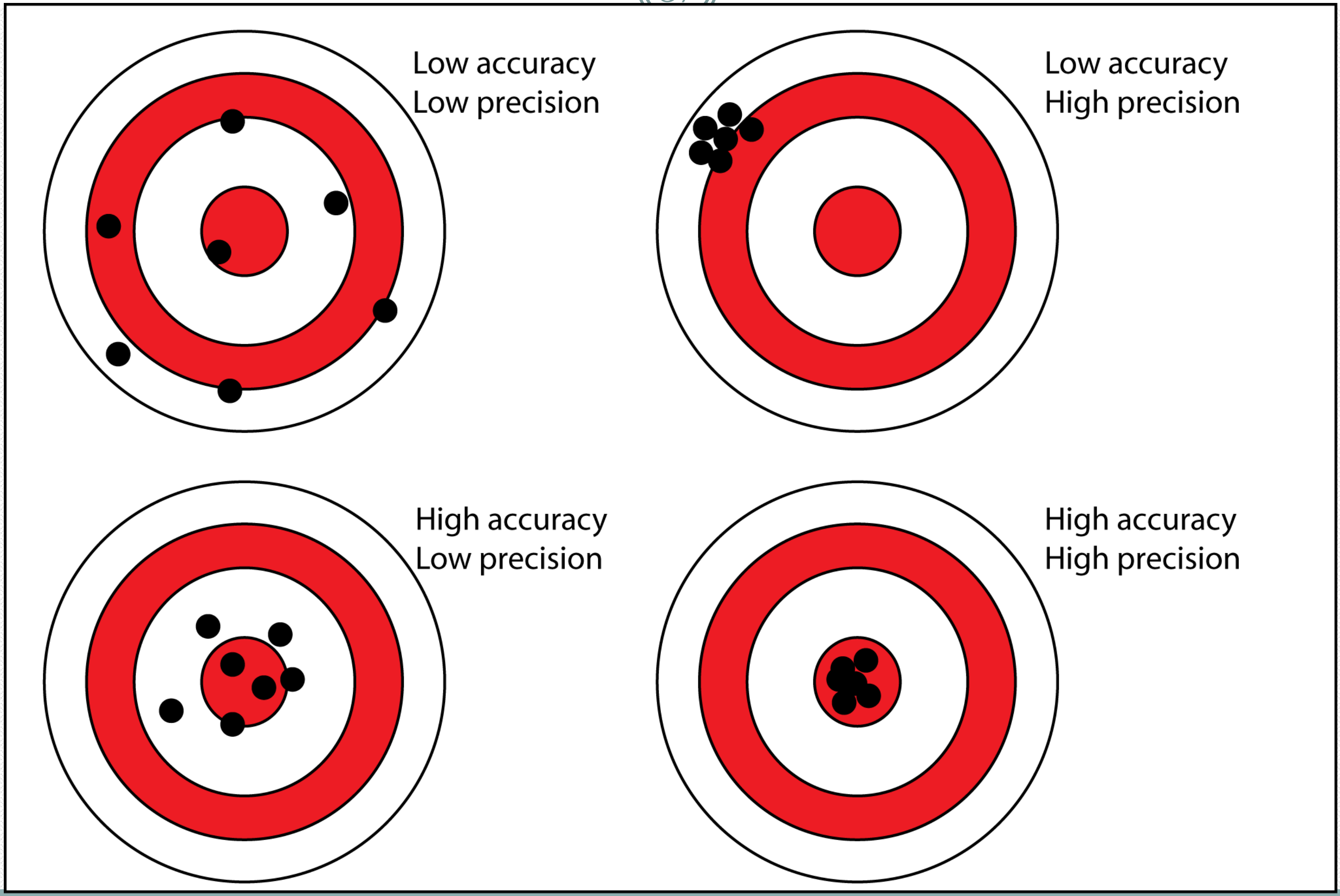


https://www.bcps.org/offices/lis/researchcourse/statistics_role.html

List of Topics

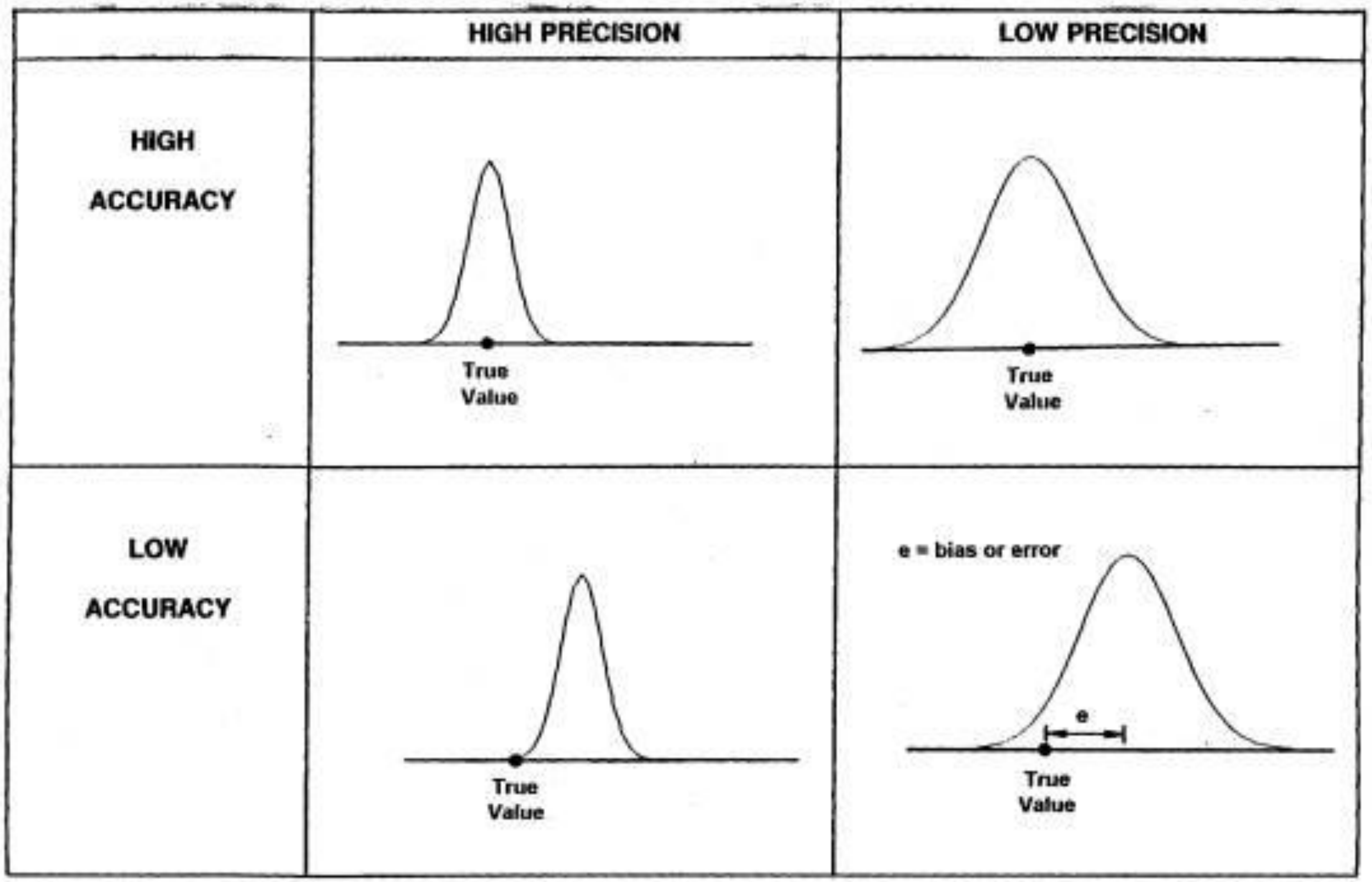
- Graphical and other means of presenting data
- Graphical Summary
  - Plots
  - Histograms
- Numerical Summary (Mean, Median, Mode etc)
- Measures of spread of data
  - Variance and Standard deviation
  - Quantifying spread
  - Chebyshev's Inequality
- Standard Deviation versus Standard Error

Accuracy vs Precision



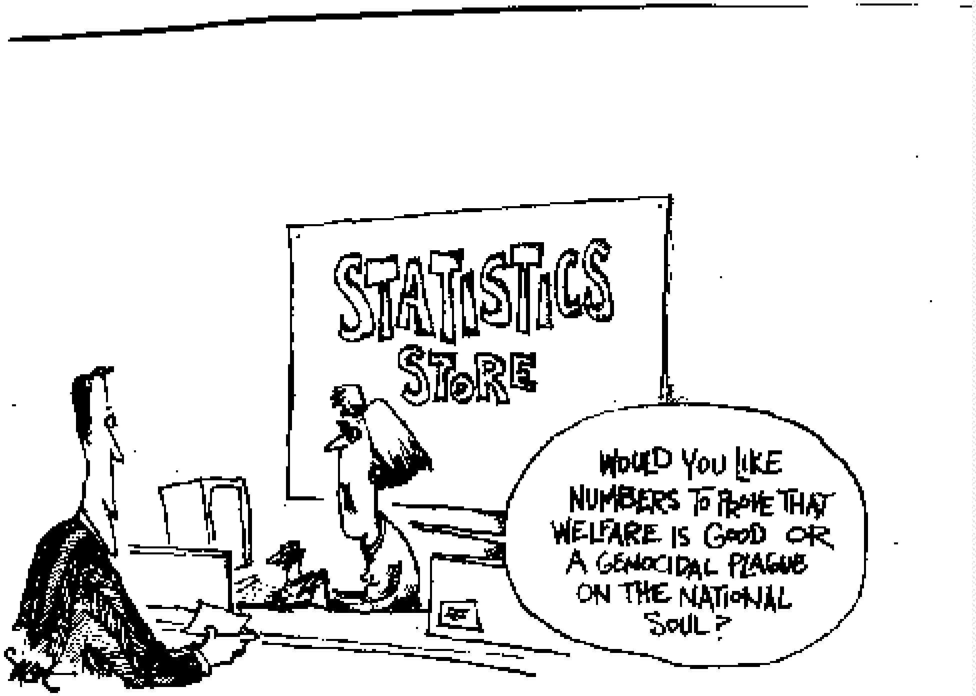
Source: https://sites.google.com/a/apaches.k12.in.us/mr-evans-science-website/accuracy-vs-precision

Accuracy vs Precision



Statistics

- Why use Statistics?



By Signe Wilkinson, Philadelphia Daily News, Cartoonists & Writers Syndicate

Get informed
Evaluate credibility of information
Make appropriate decisions

Some interesting videos on Statistics at: https://vimeo.com/113449763

 https://www.bcps.org/offices/lis/researchcourse/statistics_role.html

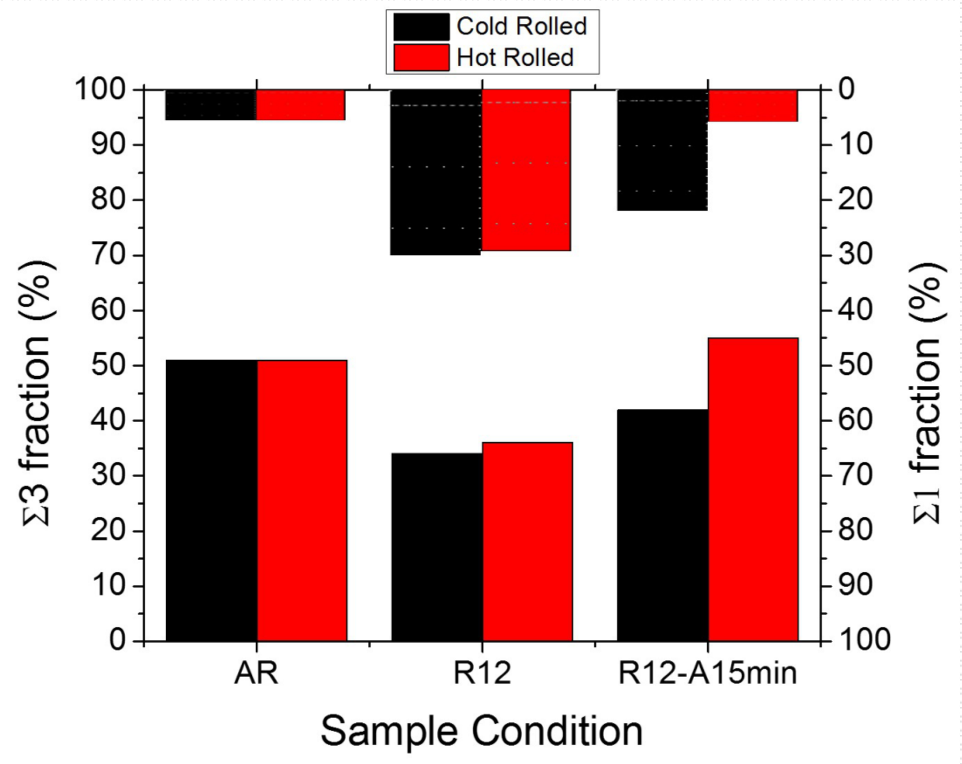
Statistics

- Why use Statistics?
- Data Set: A collection of observations
  - Population vs Sample
- Variable: A characteristic of the object
  - Univariate (height) versus Multivariate (height, weight, race…)
  - Numerical
    - Discreet (No. of employees; No. of grains)
    - Continuous (weight of boxer; Length or area of twin boundary)
  - Categorical
    - Ordinal (1st class, 2nd class, 3rd class railway coaches; Course No. MSE201, MSE301 etc)
    - Not-ordinal (Process condition-1, Process condition-2)

Summarizing data

- Comprehension in exchange of losing data
- Graphical Summary
  - Categorical variable → bar charts, pie charts
  - How not to construct charts
  - Numerical variables → Guidelines to making plots
- Numerical Summary
  - Mean (population versus sample)
  - Median
  - Mode
  - Point estimate of μ

Graphical Summary: Categorical Variable



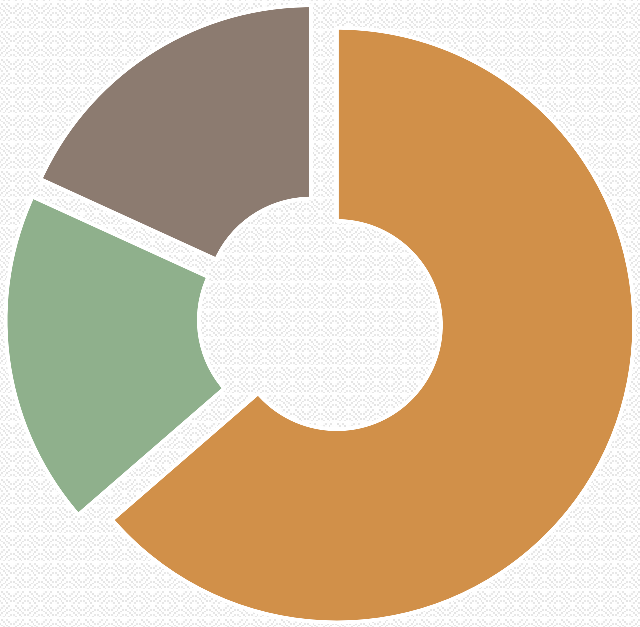
No. of Students

B. Tech. - 70

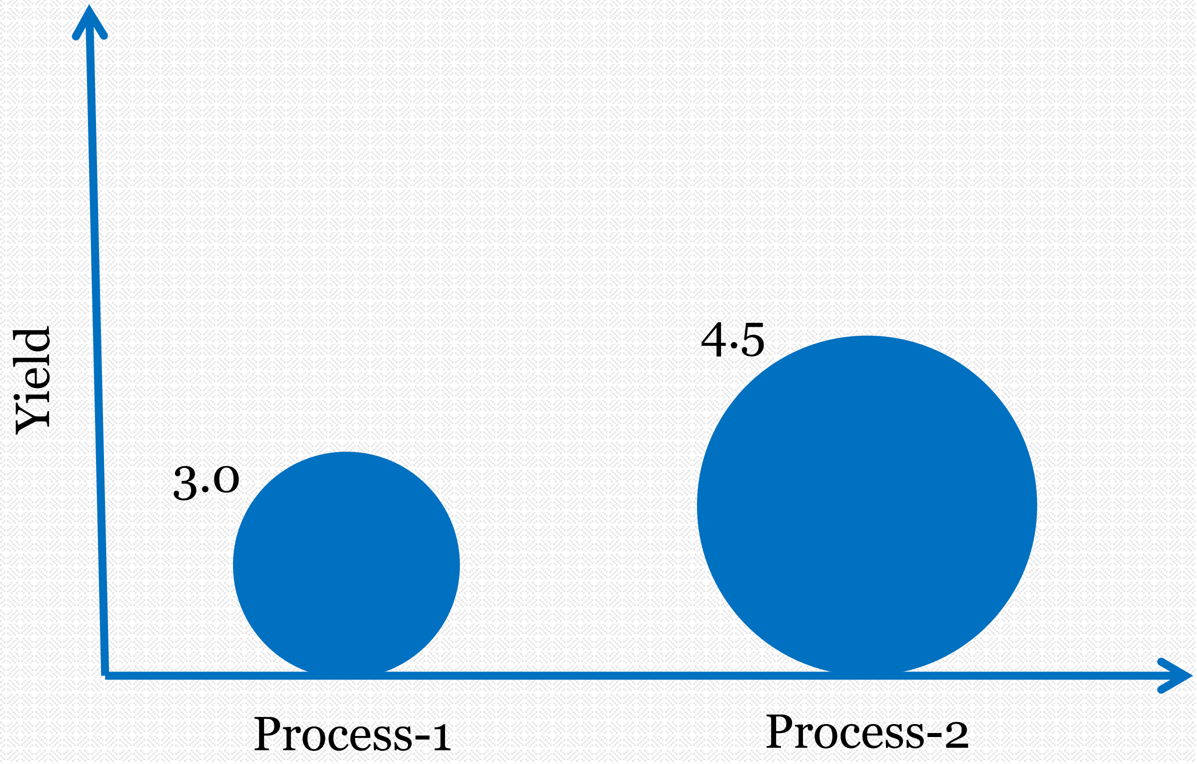
Dual Degree - 20

M. Tech. - 20

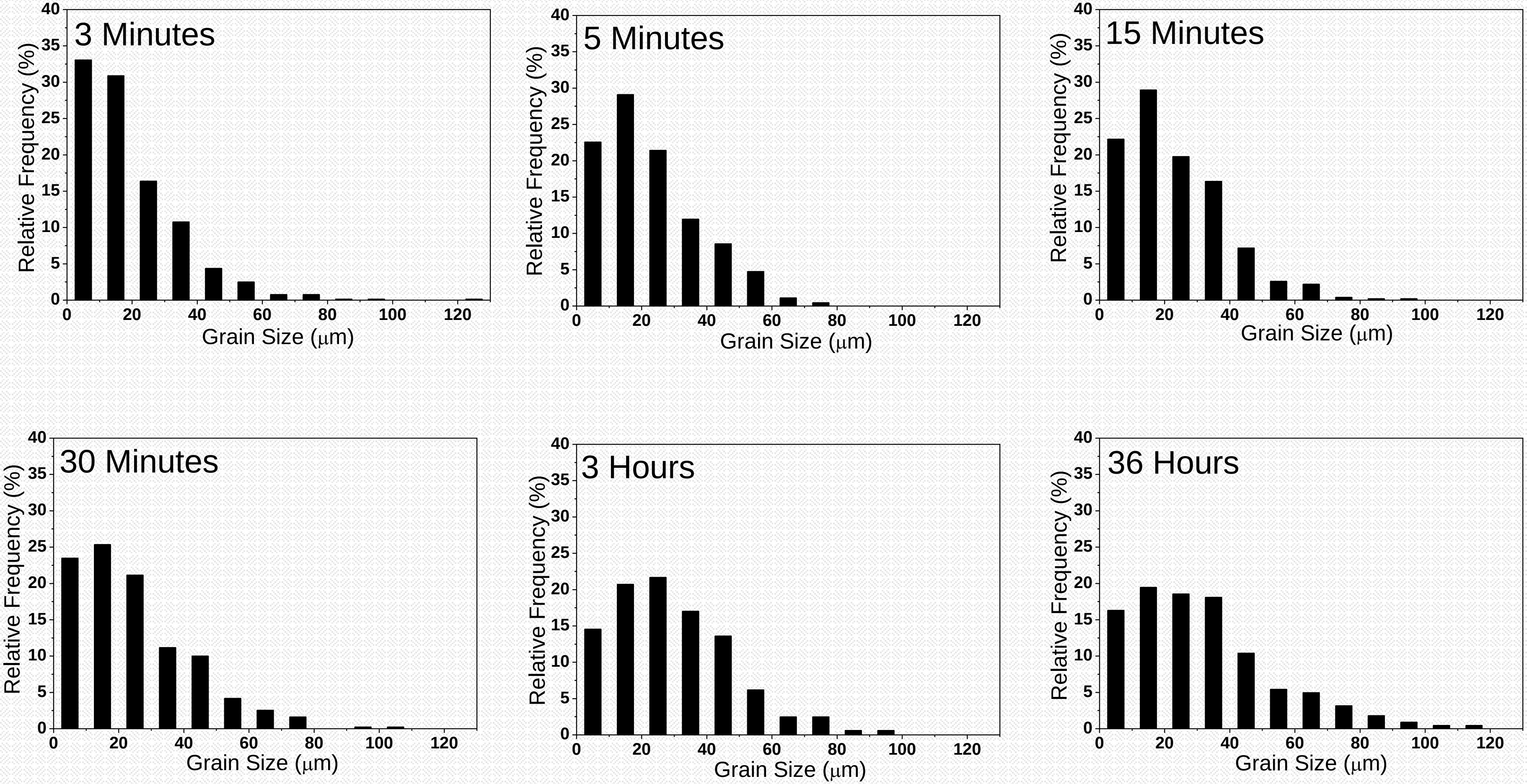
MSE



Graphical Summary: Categorical Variable

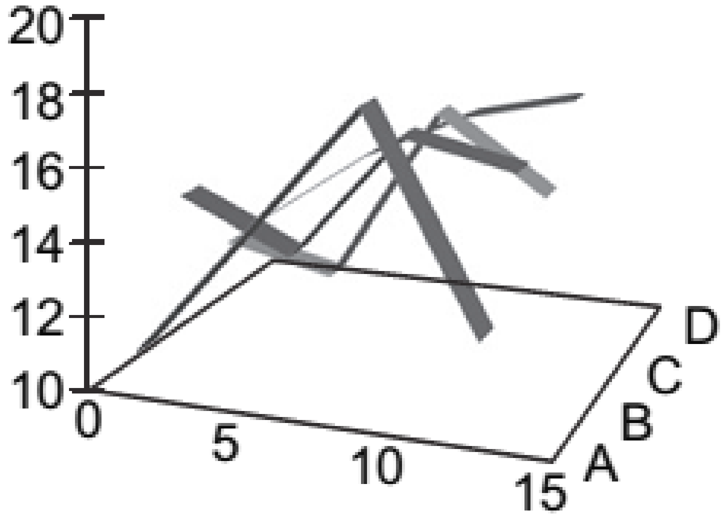


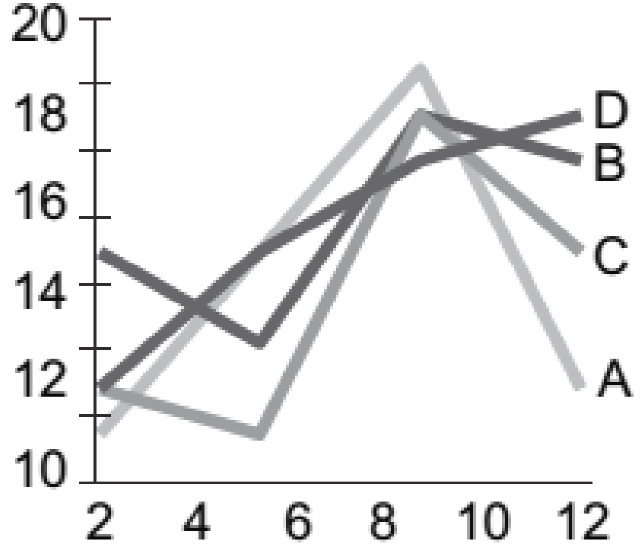
Graphical Summary: Numerical Variable



Guide for effective data presentation

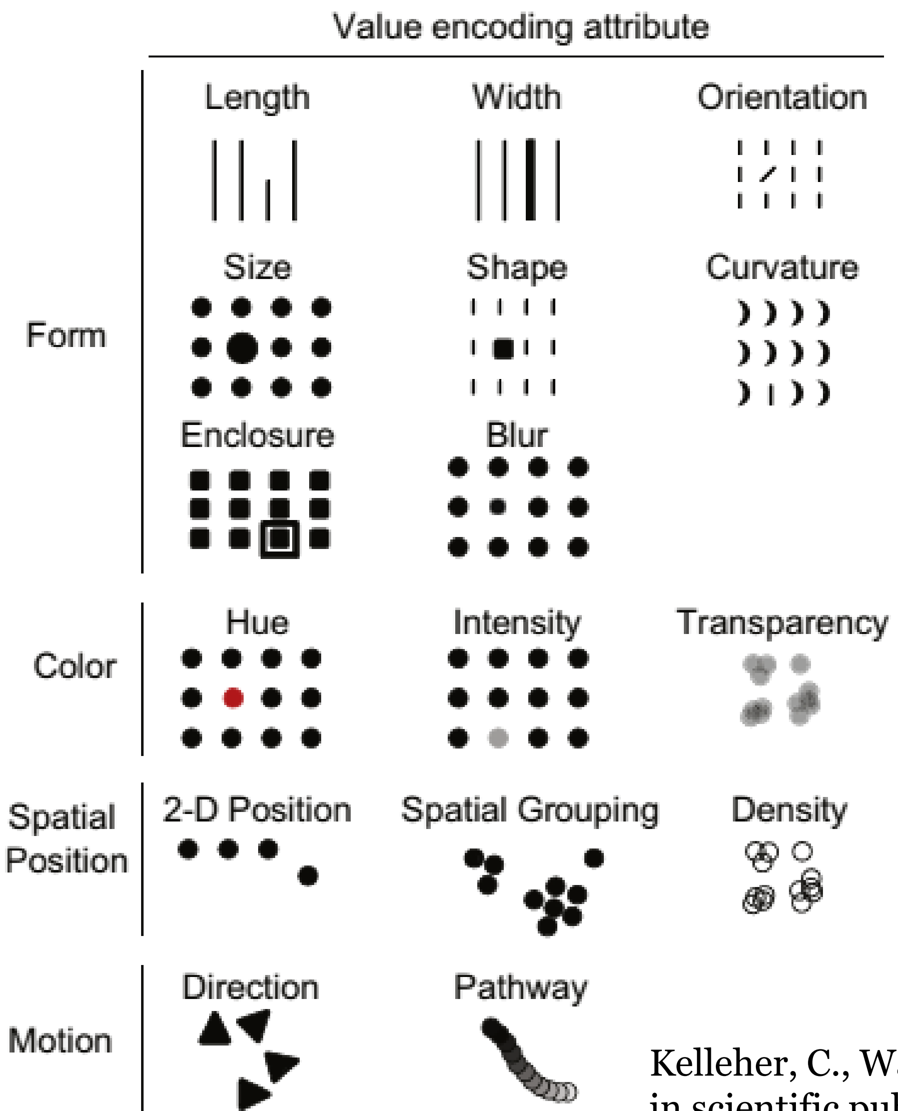
- Create the simplest graph that conveys the information (principle of less-ink)





Kelleher, C., Wagener, T., Ten guidelines for effective data visualization in scientific publications, Environmental Modelling & Software (2011)

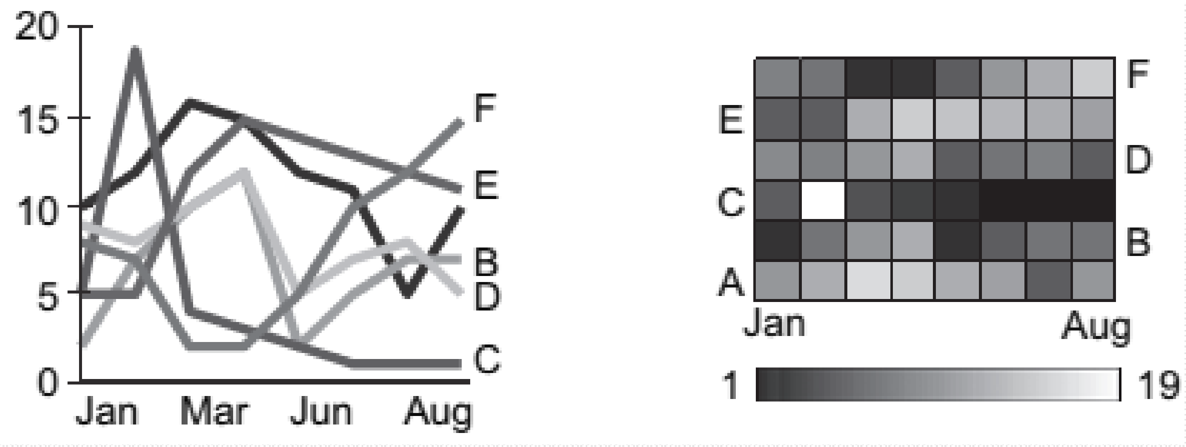
What attribute to use



- For quantitative information length and position should be used
- Qualitative information can be given by transparency, intensity, size etc.

Kelleher, C., Wagener, T., Ten guidelines for effective data visualization in scientific publications, Environmental Modelling & Software (2011)

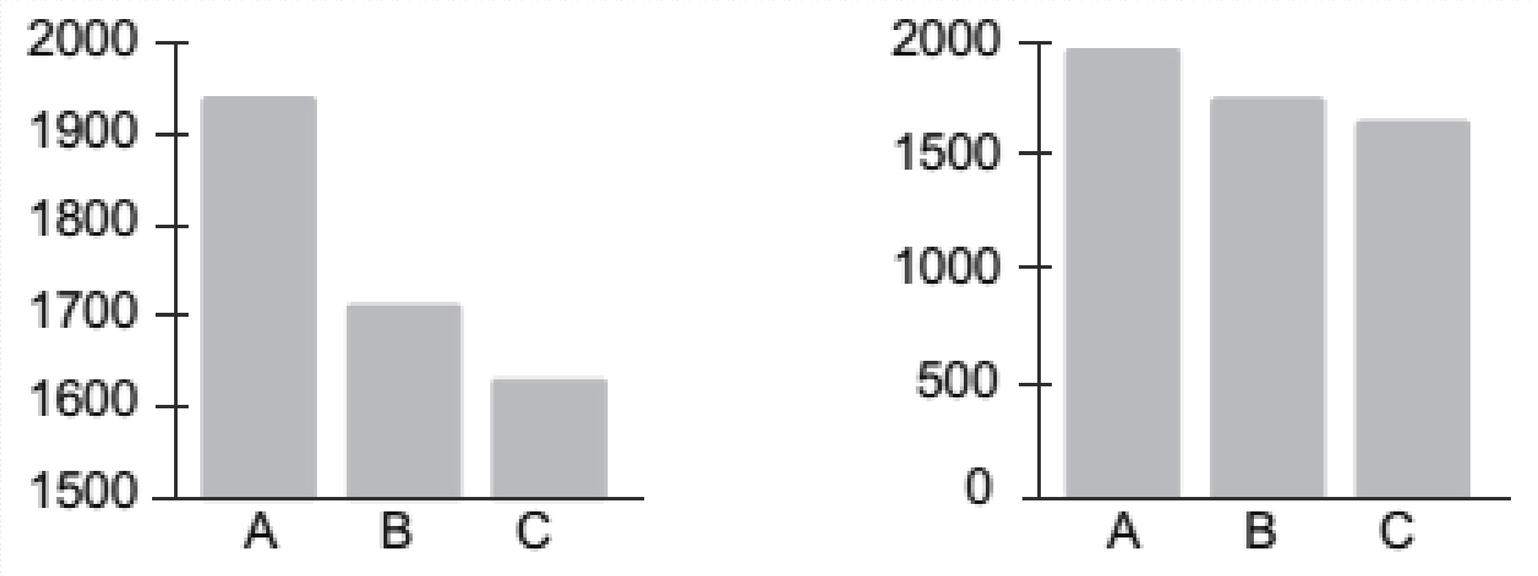
What is important pattern or detail?



- At times, it may be important to display the pattern of variation and at other times, the exact value or detail may be important
- Patterns are best represented by heat-map or bubble maps while details are always best represented by lines or bar graphs

Kelleher, C., Wagener, T., Ten guidelines for effective data visualization in scientific publications, Environmental Modelling & Software (2011)

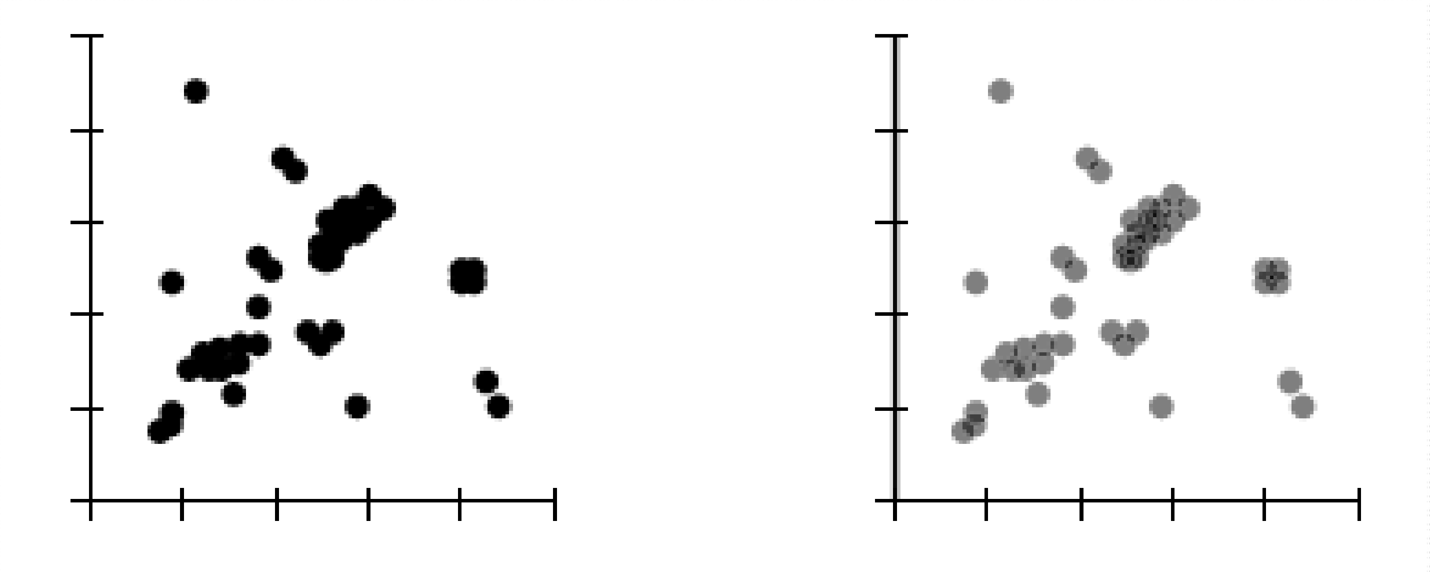
What axis range to select?



- For proper representation and comparison, always select the lowest value to be ‘0’, else it exaggerates the differences

Kelleher, C., Wagener, T., Ten guidelines for effective data visualization in scientific publications, Environmental Modelling & Software (2011)

How to represent scatter plot properly



- Scatter plot may also represent density of data points, hence utilizing transparency attribute may be useful

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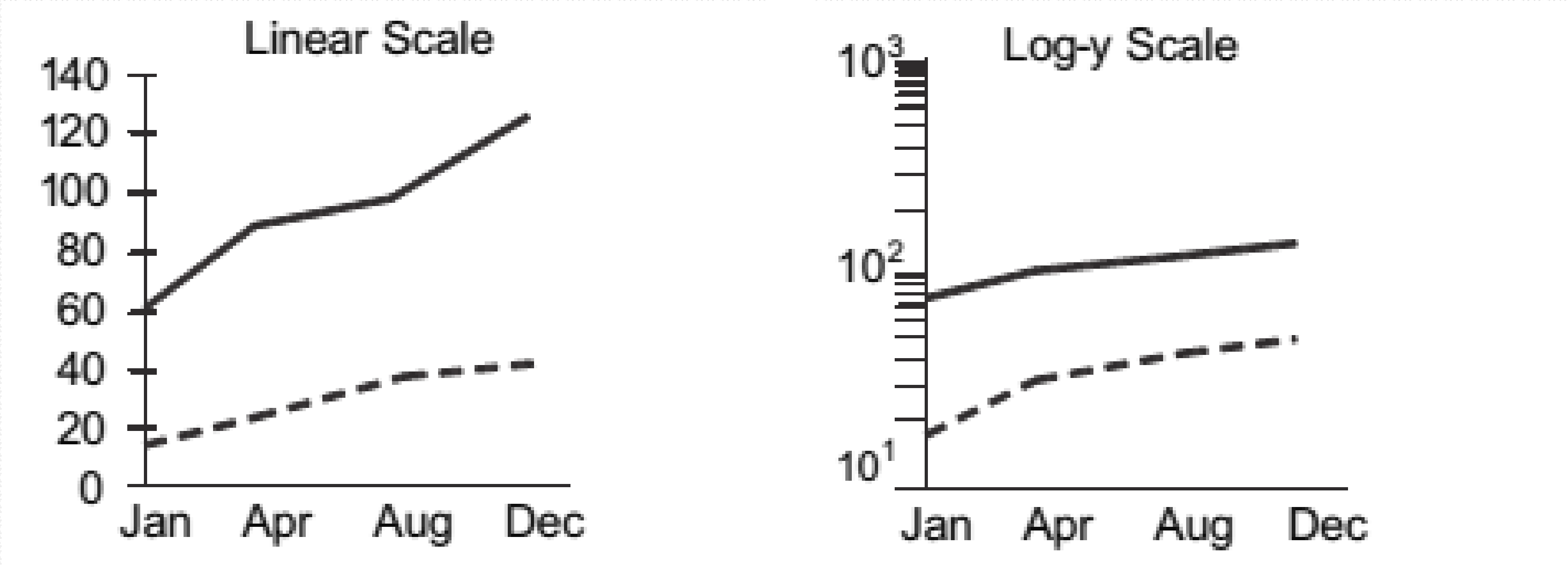
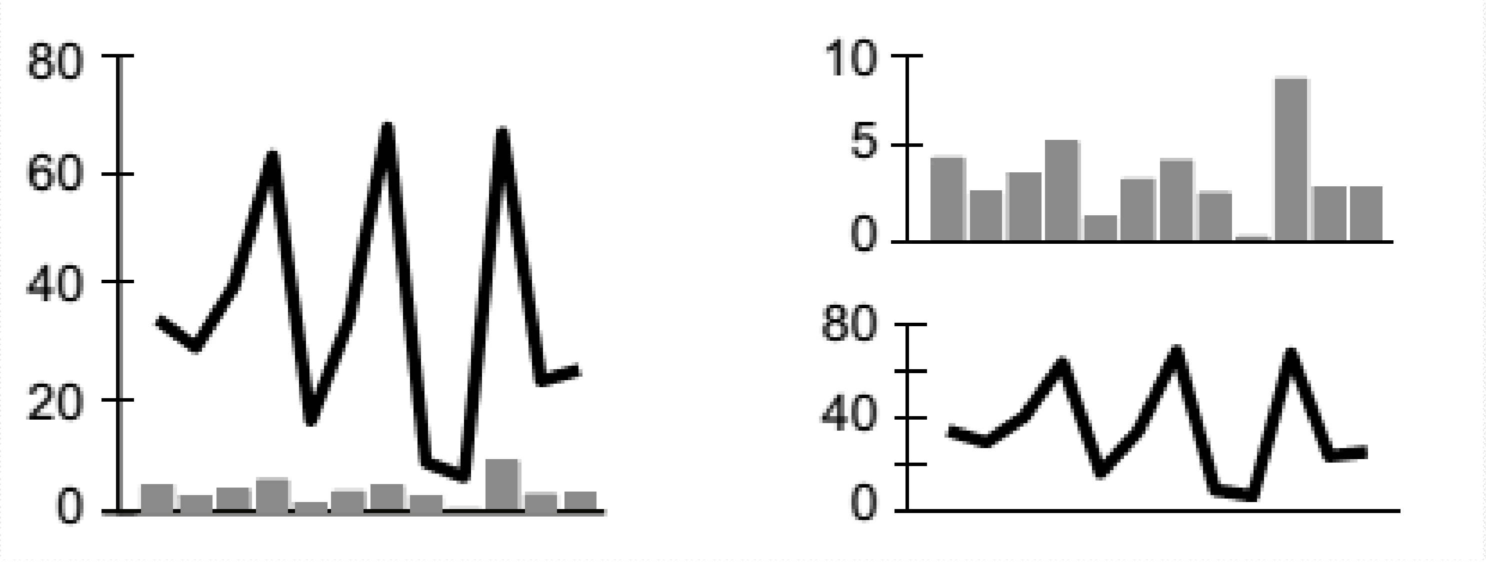
Log scale



- Rate of change with time depends on the use of Y-scale
- Log scale can remove skewness if the dataset contains very large and very small values
- Different transformations are useful under different contexts

Kelleher, C., Wagener, T., Ten guidelines for effective data visualization in scientific publications, Environmental Modelling & Software (2011)

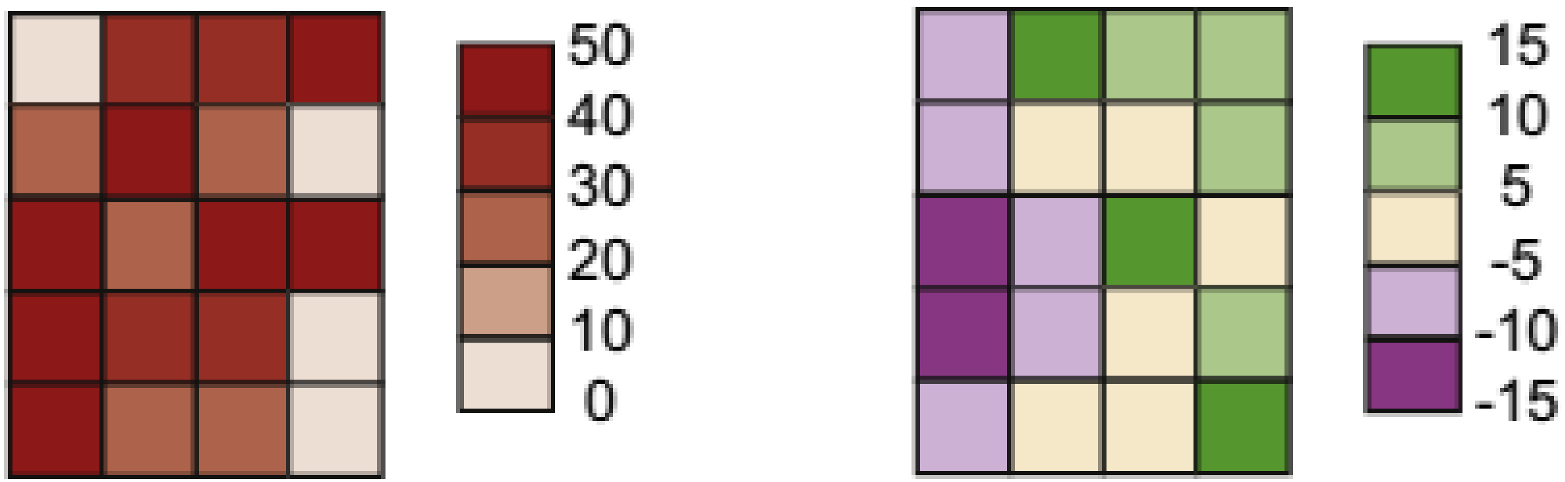
Proper selection of Y-axis



- One may need to select Y-axis properly if you are representing two data sets. One may even use two Y-axis option

Kelleher, C., Wagener, T., Ten guidelines for effective data visualization in scientific publications, Environmental Modelling & Software (2011)

Proper selection of color scheme



- Heat map may be represented in various color scheme
- Selection depends on whether you want to emphasize intensity or diversion

Kelleher, C., Wagener, T., Ten guidelines for effective data visualization in scientific publications, Environmental Modelling & Software (2011)

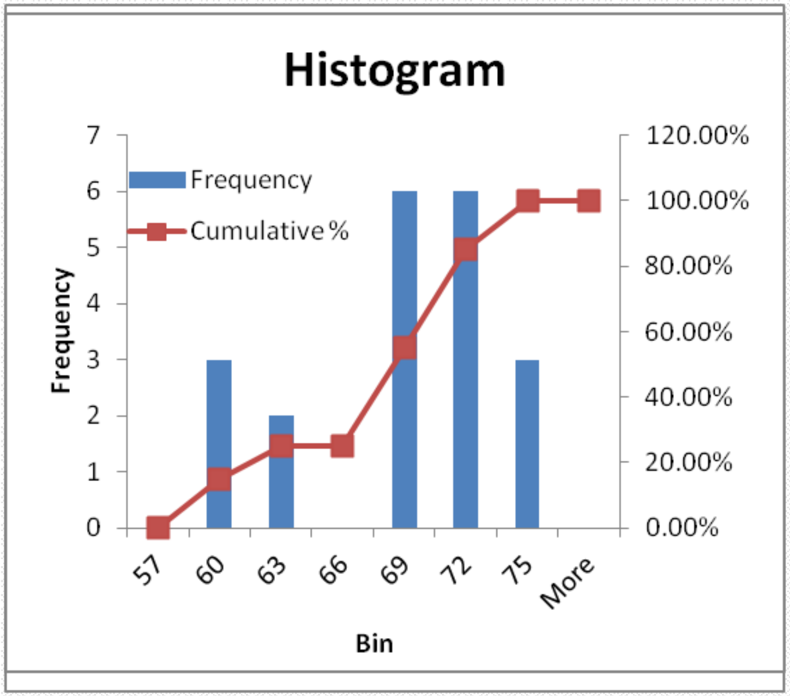
Summarizing data

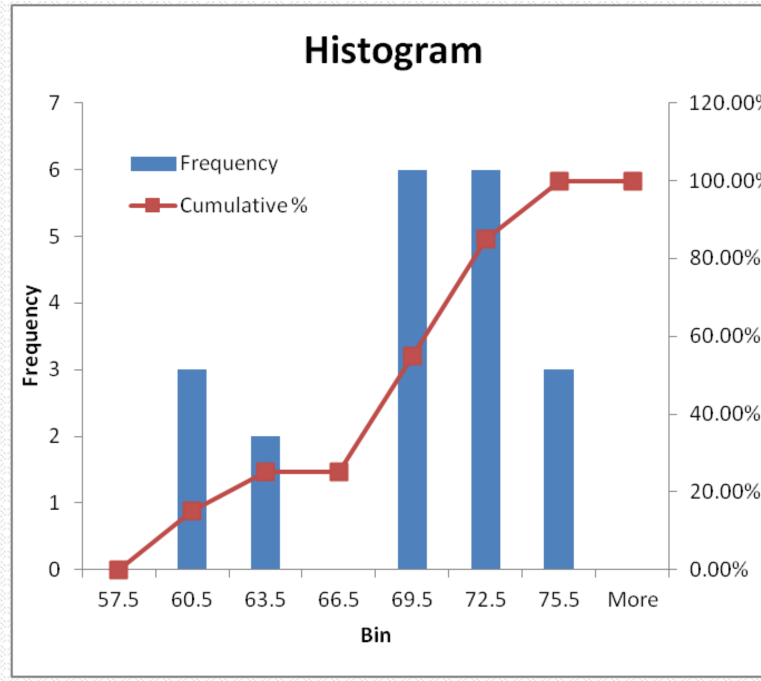
- Rules in constructing a histogram
  - Use limits for intervals that do not coincide with your raw data
  - Recommended that the intervals be of equal width
  - No of intervals: Rice Rule → $2(n^{0.33})$
  - Play with the class limits and the number of intervals to see if the overall shape of your histogram is reasonably stable
- Example in Excel
- Smoothed histogram
- Different types of histograms

Solved Example in Excel

- Height of students in a class (20) are: 59, 60, 60,62, 62, 67, 67, 67, 67, 69, 69, 70, 70, 70, 70, 71, 72, 73, 73, 75 (in inches)
- Using the Rice Rule, for n=20, we get no. of intervals = 5.37. So lets take no. of interval =6. Total range is from 59 – 75. Hence size of each bin =3.
- Now first take limits as 58.5 – 61.5, 61.5 – 64.5 etc.
- Then take limits as 57.5 – 60.5, 60.5 – 63.5 etc.

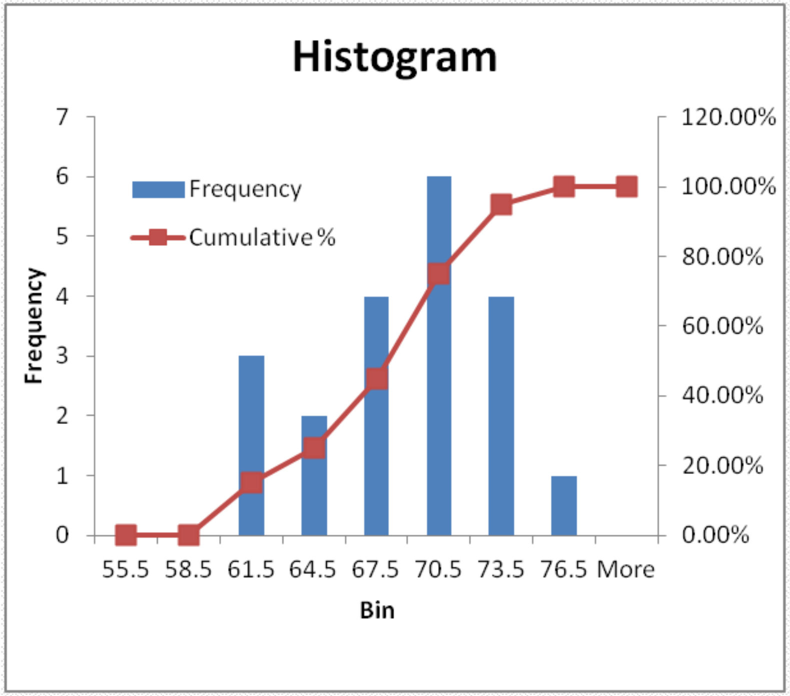
Solved Example in Excel

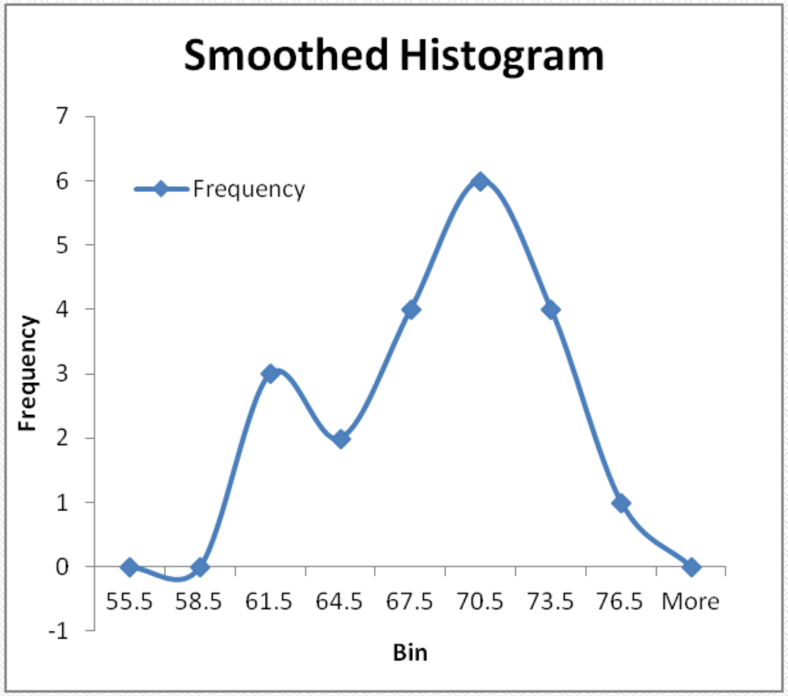




Are these two histogram plots reasonably stable?

Smoothed Histogram





Smoothed histogram or density estimate can be obtained by taking center point of each limit and connecting a curve through the top of these histograms

Numerical Summary

- Mean: average of $x_1, x_2 \ldots. x_n$

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n}$$

- Mean is greatly influenced by outliers → tendency to ignore outliers. It may be an indication of some interesting underlying phenomena
- Median: Right in the middle of observations
- Mode: Where frequency is highest

Example

- Height of students in a class (20) are: 59, 60, 60,62, 62, 67, 67, 67, 67, 69, 69, 70, 70, 70, 70, 71, 72, 73, 73, 75 (in inches)
- Find the mean (μ) of the class (population)
- Height of 5 students in front row (sample) are: 59, 62, 69, 69, 70
- Find the mean of the sample ($\bar{x}$)
- Mean is greatly influenced by outliers (add a student of height 42 inch)
- Median = (69+69)/2 = 69
- Mode = 67, 70 (70.5)

Measures of spread

- Different data set with same mean and median
  - Dataset A: -2, -1, 0, 1, 2
  - Dataset B: -10, -5, 0, 5, 10

$\bar{x}(A)=0; \quad s(A)=1.55$
$\bar{x}(B)=0; \quad s(B)=7.9$

- Inter-quartile range (Q3-Q1)
- Range (max-min)
- Standard deviation and variance (s.d. = √variance)
- Population vs Sample standard deviation

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{n} \qquad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Basic properties of mean and s.d.

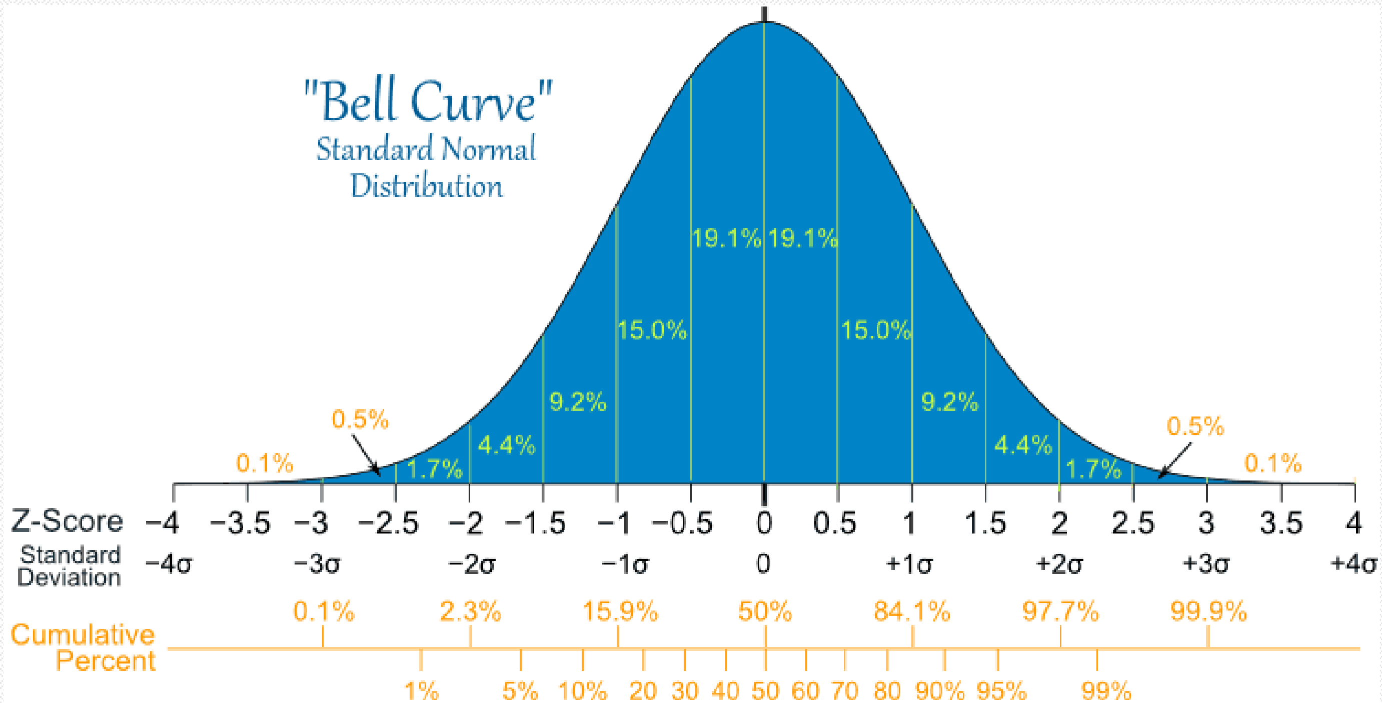
If $x_1, x_2 \ldots x_n$ have mean = $\bar{x}$ and s.d. = s, then for

- $x_1+k, x_2+k \ldots x_n+k$, mean = $\bar{x}+k$ and s.d. = s
- $cx_1, cx_2 \ldots cx_n$, mean = $c\bar{x}$ and s.d. = $|c|s$
- $cx_1+k, cx_2+k \ldots. cx_n+k$, mean = $c\bar{x} + k$ and s.d. = $|c|s$

Quantitative meaning of variance

- For normal distribution, data proportion within '±*z*' standard deviation is $erf(\frac{z}{\sqrt{2}})$

z	% data
0.5	38.3
0.674	50
1	68.27
2	95.45
3	99.73

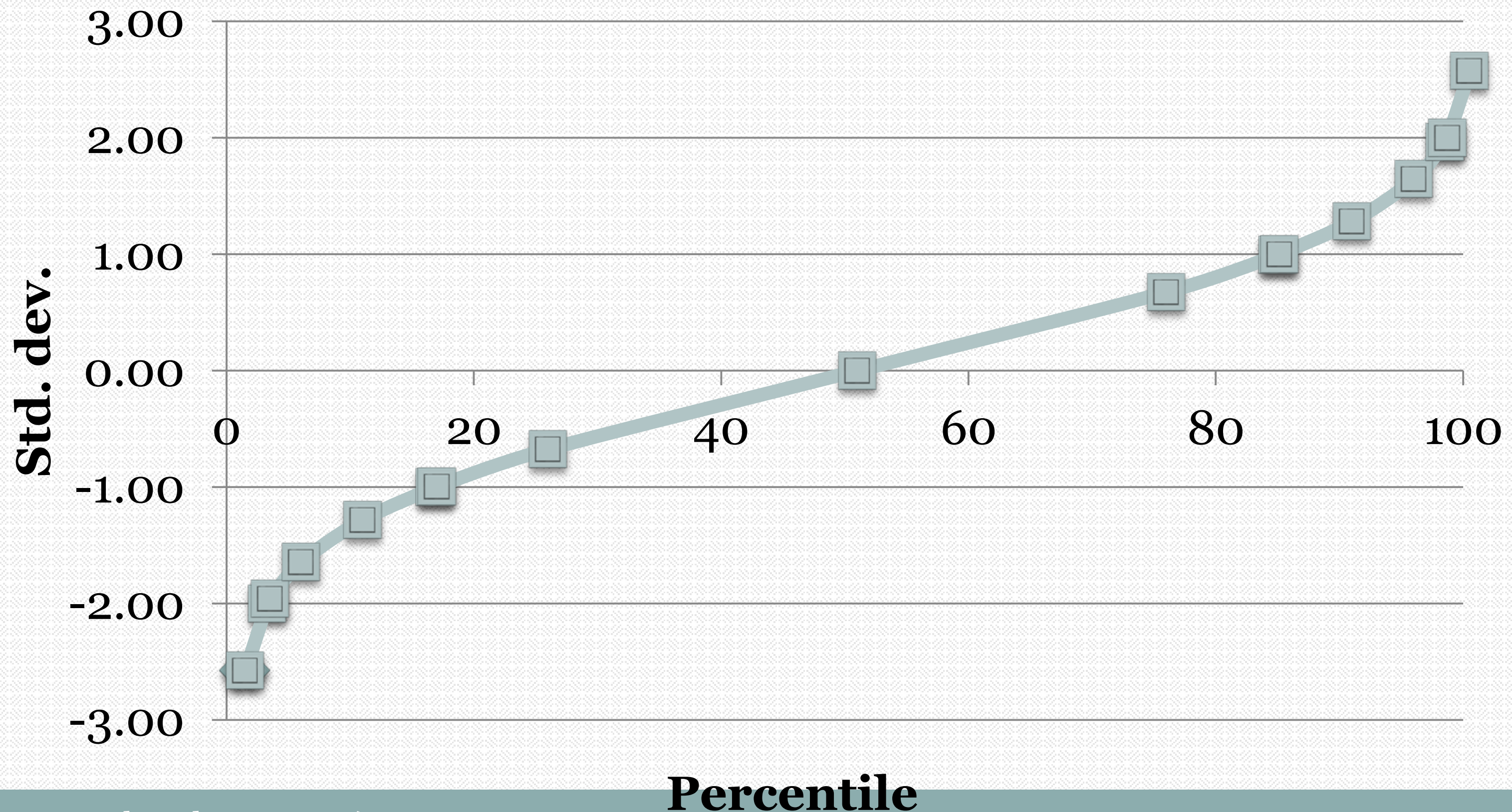


What if the data is not normally distributed? We only know $\bar{x}$ and s

www.mathsisfun.com

Quantitative meaning of variance

std-dev vs percentile data



Quantitative meaning of variance

- Chebyshev's inequality: $\bar{x} \pm e.s$ range must capture at least 100 $(1 – 1/e^2)$% of data

e	At least
1	0
2	75
3	88.89
4	93.75

- Lesser than for normal, but remember it is true for any kind of distribution, including random distribution

Example-2

- Example: Average of a midterm in a class of 55 students is 65 and s.d. =10. Cut-off for A is 85. What can you say about how many students got “A”
- $\bar{x}$ = 65; s = 10; cut-off for A = 85
- How many std. deviations away?
- $\bar{x} \pm e.10 = 85 \Rightarrow e=2 \Rightarrow$ at least 75% data within 65 ± 20 (45-85)
- % students getting more than 85% *is less than* 25% of class (0.25*55 = 13.75)
- Max no. of students getting ‘A’ = 13

Standard Error

- Standard error is the standard deviation of the sampling distribution of mean
- Different samples drawn from the same population would in general have different values of the sample mean, although there will be a true mean (for a Gaussian distribution)

Std dev versus Std error

- If a measurement which is subject only to random fluctuations, is repeated many times, approximately 68% of the measured values will fall in the range $\bar{x} \pm 1.s_x$
- If you do an experiment multiple no. of times, mean approaches real value. One can repeat the measurements to get more certain about $\bar{x}$
- Hence, a useful quantity is std dev of means (or std error), $S_{\bar{x}} = s_x / \sqrt{N}$

Example-4

- Find, mean, s.d. and s.e. for the given data sets
- Plot using error bars

Class Experiment Analysis

Lets first use data for uncoated sample

- Calculate average for each group
- Calculate average and std. dev. of raw data
- Calculate average and std. dev of mean of each group
- What should be the relation between std. dev of raw data and std dev of means?
- What can you comment on this

Class Experiment Analysis

- Plot histograms for raw data and for means
- What do you see?
- Lets look closer at the raw data
- One of the data point seems outlier
- Plot after removing this. Looks good?
- But, can we remove this data point?
- Average = 7.4; Std. dev.= 4.8
- Outlier = 24; How many std. dev away 3.45
- Can we reject it? 3.875

Class Experiment Analysis

- Now lets look at data for Red clip
- Avg. 34
- Std. Dev. 24.32
- Outlier: 78
- No. of std. dev away: 1.82

Example-4: Plotting Error bars

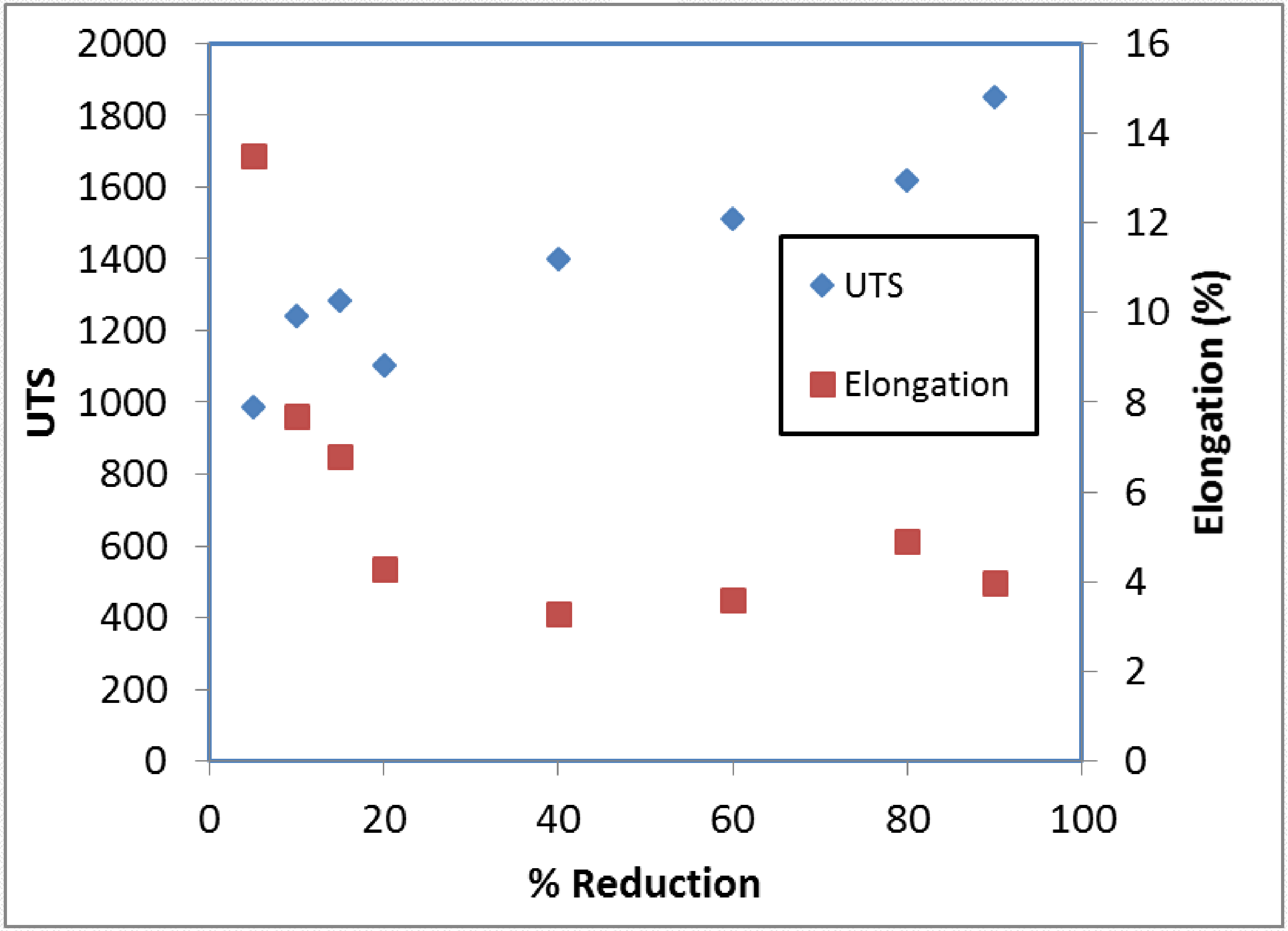
Time (hrs)	hardness-1	error-1	hardness-2	error-2
0	158.0	3.5	155.0	5.3
0.5	146.9	3.2	100.6	10.3
1	137.7	7.3	85.4	3.8
2	105.9	19.6	76.4	2.6
3	122.3	17.0	75.5	-
4	93.7	15.1	74.5	4.1
6	78.2	2.0	73.5	3.1

Example-5: Double y-axis

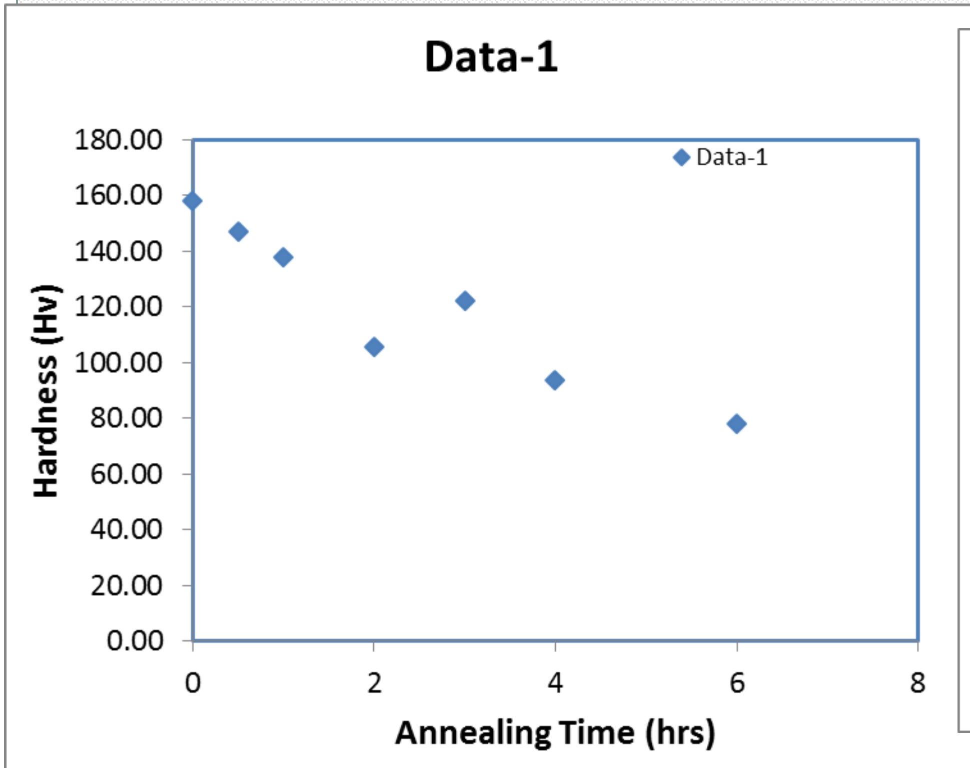
% reduction	UTS (Mpa)	Elongation
5	988	13.5
10	1239	7.7
15	1285	6.8
20	1102	4.3
40	1402	3.3
60	1511	3.6
80	1620	4.9

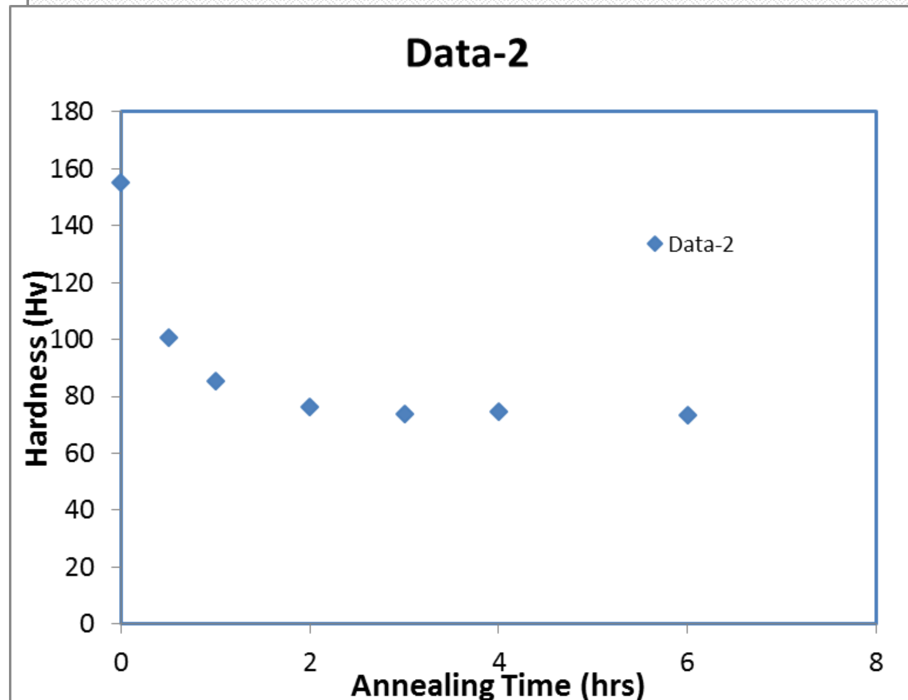
How to plot two different characteristics on one same plot?

Example-5


UTS
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
20
40
60
80
100
% Reduction
16
14
12
10
8
6
4
2
0
Elongation (%)
UTS
Elongation

Example-4 (Contd)





Summary

- Data presentation may look like a mundane task, but it involves a lot of intricacies
- The sole objective of data presentation should be to convey the full picture to the viewer without hiding any information
- Effective data presentation ensures that maximum information is conveyed in minimum 'ink'

Questions

Robust Parameter Design

Outline

- Taguchi Design of Experiments
- Robust Parameter Design
- Signal-to-Noise (SN) ratios
- The testing problem of the equality for two SN ratios

Taguchi Design of Experiments

- Robust Parameter Design, also called the Taguchi Method pioneered by Dr. Genichi TAGUCHI, greatly improves engineering productivity.
  - Comparable in importance to Statistical Process Control, the Deming approach and the Japanese concept of TQC
- Robust Parameter Design is a method for designing products and manufacturing process that are robust to uncontrollable variations.
  - Based on a Design of Experiments (Fisher's DOE) methodology for determining parameter levels
- DOE is an important tool for designing processes and products
  - A method for quantitatively identifying the right inputs and parameter levels for making a high quality product or service
- Taguchi approaches design from a robust design perspective

The Taguchi Approach to DOE

- Traditional Design of Experiments (Fisher's DOE) focused on how different design factors affect the ***average*** result level
- Taguchi's DOE (robust design)
  - *Variation* is more interesting to study than the average
  - Run experiments where controllable design factors ***and*** disturbing signal factors take on 2 or 3 levels.

Robust Design (I)

- By consciously considering the noise factors and the cost of failure in the Taguchi method helps ensure customer satisfaction.
  - Environmental variation during the product's usage
  - Manufacturing variation, component deterioration
- Noise factors (Disturbances) are events that cause the design performance to deviate from its target values
- A three step method for achieving robust design
  1. Concept design
  2. Parameter design
  3. Tolerance design
- The focus of Taguchi is on Parameter design

Robust Design (II)

Robust Parameter Design (e.g. Wu and Hamada 2000)

- A statistical / engineering methodology that aim at reducing the performance "variation" of a system.
  - The selection of control factors and their optimal levels.
- The input variables are divided into two board categories.
  - Control factor: the design parameters in product or process design.
  - Noise factor: factors whoes values are hard-to-control during normal process or use conditions
- The "optimal" parameter levels can be determined through experimentation

Signal to Noise (SN) Ratios (I)

Signal to Noise (SN) Ratios :

Performance measures (the nominal-the-best characteristic) in the most common robust parameter design

Performance measure

⇕

Taguchi's SN ratio

The product/process/system design phase involves deciding the best values/levels for the control factors. Taguchi's SN ratio for the parameter design problem is

$$\eta_T = 10 \log_{10} \left(\frac{\mu^2}{\sigma^2} \right) = -10 \log_{10} \gamma^2,$$

where γ is the coefficient of variation σ/μ (Taguchi and Phadke (1984)).

Signal to Noise (SN) Ratios (II)

SN ratios for the systems with dynamic characteristics

Signal-response systems or measurement systems to evalute the relationship of a signal M and a response y.

Model (a simple linear regression):

$$y = \beta M + \varepsilon,\ \ \varepsilon \sim N(0, \sigma^2),$$

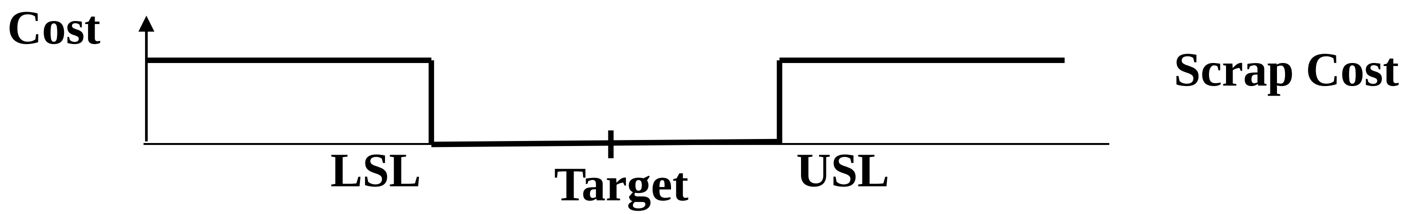
A performance measure

$$\eta = \frac{\beta^2}{\sigma^2} \quad \text{or} \quad \tilde{\eta} = 10 \log_{10} \left(\frac{\beta^2}{\sigma^2} \right) \ [\text{dB}]$$

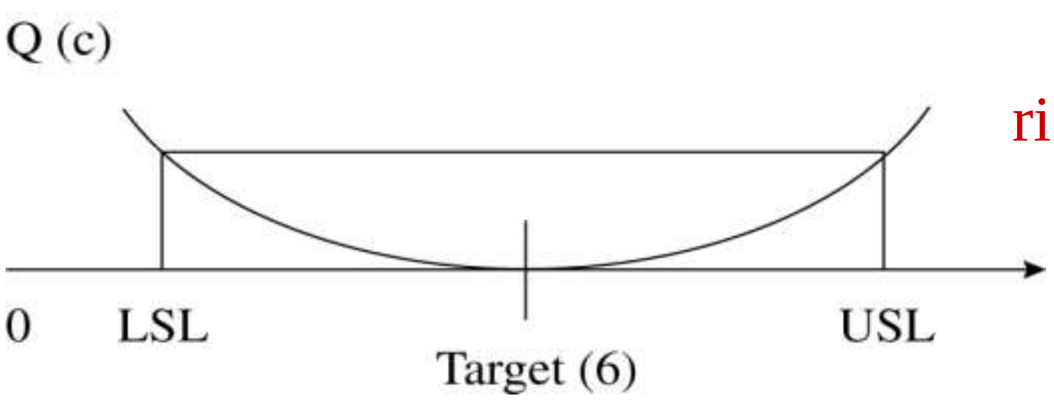
to evalute the systems with dynamic characteristics are called the dynamic SN ratio.

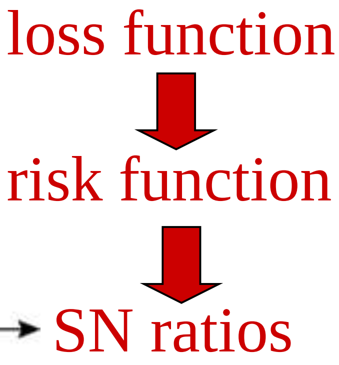
The Taguchi Quality Loss Function

- The traditional model for quality losses
  - No losses within the specification limits!



- The Taguchi loss function
  - the quality loss is zero only if we are on target





A new performance measure (I)

However, if the adopted principles of the signal-response systems are diffent and the physical quantities of the response values are different between the systems, the comparison of the Taguchi's SN ratios has no sense.

A new performance measure for the systems :

- We propose a dimensionless SN ratios (Kawamura et al. 2006).
  - Proportional model, K loss function, Dynamic SN ratios
  - The response and the signal factor values are positive real values.

A new performance measure (II)

The response and the signal factor values are positive real values.

Consider two-parameter statistical models for positive continuous observation.

- Log normal distribution
- Gamma distribution
- Inverse Gaussian distribution etc.

↑ Error distribution

A new performance measure (III)

Next, we define the dynamic characteristics SN ratios of the zero-point proportional model. For a signal factor M (given) and measurement characteristic Y (both positive)

$$Y = \beta M \cdot \varepsilon, \quad \varepsilon \sim IG(1, c^2)$$

where β and ε are the coefficient and error, respectively. In this case, the SN ratio is defined as the loss of $Y/(\beta M)$ for an arbitrary M :

$$E\left[\left(\sqrt{\frac{Y}{\beta M}} - \sqrt{\frac{\beta M}{Y}}\right)^2\right] \equiv c^2$$

K loss function

↓

K risk function

and the population SN ratio of the zero-point proportional model is assumed to be

$$\gamma_K = \frac{1}{c^2} \qquad (*)$$

A new performance measure (III)

We obtain the sample SN ratio for the nominal-the-best characteristic as follows:

$$\gamma_K (\equiv W) = \frac{1}{C^2},$$

where

$$C^2 \equiv \frac{1}{2n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\sqrt{\frac{y_j}{y_i}} - \sqrt{\frac{y_i}{y_j}} \right)^2$$

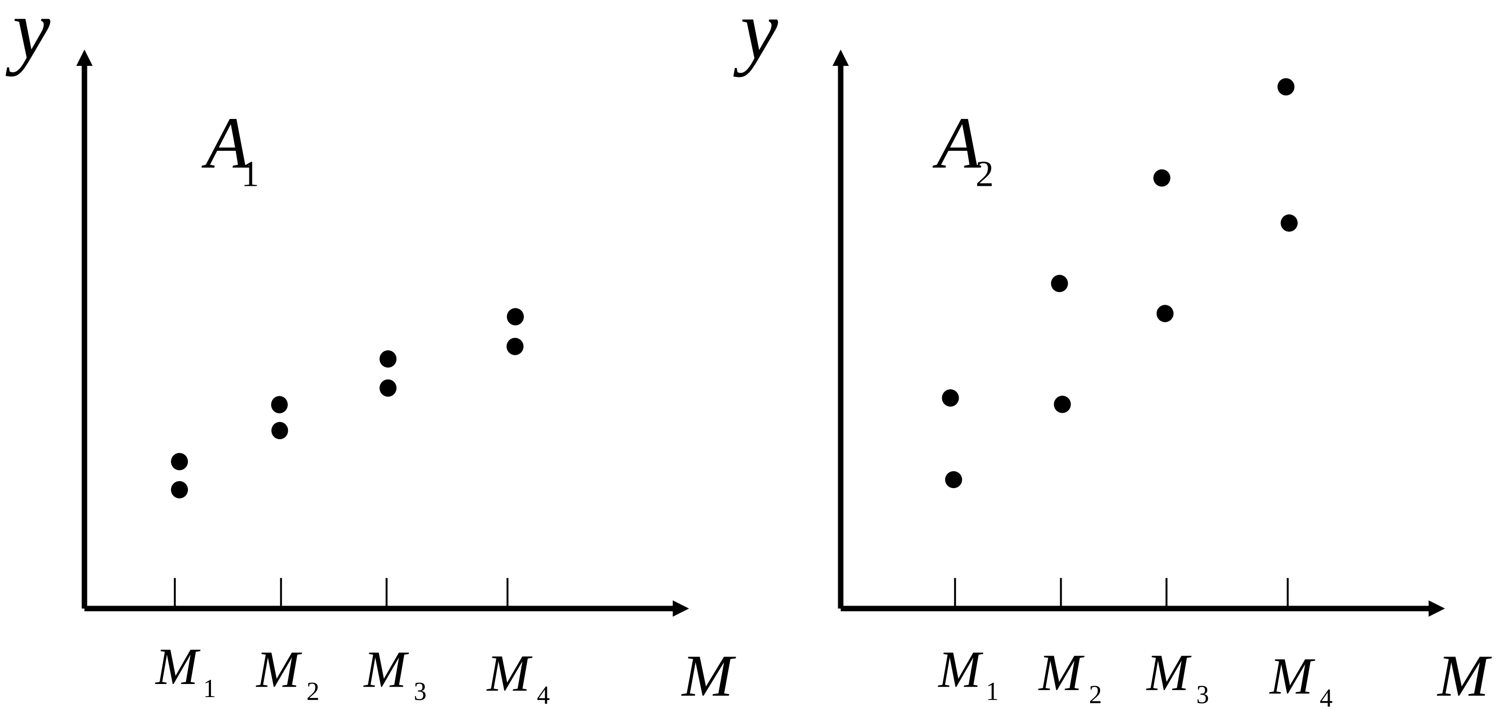
$$= \frac{\bar{y}}{\bar{y}_H} - 1,$$

⬅ Calculation !

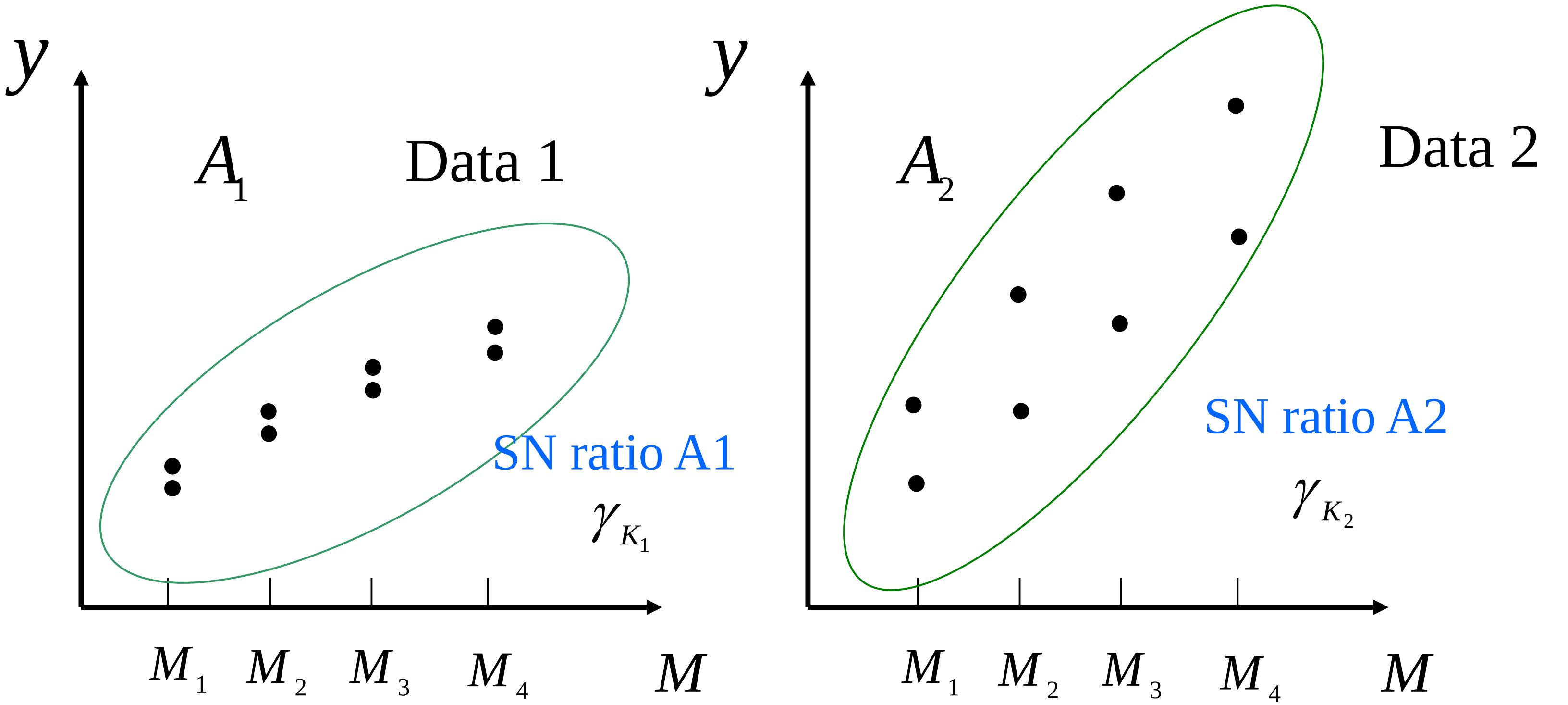
where $\bar{y}$, $\bar{y}_H$ are the sample arithmetic mean and the sample harmonic mean, respectively.

A test of the Equality for two SN ratios (I)

- We consider the testing problem of the equality for two SN ratios
  - SN ratios for the systems with Dynamic Characteristics
  - Performance comparison of the systems



A test of the Equality for two SN ratios (II)



Which performance is good ?

⇨ Testing homogeneity of SN ratios

A test of the Equality for two SN ratios (III)

Cosider the testing problem of the equality for two SN raitos, η_{K_1} and η_{K_2}. The null hypothesis and the alternative hypothesis are

$$H_0 : \eta_{K_1} = \eta_{K_2}$$
$$H_1 : \eta_{K_1} \neq \eta_{K_2}$$

Null hypothesis

respectively. To test these these hypotheses, we construct a following rejection rule from the approximation test:

$$\text{reject } H_0 \Leftrightarrow |\xi(W_1) - \xi(W_2)|/\sqrt{2} \geq z_{\alpha/2},$$

where $z_{\alpha/2}$ is upper $100\alpha/2\%$ point of the standard normal distribution, and $\xi(W_i)$ is

A Variance Stabilizing Transformation

$$\xi(W_i) = \frac{1}{\sqrt{c}} \log\left(2\sqrt{c(a + bW_i + cW_i^2)} + 2cW_i + b\right), \quad i = 1, 2,$$

where $a = 1/(n-3)(n-5)$, $b = 1/(n-5)$, $c = 2/(n-5)$.

↓

Approximation Test

A numerical example (I)

We analyze the data give in Table 1. These data were measured in order to comparare the SN ratios of two methds A_1 and A_2 for microanalysis of cadmium.

- M is a signal factor and M_k means that the cadmium content of a sample is k ppm ($k = 1, 2, \ldots, 5$).
- R_1 and R_2 represent two analysts (replications).

Data :

Percentage of cadmium permeability

		M_1	M_2	M_3	M_4	M_5
A_1	R_1	12.0	23.0	33.0	42.5	52.0
	R_2	11.0	22.5	34.0	44.0	54.0
A_2	R_1	3.5	7.0	10.5	14.5	18.0
	R_2	4.0	7.5	12.0	15.5	19.5

A numerical example (II)

Result :

$$W_1 = \gamma_{A_1} = 663.03 \ (28.22 \text{ dB}),$$
$$W_2 = \gamma_{A_2} = 364.73 \ (25.62 \text{ dB}),$$
$$a = 0.0286,, \ b = 0.2, \ c = 0.4$$
$$\xi(W_1) = 10.644, \ \xi(W_2) = 9.760$$

From these values, we obtain the values of test statistic :

$$\frac{|11.4192 - 10.474|}{\sqrt{2}} = 0.667 < z_{\alpha/2}$$

Significant level 1%

Not significant !

In this example, the significant difference of the SN ratios between A1and A2 is not shown.

Lecture – 4 - What is robust design?

Dr. Genichi Taguchi, a mechanical engineer, who has won four times Deming Awards, introduced the loss function concept, which combines cost, target, and variation into one metric. He developed the concept of robustness in design, which means that noise variables (or nuisance variables or variables which are uneconomical to control) are taken into account to ensure proper functioning of the system functions. He emphasized on developing design in presence of noise rather than eliminating noise.

Loss Function

Taguchi defined quality as a loss imparted to society from the time a product is shipped to customer. Societal losses include failure to meet customer requirements, failure to meet ideal performance, and its harmful side effects.

Assuming the target [tau (τ)] is correct, losses are those caused by a product's critical performance characteristics, if it deviats from the target. The importance of concentrating on "hitting the target" is shown by Sony TV sells example. In spite of the fact that the design and specifications were identical, U.S. customers preferred the color density of shipped TV sets produced by Sony-Japan over those produced by Sony-USA. Investigation of this situation revealed that the frequency distributions were markedly different, as shown in **Figure 3-13**. Even though Sony-Japan had 0.3% outside the specifications, the distribution was normal and centered on the target with minimum variability as compared to Sony-USA. The distribution of the Sony-USA was uniform between the specifications with no values outside specifications. It was clear that customers perceived quality as meeting the target (Sony-Japan) rather than just meeting the specifications (USA). Ford Motor also had a similar experience with their transmissions.

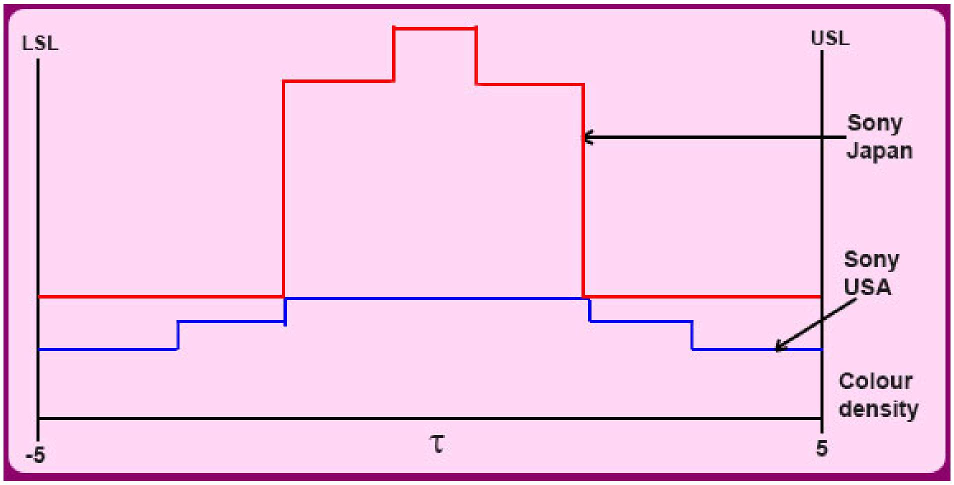


Figure 3-13 Distribution of color density for Sony-USA and Sony-Japan

Out of specification is the common measure of quality loss in Goal post mentality [**Figure 3-14 (a)**]. Although this concept may be appropriate for accounting, it is a poor concept for various other areas. It implies that all products that meet specifications are good, whereas those that do not are bad. From the customer's point of view, the product that barely meets the specification is as good (or bad) as the product that is barely just out-of-specification. Thus, it appears that wrong measuring system for quality loss is being used. The Taguchi's loss function [**Figure 3-14 (b)**] corrects for the deficiency described above by combining cost, target, and variation into one single metric.

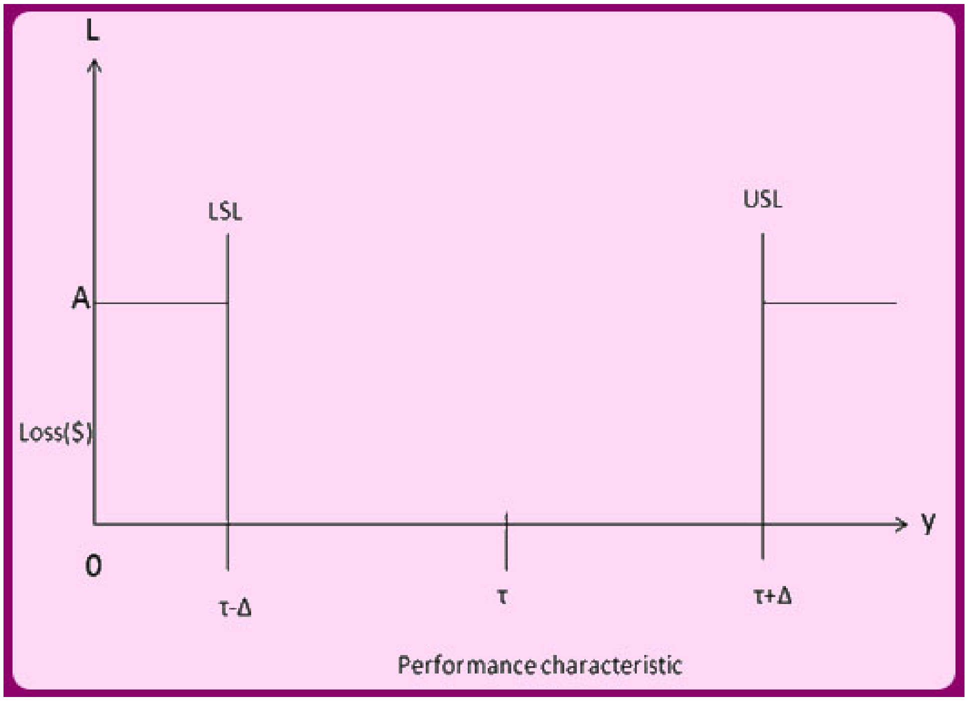


Figure 3-14(a): Discontinuos Loss Function (Goal Post Mentality)

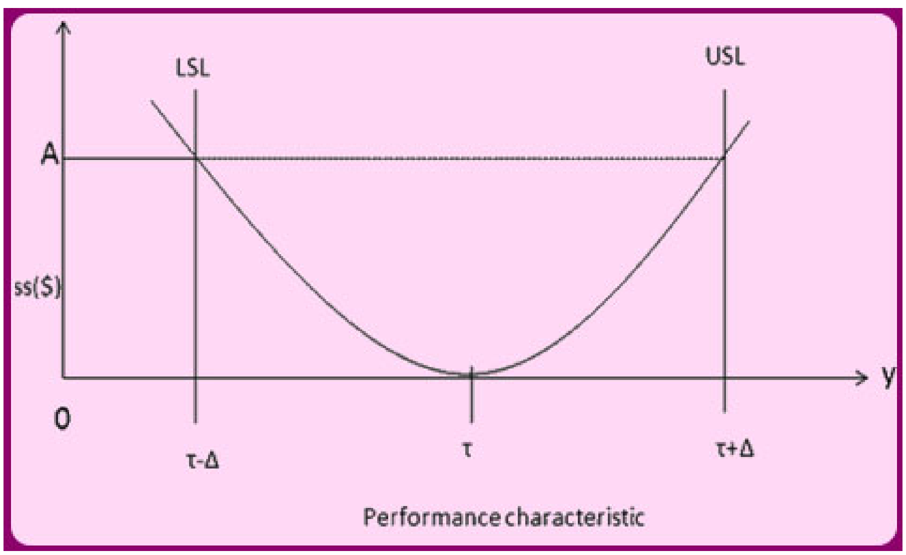


Figure 3-14(b): Continuous Quadratic Loss function (Taguchi Method)

Figure 3-14(a) shows the loss function that describes the Sony-USA situation as per 'Goal Post Mentality' considering NTB (Nominal-the-Best)-type of quality charecteristic. Few performance characteristics considered as NTB are color density, voltage, bore dimensions, surface finish. In NTB, a target (nominal dimension) is specified with a upper and lower specification, say diameter of a engine cylinder liner bore. Thus, when the value for the performance characteristic, y, is within specifications the quality loss is *$0*, and when it is outside the specifications the loss is *$A*. The quadratic loss function as shown in **Figure 3-14(b)** describes the Taguchi method of definining loss function. In this situation, loss occurs as soon as the performance characteristic, y, departs from the target, τ .

The quadratic loss function is described by the equation

$$L = k\left(y - \tau\right)^2,$$

Where L = cost incurred as quality deviates from the target (τ)

y is the performance characteristic, k = quality loss coefficient.

The loss coefficient is determined by setting

$$k = A / \left(y - \tau\right)^2 = A / \Delta^2$$

Assuming, the specifications (NTB) is10 ± 3 for a particular quality characteristic and the average repair cost is $230, the loss coefficient is calculated as,

$$k = A / \Delta^2 = 230 / 3^2 = 25.6$$

Thus, L = 25.6 (for y= 10) and at L=102.4 (for y = 12),

$$\begin{aligned} L &= 25.6\left(y - 10\right)^2 \\ &= 25.6\left(12 - 10\right)^2 \\ &= \$102.40 \end{aligned}$$

Average or Expected Loss

The loss described above assumes that the quality characteristic is static. In reality, one cannot always hit the target. It will vary due to presence of noise, and the loss function must reflect the variation of many pieces rather than just single piece. An equation can be derived by summing the individual loss values and dividing by their number to give

$$\bar{L} = k\left[\sigma^2 + \left(\bar{y} - \tau\right)^2\right]$$

Where $\bar{L}$ = the average or expected loss, σ is the process variability of y charecteristic, $\bar{y}$ is the average dimension coming out of the process.

Because the population standard deviation, σ , is unknown, the sample standard deviation, s, is used as a substituted. This action will make the variability value somewhat larger. However, the average loss (**Figure 3-15**) is quite conservative in nature.

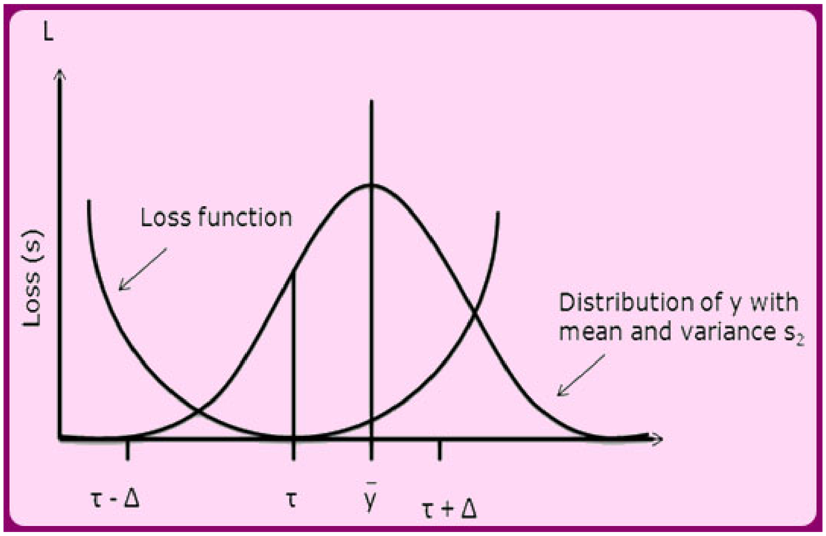


Figure 3-15 Average or Expected Loss

The loss can be lowered by reducing the variation, and adjusting the average, y, to bring it on target.

Let is compute the average loss for a process that produces shafts. The target value , say 6.40 mm and the loss coefficient is 9500. Eight samples give reading of 6.36, 6.40, 6.38, 6.39, 6.43, 6.39, 6.46, and 6.42. Thus,

s = 0.0315945 $\bar{y} = 6.40375$

$$\bar{L} = k\left[s^2 + (\bar{y} - \tau)^2\right]$$
$$= 9500\left[0.0315945^2 + (6.40375 - 6.40)^2\right]$$
$$= \$9.62$$

There are two other loss functions that are quite common, smaller-the-better and larger- the-better. In smaller-the-better type, the lesser the value is preferred for the characteristic of interest, say defect rate, expected cost, and engine oil consumption. **Figure 3-16** illustrates the concept.

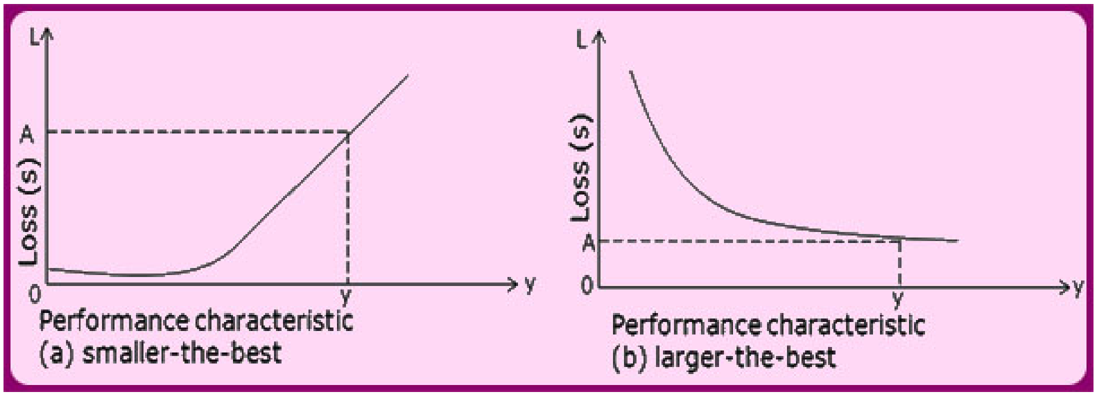


Figure 3-16 : (a) Smaller –the –Better and (b) Larger–the- Better-type of Loss Function

To summarize the equations for the three common loss functions,

Nominal the best

$L = k(y - \tau)^2$ Where $k = A / \Delta^2$

$\bar{L}$ =k (MSD) where MSD= $\left[\Sigma + (\bar{y} - \tau)^2\right] / n$

$\bar{L} = k\left[\sigma^2 + (\bar{y} - \tau)^2\right]$

Smaller the better

$L = ky^2$ where $k = A / y^2$

$\bar{L}$ =k(MSD) where MSD= $\left[\Sigma y^2\right] / n$

$\bar{L} = k\left[\bar{y}^2 + \sigma^2\right]$

Larger the better

$L = k(1 - y^2)$ where $k = Ay^2$

$\bar{L}$ =k(MSD) where MSD= $\left[\Sigma(1 / y^2)\right] / n$

$\bar{L} = k\left[\Sigma(1 / y^2)\right] / n$

In case of larget-the-better, higher value is preferred for the characteristic of interest. Few examples of performance characteristics considered as larget-the-better are bond strength of adhesives, welding strength, tensile strength, expected profit.

Orthogonal Arrays

Taguchis method emphasized on highly fractionated factorial design matrix or Orthogonal arrays (OA) [http://en.wikipedia.org/wiki/Orthogonal_array] for experiment. This arrays are developed by Sir R. A. Fischer and with the help of Prof C R Rao (http://en.wikipedia.org/wiki/C._R._Rao) of Indian Statistical Institute, Kolkata. A L8 orthogonal array is shown below. An orthogonal array is a type of experiment where the columns for the independent variables are "orthogonal" or "independent" to one another.

Table 3-4 L8 Orthogonal Array

Orthogonal Array OA8

TC	1	2	3	4	5	6	7
1	1	1	1	**1**	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	1	1	1	1	2
6	2	1	1	2	2	2	1
7	2	2	2	1	1	2	1
8	2	2	2	2	2	1	2

The 8 in the designation OA8 (**Table 3-4**) represents the number of experimental rows, which is also the number of treatment conditions (TC). Across the top of the orthogonal array is the maximum number of factors that can be assigned, which in this case is seven. The levels are designated by 1 and 2. If more levels occur in the array, then 3, 4, 5, and so forth, are used. Other schemes such as -1, 0, and +1 can be used. The orthogonal property of an OA is not compromised by changing the rows or the columns. Orthogonal arrays can also handle dummy factors and can be accordingly modified. With the help of OA the number of trial or experiments can be drastically reduced.

To determine the appropriate orthogonal array, we can use the following procedure,

Step-1 Define the number of factors and their levels.

Step-2 Determine the degrees of freedom.

Step-3 Select an orthogonal array.

Step-4 Consider any interactions (if required).

To understand required df, we assuming four two-level (leveled as 1 and leveled as 2) factors, A, B, C, D, and two suspected interactions, BC and CD, determine the degrees of freedom, df.
At least seven treatment conditions are needed for the two-level,

$$df=4(2-1)+2(2-1)(2-1)+1=7$$

Selecting the Orthogonal Array

Once the degrees of freedom are known, factor levels are identified, and possible interaction to be studied, the next step is to select the orthogonal array (OA). The number of treatment conditions is equal to the number of rows in the OA and must be equal to or greater than the degrees of freedom. **Table 3-5** shows the orthogonal arrays that are available, up to OA36. Thus, if the number of degrees of freedom is 13, then the next available OA is OA16. The second column of the table has the number of rows and is redundant with the designation in the first column. The third column gives the maximum number of factors that can be used, and the last four columns give the maximum number of columns available at each level.

Table 3-5 Required Orthogonal Array

Orthogonal Array Information

OA	Number of rows	Maximum number of factors	Maximum number of columns			
			2-level	3-level	4-level	5-level
OA2	4	3	3	-	-	-
OA8	8	7	7	-	-	-
OA9	9	4	-	4	-	-
OA12	12	11	11	-	-	-
OA16	16	15	15	-	-	-
OA16	16	5	-	-	5	-
OA18	18	8	1	7	-	-
OA25	25	6	-	-	-	6
OA27	27	13	-	13	-	-
OA32	32	31	31	-	-	-
OA32	32	10	1	-	9	-
OA36	36	23	11	12	-	-
OA36	36	16	3	13	-	-
. . .	. . .	. . .	. . .	. . .	. . .	. . .

Analysis of the table shows that there is a geometric progression for the two-level arrays of OA4, OA8, OAI6, OA32, ... , which is 2^2, 2^3, 2^4, 2^5, ... For the three-level arrays of OA9, OA27, OA8I, ... , it is 3^2, 3^3, 3^4, ... Orthogonal arrays can also be modified.

Interaction Table

Confounding is the inability to distinguish among the effects of one factor from another factor and/or interaction. In order to prevent confounding, one must know which columns to use for the factors in Taguchi method. This knowledge is provided by an interaction table, which is shown in **Table 3-6**.

Table 3-6 Interaction Table for OA8

Interaction Table for OA8

Column	1	2	3	4	5	6	7
1	(1)	3	2	5	4	7	6
2		(2)	1	6	7	4	5
3			(3)	7	6	5	4
4				(4)	1	2	3
5					(5)	3	2
6						(6)	1
7							(7)

Let's assume that factor A is assigned to column 1 and factor B to column 2. If there is an interaction between factors A and B, then column 3 is used for the interaction, AB. Another factor, say, C, would need to be assigned to column 4. If there is an interaction between factor A (column 1) and factor C (column 4), then interaction AC will occur in column 5. The columns that are reserved for interactions are used so that calculations can be made to determine whether there is a strong interaction. If there are no interactions, then all the columns can be used for factors. The actual experiment is conducted using the columns designated for the factors, and these columns are referred to as the design matrix. All the columns are referred to as the design space.

Linear Graphs

Taguchi developed a simpler method to work with interactions by using linear graphs.

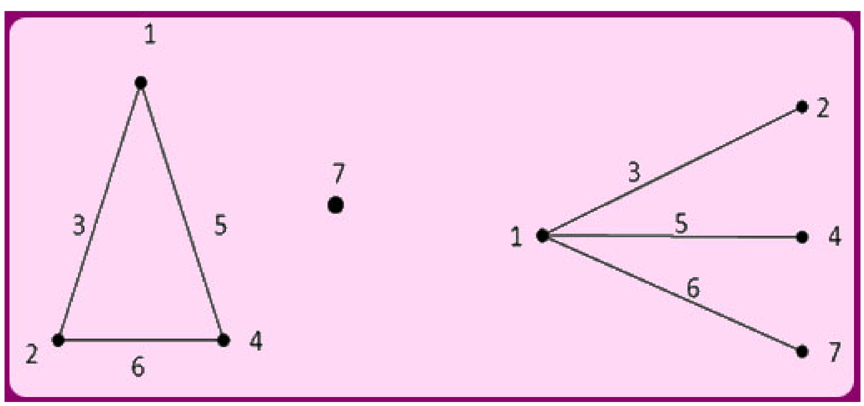


Figure 3-17 Two linear Graphs for OA8

Two linear graph are shown in **Figure 3-17** for OA8. They make it easier to assign factors and interactions to the various columns of an array. Factors are assigned to the points. If there is an interaction between two factors, then it is assigned to the line segment between the two points.

For example, using the linear graph on the left in the figure, if factor B is assigned to column 2 and factor C is assigned to column 4, and then interaction BC is assigned to column 6. If there is no interaction, then column 6 can be used for a factor.

The linear graph on the right can be used when one factor has three two-level or higher order interactions. Three-level orthogonal arrays must use two columns for interactions, because one column is for the linear interaction and one column is for the quadratic interaction. The linear graphs-and, for that matter, the interaction tables-are not designed for three or more factor interactions, which are rare events. Linear graphs can also be modified. Use of the linear graphs requires some trial-and-error activity

Interactions

Interactions simply means relationship existing between different X-factors/X with noise variables considered for experiment. **Figure 3-18** shows graphical relationship between any two factors. At (a) there is no interaction as the lines are parallel; at (b) there is little interaction existing between the factors; and at (c) there is a strong evidence of interaction. The graph is constructed by plotting the points A_1B_1 A_2B_2, A_2B_1 and A_2B_2.

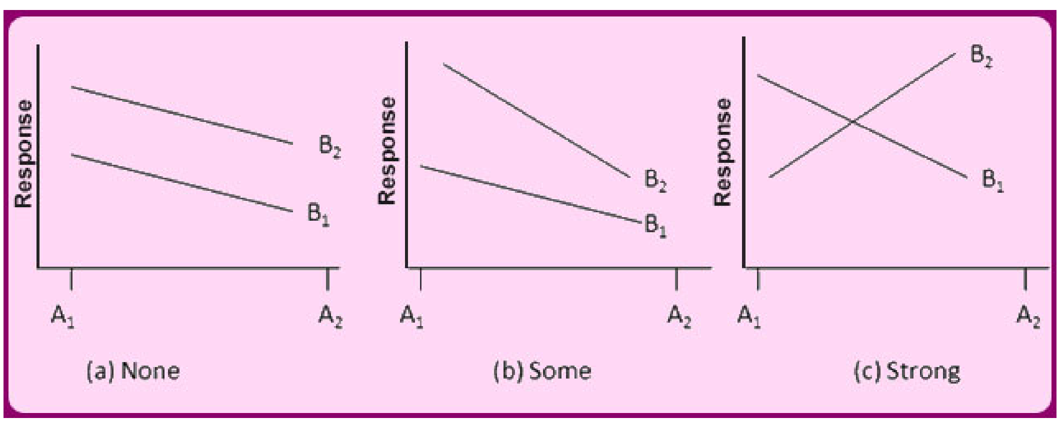


Figure 3-18 Interaction between Two Factors

Signal-to-Noise (SIN) Ratio

The important contribution of Taguchi is proposing the signal-to-noise (S/N) ratio. It was developed as a proactive equivalent to the reactive loss function. When a person puts his/her foot on the brake pedal of a car, energy is transformed with the intent to slow the car, which is the signal. However, some of the energy is wasted by squeal, pad wear, and heat. **Figure 3-19** emphasizes that energy is neither created nor destroyed.

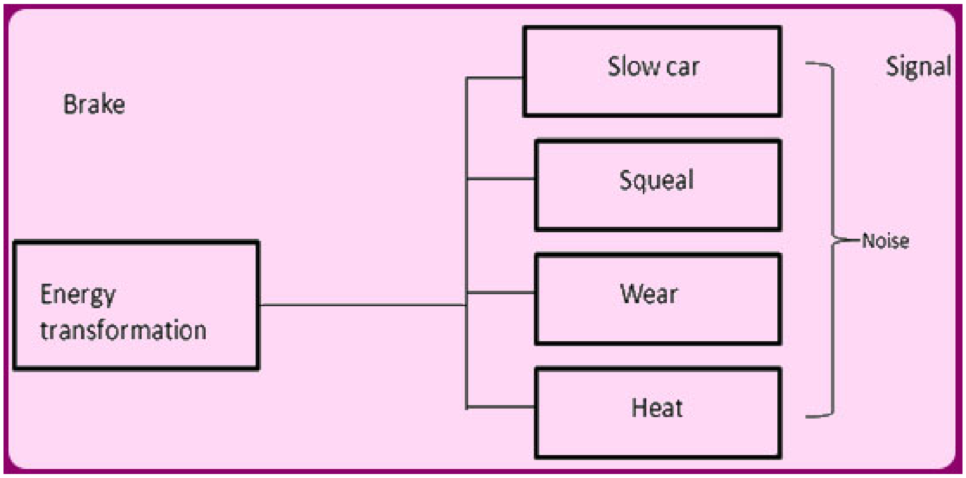


Figure 3-19 Concept of Signal-to-Noise (S/N) Ratio

Signal factors (Y) are set by the designer or operator to obtain the intended value of the response variable. Noise factors (S_2) are not controlled or are very expensive or difficult to control. Both the average, y, and the variance, $s^{2,}$ need to be controlled with it single figure of merit. In elementary form, S/N is $\bar{y} / s$, which is the inverse of the coefficient of variation and a unitless value. Squaring and taking the log transformation gives

$$S / N_N = \log_{10}\left(\bar{y}^2 - s^2\right)$$

Adjusting for small sample sizes and changing from Bels to decibels for NTB type gives

$$S / N_N = 10 \log_{10}\left[\left(\bar{y}^2 - s^2\right) - (1 / n)\right]$$

There are many different S/N ratios. The equation for nominal-the-best was given above. It is used wherever there is a nominal or target value and a variation about that value, such as dimensions, voltage, weight, and so forth. The target (τ) is finite but not zero. For robust (optimal) design, the S/N ratio should be maximized. The-nominal-the-best S/N value is a maximum when the average is near target and the variance is small. Taguchi's two-step optimization approach is to identify factors (X) which reduces variation of Y, and then bring the average (Y) on target by a different set of factor (X). The he S/N ratio for a process that has a temperature average of 21°C and a sample standard deviation of 2°C for four observations is given by

$$
\begin{aligned}
S/N_N &= 10\log_{10}\left[\left(\bar{y}^2 - s^2\right) - \left(1/n\right)\right] \\
&= 10\log_{10}\left[\left(21^2 - 2^2\right) - \left(1/4\right)\right] \\
&= 20.41 \text{ dB}
\end{aligned}
$$

The adjustment for the small sample size has little effect on the answer. If it had not been used, the answer would have been 20.42 dB.

Smaller-the-Better

The S/Ns ratio for smaller-the-better is used for situations where the target value (τ) is zero, such as computer response time, automotive emissions, or corrosion. The S/N equation used is

$$S/N_S = -10\log_{10}\left[MSD\right] = -10\log_{10}\left[\left(\Sigma y^2\right)/n\right]$$

The negative sign ensures that the largest S/N value gives the optimum value for the response variable and, thus a robust design. Mean square deviation (MSD) is given to show the relationship with the loss function.

Larger-the-Better

The S/N ratio for larger-the-better type of characteristic is given by

$$S/N_L = -10\log_{10}\left[MSD\right] = -10\log_{10}\left[\Sigma\left(1/y^2\right)/n\right]$$

Let us consider a battery life experiment. For the existing design, the lives of three AA batteries are calculated as 20, 22, and 21 hours. A different design produces batteries life of 17, 21, and 25 hours. To understand which is a better design (E or D) and by how much, we can use the S/N ratio calculation. As it is a larger-the-better (LTB) type of charectersitic (Response), the calculation are

$$S/N_E = -10\log_{10}\left[\Sigma\left(1/y^2\right)/n\right]$$
$$= -10\log_{10}\left[\left(\frac{1}{20^2}+\frac{1}{22^2}+\frac{1}{25^2}+\right)/3\right]$$
$$= 26.42 \text{ dB}$$

$$S/N_D = -10\log_{10}\left[\Sigma\left(1/y^2\right)/n\right]$$
$$= -10\log_{10}\left[\left(\frac{1}{17^2}+\frac{1}{21^2}+\frac{1}{25^2}+\right)/3\right]$$
$$= 26.12 \text{ dB}$$

$$\Delta = \left|26.42 - 26.12\right| = 0.3 \; db$$

The different design is 7% better than existing design. More data will be required to confirm the result and so-called ‘Confirmatory trials’.

Although the metric signal-to-noise ratio have achieved good practical results, they are yet to be accepted universaly as a valid statistical measure. The controversy is on meaures and shape of loss function. However, Taguchi’s concept has resulted in a paradigm shift in the concept of product quality and can optimize without any empirical regression modelling concept.

It is also to be noted that inner (controllable factors) and outer array (for noise variable) design is recommended by Taguchi to understand the best setting for Robust Design, which many a times researchers omit for easy of experimentation. This practice may be avoided. Engineering knowledge and idea of interaction is essential to get the best benefit out of OA design. For further details on Taguchi method, reader may refer the books written by P J Ross (1996), A Mitra (2008),Besterfield et at. (2004) and M Phatke (1995).

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Application Of Taguchi Method For Optimization Of Process Parameters In Improving The Surface Roughness Of Lathe Facing Operation

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ABSTRACT: *Taguchi Method is a statistical approach to optimize the process parameters and improve the quality of components that are manufactured. The objective of this study is to illustrate the procedure adopted in using Taguchi Method to a lathe facing operation. The orthogonal array, signal-to-noise ratio, and the analysis of variance are employed to study the performance characteristics on facing operation. In this analysis, three factors namely speed; feed and depth of cut were considered. Accordingly, a suitable orthogonal array was selected and experiments were conducted. After conducting the experiments the surface roughness was measured and Signal to Noise ratio was calculated. With the help of graphs, optimum parameter values were obtained and the confirmation experiments were carried out. These results were compared with the results of full factorial method.*

Keywords: *Annova, Design of Experiments, Facing Operation, Orthogonal Array, S/N Ratio, Taguchi Method.*

I. Introduction

Taguchi method is a statistical method developed by Taguchi and Konishi [1]. Initially it was developed for improving the quality of goods manufactured (manufacturing process development), later its application was expanded to many other fields in Engineering, such as Biotechnology [2] etc. Professional statisticians have acknowledged Taguchi's efforts especially in the development of designs for studying variation. Success in achieving the desired results involves a careful selection of process parameters and bifurcating them into control and noise factors. Selection of control factors must be made such that it nullifies the effect of noise factors. Taguchi Method involves identification of proper control factors to obtain the optimum results of the process. Orthogonal Arrays (OA) are used to conduct a set of experiments. Results of these experiments are used to analyze the data and predict the quality of components produced.

Here, an attempt has been made to demonstrate the application of Taguchi's Method to improve the surface finish characteristics of faced components that were processed on a lathe machine. Surface roughness is a measure of the smoothness of a products surface and it is a factor that has a high influence on the manufacturing cost. Surface finish also affects the life of any product and hence it is desirable to obtain higher grades of surface finish at minimum cost.

II. Approach to Product/Process Development

Many methods have been developed and implemented over the years to optimize the manufacturing processes. Some of the widely used approaches are as given below:

1.1 Build-Test-Fix

The "Build-test-fix" is the most primitive approach which is rather inaccurate as the process is carried out according to the resources available, instead of trying to optimize it. In this method the process/product is tested and reworked each time till the results are acceptable.

1.2 One Factor at a Time

The "one-factor-at-a-time" approach is aimed at optimizing the process by running an experiment at one particular condition and repeating the experiment by changing any other one factor till the effect of all factors are recorded and analyzed. Evidently, it is a very time consuming and expensive approach. In this process, interactions between factors are not taken in to account.

1.3 Design of Experiments

The Design of Experiments is considered as one of the most comprehensive approach in product/process developments. It is a statistical approach that attempts to provide a predictive knowledge of a complex, multi-variable process with few trials. Following are the major approaches to DOE:

1.3.1 Full Factorial Design

A full factorial experiment is an experiment whose design consists of two or more factors, each with a discrete possible level and whose experimental units take all possible combinations of all those levels across all such factors. Such an experiment allows studying the effect of each factor on the response variable, as well as on the effects of interactions between factors on the response variable. A common experimental design is the one with all input factors set at two levels each. If there are k factors each at 2 levels; a full factorial design has 2^k runs. Thus for 6 factors at two levels it would take 64 trial runs.

1.3.2 Taguchi Method

The Full Factorial Design requires a large number of experiments to be carried out as stated above. It becomes laborious and complex, if the number of factors increase. To overcome this problem Taguchi suggested a specially designed method called the use of orthogonal array to study the entire parameter space with lesser number of experiments to be conducted. Taguchi thus, recommends the use of the loss function to measure the performance characteristics that are deviating from the desired target value. The value of this loss function is further transformed into signal-to-noise (S/N) ratio. Usually, there are three categories of the performance characteristics to analyze the S/N ratio. They are: nominal-the-best, larger-the-better, and smaller-the-better.

III. Steps Involved in Taguchi Method

The use of Taguchi's parameter design involves the following steps [3].

a. Identify the main function and its side effects.
b. Identify the noise factors, testing condition and quality characteristics.
c. Identify the objective function to be optimized.
d. Identify the control factors and their levels.
e. Select a suitable Orthogonal Array and construct the Matrix
f. Conduct the Matrix experiment.
g. Examine the data; predict the optimum control factor levels and its performance.
h. Conduct the verification experiment.

IV. Approach to the Experimental Design

In accordance with the steps that are involved in Taguchi's Method, a series of experiments are to be conducted. Here, facing operation on mild steel components using a lathe has been carried out as a case study. The procedure is given below.

4.1 Identification of Main Function and its side effects

Main function: Facing Operation on MS work piece using lathe machine.
Side effects : Variation in surface finish.

Before proceeding on to further steps, it is necessary to list down all the factors that are going to affect or influence the facing process and from those factors one has to identify the control and noise factors. The "Factors" that affect facing operation on a lathe machine are listed in the table 4.1.

Table 4.1: Factors that affect facing operation

Control factors	Noise Factors
Cutting speed	Vibration
Depth of cut	Raw material variation
Feed rate	Machine Condition
Nose radius	Temperature
Coolant	Operator Skill

After listing the control and the noise factors, decisions on the factors that significantly affect the performance will have to be ascertained and only those factors must be taken in to consideration in constructing the matrix for experimentation. All other factors are considered as Noise Factors.

4.2 Identifying the Testing Conditions and Quality Characteristics To Be Observed

Quality Characteristic: Surface finish
Work piece material: Mild Steel

Cutting tool: Tungsten: Carbide Tipped tool
Operating Machine: Lathe machine
Testing Equipment: Portable surface tester

4.3 Identify The Objective Function

Objective Function: Smaller-the-Better

S/N Ratio for this function: $\eta = -10 \log_{10}\left(\frac{1}{n}\sum_{i=1}^{n} y^2{}_i\right)$ [4]

Where, n= Sample Size, and y= Surface Roughness in that run.

4.4 Identifying the Control Factors and their levels

The factors and their levels were decided for conducting the experiment, based on a "brain storming session" that was held with a group of people and also considering the guide lines given in the operator's manual provided by the manufacturer of the lathe machine. The factors and their levels are shown in table 4.2.

Table 4.2 Selected Factors and their Levels.

FACTORS	LEVELS		
	1	**2**	**3**
Cutting speed(v, rpm)	960	640	1280
Depth of cut(t, mm)	0.3	0.2	0.4
Feed rate(f, mm/min)	145	130	160

4.5 Selection of Orthogonal Array

To select an appropriate orthogonal array for conducting the experiments, the degrees of freedom are to be computed. The same is given below:

Degrees of Freedom: 1 for Mean Value, and
8= (2x4), two each for the remaining factors

Total Degrees of Freedom: 9

The most suitable orthogonal array for experimentation is L9 array as shown in Table 4.3[5]. Therefore, a total nine experiments are to be carried out.

Table 4.3 Orthogonal Array (OA) L9

Experiment No.	Control Factors		
	1	**2**	**3**
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	3
5	2	2	1
6	2	3	2
7	3	1	2
8	3	2	3
9	3	3	1

4.6 Conducting The Matrix Experiment

In accordance with the above OA, experiments were conducted with their factors and their levels as mentioned in table 4.2. The experimental layout with the selected values of the factors is shown in Table 4.4. Each of the above 9 experiments were conducted 5 times (45 experiments in all) to account for the variations that may occur due to the noise factors. The surface roughness (Ra) was measured using the surface roughness tester. The table 4.5 shows the measured values of surface roughness obtained from different experiments.

Table 4.4 OA with Control Factors

Experiment No.	Control Factors		
	V(rpm)	**t(mm)**	**F(mm/min)**
1	960	0.3	145
2	960	0.2	130
3	960	0.4	160
4	640	0.3	160

5	640	0.2	145
6	640	0.4	130
7	1280	0.3	130
8	1280	0.2	160
9	1280	0.4	145

Table 4.5 Measured values of surface roughness

Experiment No.	Surface Roughness (R_a, □m)					
	1	**2**	**3**	**4**	**5**	**Mean**
1	2.35	2.43	1.94	2.91	2.77	2.48
2	2.5	3.6	2.66	2.98	2.64	2.876
3	2.43	2.82	4.01	2.96	4.1	3.264
4	2.24	3.38	2.45	4.05	4.79	3.382
5	2.54	3.67	2.70	4.25	4.37	3.506
6	4.76	4.25	3.19	3.36	4.35	3.982
7	2.04	2.49	3.84	1.71	3.79	2.834
8	4.4	2.5	3.15	3.24	3.1	3.278
9	3.94	2.19	2.31	2.44	3.30	3.306

4.7 Examination of Data

The following are the experimental results of the work carried out.

4.7.1 Experimental Details

Since the objective function (Surface Finish) is smaller-the-better type of control function, was used in calculating the S/N ratio. The S/N ratios of all the experiments were calculated and tabulated as shown in Table 4.6.

Table 4.6 Tabulated S/N ratios

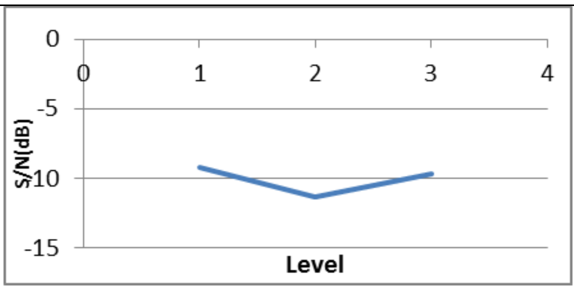
Experiment No.	S/N Ratio (dB)
1	-7.9702
2	-9.2568
3	-10.4539
4	-10.9196
5	-11.0971
6	-12.1010
7	-9.2385
8	-10.4642
9	-9.2941

The S/N ratio for the individual control factors are calculated as given below:
$S_{s1}=(\eta_1+\eta_2+\eta_3)$, $S_{s2}=(\eta_4+\eta_5+\eta_6)$ & $S_{s3}=(\eta_7+\eta_8+\eta_9)$
$S_{f1}=(\eta_1+\eta_4+\eta_7)$, $S_{f2}=(\eta_2+\eta_5+\eta_8)$ & $S_{f3}=(\eta_3+\eta_6+\eta_9)$
$S_{t1}=(\eta_1+\eta_5+\eta_9)$, $S_{t2}=(\eta_2+\eta_6+\eta_7)$ & $S_{t3}=(\eta_3+\eta_4+\eta_8)$
For selecting the values of η_1, η_2, η_3 etc. and to calculate S_{s1}, S_{s2} & S_{s3} see table 4.3.
η_k is the S/N ratio corresponding to Experiment k.
Average S/N ratio corresponding to Cutting Speed at level 1 = $S_{s1}/3$
Average S/N ratio corresponding to Cutting Speed at level 2 = $S_{s2}/3$
Average S/N ratio corresponding to Cutting Speed at level 3 = $S_{s3}/3$
j is the corresponding level each factor. Similarly S_{fj} and S_{tj} are calculated for feed and depth of cut.
The average of the signal to noise ratios is shown in table 4.7.
Similarly S/N ratios can be calculated for other factors.

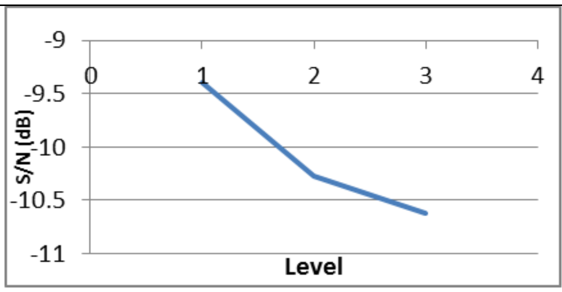
Table 4.7: Average S/N Ratios for each factor

Level	Speed		Feed		Depth of Cut	
	Sum (S_{sj})	**Avg S/N ratio**	**Sum (S_{fj})**	**Avg S/N ratio**	**Sum (S_{tj})**	**Avg S/N ratio**

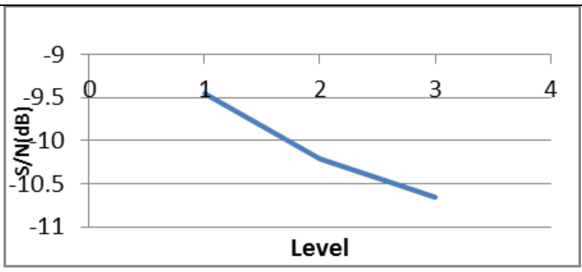
1	-27.6809	-9.2268	-28.3614	-9.3909	-28.1283	-9.45
2	-34.1177	-11.3722	-30.5963	-10.273	-30.8181	-10.21
3	-28.9968	-9.68	-31.8377	-10.616	-31.849	-10.65



(a). Speed



(b). Feed Rate



(c). Depth of Cut

For calculating the Surface Roughness the objective function, "smaller-the-better" type was used as shown.

$$\eta = -10\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n} y^2_i\right)$$

The factor levels corresponding to the highest S/N ratio were chosen to optimize the condition. From these linear graphs it is clear that the optimum values of the factors and their levels are as given in table 4.8.

Fig-1 Charts Showing Parameter Level v/s S/N Ratio

Table 4.8 Optimum values of factors and their levels

Parameter	**Optimum Value**
Speed (rpm)	960
Feed Rate (mm/min)	145
Depth of cut (mm)	0.3

4.7.2. Full Factorial Analysis

A full factorial analysis consists of conducting experiments taking into account all the possible combinations of the factors and their levels. As far as the following experiments are concerned the 3 factors i.e.; speed, feed, and depth of cut were considered at 3 different levels as shown in Table 4.9. These were compared with the results of the fractional factorial that was conducted using Taguchi method.

Table 4.9 Full Factorial Experiment Matrix

Experiment No.	**Parameters**			**Mean Surface Roughness, R_a(□m)**
	Speed(rpm)	**Depth of cut(mm)**	**Feed (mm/min)**	
1	960	0.3	145	2.48
2	960	0.2	145	3.26
3	960	0.4	145	2.57
4	960	0.3	130	2.62
5	960	0.2	130	2.876
6	960	0.4	130	2.87
7	960	0.3	160	2.74
8	960	0.2	160	4.35
9	960	0.4	160	3.264
10	640	0.3	145	3.82
11	640	0.2	145	3.506

12	640	0.4	145	3.41
13	640	0.3	130	2.96
14	640	0.2	130	3.45
15	640	0.4	130	3.982
16	640	0.3	160	3.382
17	640	0.2	160	5.04
18	640	0.4	160	4.25
19	1280	0.3	145	4.02
20	1280	0.2	145	4.03
21	1280	0.4	145	3.306
22	1280	0.3	130	2.834
23	1280	0.2	130	4.14
24	1280	0.4	130	3.3
25	1280	0.3	160	2.6
26	1280	0.2	160	3.278
27	1280	0.4	160	2.76

4.7.3 Comparison of full factorial analysis with Taguchi parameter design:

It is evident from the results of the full factorial analysis shown in Table 4.9, the best surface finish characteristics obtained were at 960 rpm, 0.3 mm depth of cut and 145 mm/min feed rate. From Taguchi parameter design the optimum parameter levels obtained were also the same (see Table 4.8). Thus, it can be noted that Taguchi parameter design will also give accurate results with lesser number of experiments to be performed.

4.8 Confirmation Experiment

The following table 4.10 shows confirmation experiments conducted using 960 rpm, 0.3 mm depth of cut and 145 mm/min feed rate. Total five sets of experiments were conducted and their surface roughness values were checked. It can be seen that the results are consistent.

Table 4.10 Confirmation Experiment

Experiment No.	**Surface Roughness, R_a (µm)**
1	2.43
2	2.23
3	2.86
4	2.51
5	2.21
Mean	2.448

V. ANNOVA AND ITS SIGNIFICANCE

Analysis of variance (ANOVA) is used to evaluate the response magnitude in (%) of each parameter in the orthogonal experiment. It is used to identify and quantify the sources of different trial results from different trial runs (i.e. different cutting parameters). The basic property of ANOVA is that the total sums of the squares (total variation) is equal to the sum of the SS (sums of the squares of the deviations) of all the condition parameters and the error components, i.e., adding the variations of each factors,

$$SS_T = SS_S + SS_f + SS_t + SS_e \qquad \text{(Eqn. 5.1)}$$

$$SS_T = \sum_1^n y_i^2 - \frac{G^2}{n} \qquad \text{(Eqn. 5.2)}$$

Where, G = *is the sum of the resulting data of all trial runs;* and *n* is the total number of the trial runs .

$$SS_k = \sum_{j=1}^{t} \left(\frac{Sy_j^2}{t}\right) - \frac{G^2}{n} \qquad \text{(Eqn. 5.3)}$$

Where k represents one of the tested parameters; j is level number of this parameter; Sy_j is sum of all trial results involving this parameter *k* at level j; *n* is the total number of trial runs. The following table 5.1 shows the results of the ANOVA.

Table 5.1 Sum of all squares of all deviations

Parameter	**DOF**	**SS**	**SS%**
Speed, S	2	7.7329	58.49
Feed, f	2	2.0689	15.65
Depth of cut, t	2	2.4541	18.56
Noise, e	2	0.9655	7.3
Total	8	13.2214	100

It can be seen from this table that for the surface finish (Ra), the contribution of cutting speed (58.49%) is more significant than depth of cut which is (18.56%). These factors are more significant than the feed rate (15.65%). It is clear that the effect of noise factor (7.3%) on surface finish is very low as compared to the control factors.

VI. CONCLUSION

This paper illustrates the application of the parameter design (Taguchi method) in the optimization of facing operation. The following conclusions can be drawn based on the above experimental results of this study:

- Taguchi's Method of parameter design can be performed with lesser number of experimentations as compared to that of full factorial analysis and yields similar results.
- Taguchi's method can be applied for analyzing any other kind of problems as described in this paper.
- It is found that the parameter design of the Taguchi method provides a simple, systematic, and efficient methodology for optimizing the process parameters.

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